Quantitative Measurement of Supersonic Flow Field around an Asymmetric Cone Model with CGBOS Method and 3D-CT Reconstruction

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Abstract
In this report the novel and quantitative measurement technique of the image distortion related to the density gradient around an asymmetric cone model using colored-grid background oriented schlieren (CGBOS) method is presented. The CGBOS method enables us to obtain two-dimensional quantitative integrated distribution data of refractive index gradient around a test model in supersonic flow by calculating the distortion of background image. We try to reconstruct the three-dimensional density information from CGBOS technique and Computed Tomography technique.

Key words
Background Oriented Schlieren, Colored Grid, Shock Wave, Computed Tomography, Algebraic Reconstruction Technique, Supersonic Flow

1. Introduction
Today, knowing the flow field around the aerospace vehicles by quantitative measurement technique is very important, because of the rapid development of the high-speed airplane and rocket. Under the high expectation of aerospace transportation system, the shock and expansion waves, and their interaction with boundary layer around the aircraft have become especially important. Our objective is to measure and visualize quantitatively such supersonic flow fields around a vehicle model to clarify these problems.

In this report, the quantitative measurement technique using colored-grid background oriented schlieren (CGBOS) method is proposed and discussed. The BOS method is one of the novel and advanced techniques that was introduced by G. E. A. Meier [1]. The BOS method utilizes the same physical background as conventional schlieren visualization method, as it applies the deviation of the light pass through the density gradient of the flowing medium by the variation of refractive index. The CGBOS technique needs only a light source, a colored-grid background image, a digital camera and a computer. If there is some density change around the model, the light path refracts and the image taken by the camera would be distorted.

In this investigation the test model is an asymmetric cone. The lower side of the model is cut to an inclined plane surface. The experiment of the flow field around the asymmetric cone model was performed at Mach number M=2.0 of the blow-down-type supersonic wind tunnel of JAXA-ISAS. The asymmetric cone model was rotated from 0 to 90 degrees by every 5 degrees, and we succeeded to take CGBOS images for each rotation angles θ. In addition, we have tried to obtain the quantitative density gradient information by using analysis of the background image from CGBOS images and the CT reconstruction technique.

2. Optical Measurement Method

2.1 Background oriented schlieren (BOS)
Background Oriented Schlieren (BOS) technique is one of the quantitative measurement techniques developed from Schlieren method. We can observe and even obtain the quantitative data of flow field with shock wave by the BOS technique.

The basic idea of the BOS technique is shown in Fig.1. To use the BOS technique, we only need a digital camera, a light source, and a background image. The principle of BOS is also simple. If there is density change in the observation area between the digital camera and the background, by clicking shutter, we obtain the picture which shows the distorted image of the background. It is because that the light refracts when it passes through the density change and the refracted light is recorded in the picture. Analyzing how the picture was distorted, we can obtain the information about density change [1, 2].

The relationship between the integration of refractive index gradient and distortion of background image is given by Eq. (1), where \( L_b \) is the distance between test model and background image, \( L_c \) is the distance between test model and digital camera, \( f \) is the focal length of digital camera, \( n \) is the refractive index, \( ε \) is the deflection angle, and \( Δh \) is the distortion of background on the image of digital camera. \( J \) is the integrated refractive index gradient. The relationship between the refractive index and density is given by Eq. (2) where \( ρ \) is the density and \( G \) is Gladstone-dale constant.

The top view of BOS optical setup in the experiment is shown in Fig.2, and it consists of a digital camera (EOS Kiss Digital X, Canon), a background image, metal halide lamps. A steel asymmetric model was installed in the supersonic wind tunnel and its test section is 600mm×600mm square. The \( L_b \) is set to 0.66m, \( L_c \) is 5.16m and \( f \) is 439mm.

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2.2 Conventional schlieren method

Schlieren method is the conventional flow visualization technique which enables us to obtain the optically expressed image of density gradient integration qualitatively. The light source is set to focus at the point of the first concave mirror to obtain the parallel light at the test section. Therefore the light can be assumed to be the parallel light to the optical axis. The parallel light passing through the test section is set to focus at the second concave mirror. The color filter is set on the focal point. If there are density gradients in the test section, an optical toned image can be observed. In supersonic visualization experiment we also used the conventional schlieren setup originally settled the supersonic wind tunnel of the Institute of Space and Astronautical Science of the Japan Aerospace Exploration Agency (JAXA-ISAS). The schlieren optical setup is shown in Fig.3.

3. Measurement

3.1 CGBOS background image and wind tunnel

The BOS technique is usually used with dot pattern images as background, and PIV (Particle Image Velocimetry) technique is applied to analysis [1, 2]. Apart from these reports, we propose to use colored-grid BOS (CGBOS) technique with a colored-grid pattern as a background. We calculate the displacement of the grid line distortion and obtain the integrated information about refractive index gradient change [3]. Each line’s width is 0.56mm for vertical red lines, 0.56mm for horizontal green lines and they are printed at 0.84mm intervals.

3.2 Test model (asymmetric cone)

The test model is an asymmetric cone model made of steel which is shown in Fig.4. The model has cone nose with semi angle of 20°, 82mm length and 60mm maximum diameter. The cone nose is set to the cylindrical rod and total length of the model is 292mm. The lower part of the cone is cut to a plane surface of 14° inclination as shown in Fig.4. The definition of the rotation angle is shown in Fig.5. We define $\theta=0$ degree where cut surface of the cone edge is horizontal to the ground. We rotated the model from $\theta=0$ degree to 90 degrees with 5 degrees interval in the experiment.

4. Result for BOS Displacement of Grid

4.1 Schlieren method

The visualized image using conventional color schlieren method in our supersonic experiment is shown in Fig.6. The three colors filter in which colors are blue, green and red is used in the experiment to recognize where the light refraction occur. Where the light refract upper side is colored in blue, and down side is colored in red. As visualized result, a bow shock and expansion waves from the model are clearly observed. In the rotation angle $\theta=0$ degrees, the flat section of the model was set up to upper side direction, and consequently expansion waves are asymmetric.
4.2 CGBOS method
After trying our CGBOS experiment to the flow, we obtained the distorted image as in Fig.7. The colored-grid of our experiment is very convenient for the separation of horizontal and vertical strip patterns when the digital color separation technique is applied. Image of Fig.8 is extracted only green horizontal stripes. Zoom-up images around the top of the model tip from CGBOS test image and extracted image are shown in Fig.9 (a) and (b).

The calculated gray-scale CGBOS quantitative image is also shown in Fig.10. This image represents displacement of background image corresponding to the integrated density gradient \( I \). The distribution of \( I \) is indicated in 8-bit gray scale and gray level is corresponding the color bar indicated in left upper side. We can recognize a bow shock and expansion waves clearly. Using CGBOS method and image processing, not only the 2D measurements of flow as same as the conventional schlieren images, but also on the quantitative integration can be obtained.

4.3 Three-dimensional CT Reconstruction
In the supersonic wind tunnel experiment, we have obtained 19 BOS images from 0 to 90 degrees with rotation of the test model at interval 5 degrees. After analyzing, three-dimensional density information was obtained quantitatively. The 19 BOS images have changed into the calculated density gradient information by CT reconstruction. Three-dimensional density information have reconstructed by Algebraic Reconstruction Technique (ART). ART is one of the CT techniques that uses algebraic iteration approximation. The ART is useful for reconstructing from small number of projections [4]. The basic equation of ART iteration is shown in Eq. (3). After CT reconstruction, distributed 3-D data of density gradient \((\partial \rho / \partial r)\) is obtained.

\[
f^{m+1}(x,y) = f^m(x,y) + \frac{P(X,\theta) - R^m(X,\theta)}{C(X,\theta)}
\]  

(3)

Some cross-sectional figures of reconstructed density gradient are shown in Fig.11. In this figure (a) is the cross sections of y-z plane, (b) is x-y cross section and (c), (d) are x-z cross sections. We can recognize the shock wave and model section from (a) and this image shows the distribution of density gradient around the model. From (c) and (d) images, we can also notice the 3-D structures of shock wave, expansion wave and part of the asymmetric model clearly. In our satisfactory CT density results, the remained problem is the occurrence of the artifacts shown in Fig.11 (c) or (d). It would be caused by the discrepancy of model position in BOS test images for some projection angles. Or amount of projection data which is 5 degrees interval is not enough. The large size image of distribution of density gradient on x-y plane from 3-D reconstructed density information is shown in Fig.12. The plot of density gradient extracted from the line of A-A’ in Fig.12 is shown in Fig.13. Various density gradients due to the shock wave
and artifacts are displayed quantitatively with the change of density gradient in Fig. 13. By the artifact, the graph of density gradient in the position of around the model is fluctuating. Though the analysis of separated vertical (red) stripes from CGBOS images is under going, the density gradient data from vertical and horizontal stripes will be combined and the full 3-D density distribution will be obtained by solving Poisson equation [5].

5. Conclusion

From the digital processing of the distortion of stripes extracted from colored-grid pattern image, the quantitative CGBOS images have been obtained and it shows a good agreement with the conventional schlieren results.

1) As quantitative CGBOS image, the displacement of stripes have been estimated and the integrated refractive index gradient of $I$ is obtained quantitatively.

2) In addition to the 2-D CGBOS image, the 3D reconstructed image is successfully obtained by CT reconstruction with ART.

3) In 3-D reconstructed density field image, some artifacts are seen around the model. Some improvements on CT reconstruction technique will be necessary to obtain more clear 3-D results.

Nomenclature

$\rho$ density, kg/m$^3$

$\rho_0$ density at free stream, kg/m$^3$

$G$ Gladstone-dale constant

$L_a$ distance between model and background image, m

$L_c$ distance between model and digital camera, m

$f$ focal length of digital camera, m

$n$ refractive index

$\varepsilon$ deflection angle

$I$ integrated refractive index

$\Delta h$ distortion of background on the image, m

$f(x,y)$ approximation of ART

$P(x, \theta)$ projection data of $\theta$

$R(x, \theta)$ quasi-projection data of $\theta$

$C(x, \theta)$ number of pixels in light source

$m$ number of calculation

References


