Initial Unsteadiness Effect on Transition to Turbulence in Constant Acceleration Pipe Flow

Kazuyoshi NISHIHARA¹, Hisanori SAKA¹, Yoshiaki UEDA², Hirofumi OOYABU² and Manabu IGUCHI²

¹ Graduate School of Engineering, Osaka Electro-Communication University, Osaka 572-8630, Japan
² Graduate School of Engineering, Hokkaido University, Hokkaido 060-8628, Japan

(Received 6 January 2012; received in revised form 20 March 2012; accepted 26 April 2012)

Abstract
Effect of initial unsteadiness on transition to turbulence in constant acceleration pipe flow has been investigated experimentally together with the laminar analytical solution. The acceleration pattern in the present study is a combination of a constant deceleration after the steady regime and then a constant acceleration. Three kinds of the zero velocity period between the constant deceleration and acceleration are selected as (Run 1): 0.0 s, (Run 2): 0.5 s, (Run 3): 5.0 s, and the results are compared with that for a constant acceleration from rest (Run 0). The fully developed axisymmetric Navier-Stokes equations are solved for the present target flow to compare the experimental results before the appearance of turbulence. As a result, the zero velocity period could not distinctly affect transition to turbulence except for \( t_1 = 0.0 \) s within the present experimental conditions.

Keywords
Acceleration, Transition to Turbulence, Unsteady Flow, Circular Pipe, Flow Control

1. Introduction
The present target study on transition to turbulence in a pipe flow is important for a wide range of applications including the design of water-supply systems and oil pipeline and the analysis of blood flow in arteries. In particular, a deceleration pattern is known to affect transition to turbulence in a later acceleration period in the problem of a water hammer. In this study, a particular attention is given for investigating that imposition of unsteadiness on a pipe flow can suppress the flow to remain laminar even at high Reynolds numbers.

An accelerating pipe flow is an interesting issue on the study of the transition process to turbulence. In our previous studies [1, 2], transition to turbulence in a pipe flow was found to be significantly delayed by imposing constant acceleration on the flow until the cross-sectional mean velocity reached a certain constant value. Such a flow with a constant acceleration is generated using a newly developed unsteady flow generation system that a butterfly valve driven by a stepper motor, which follows pre-programmed rotation speed, can produce an arbitrary-accelerating flow in a brass pipe.

Fig. 1 Schematic diagram of the histories of cross-sectional mean velocity

In this study, we investigate initial unsteadiness effect on transition to turbulence due to a constant deceleration and acceleration of a pipe flow whose cross-sectional mean velocity \( u_m \) is controlled as shown in Fig. 1, i.e., the steady pipe flow is constantly decelerated to \( u_m = 0 \) at \( t = t_1 \) and, after \( t_2 = t_2 - t_1 \), \( u_m \) is constantly accelerated again until \( t = t_3 \) and then the acceleration is stopped when the value of \( u_m \) reaches the final velocity of the steady flow \( u_{m, st} \). In Fig. 1, \( t_1 \) indicates initiation time of turbulence after \( t = t_2 \), and \( T_{lag} \) also indicates time lag from \( t_2 \). In addition, this study derives the laminar analytical solution for the present pipe flow along the line of our previous paper [3], and compares the experimental results with it.

2. Experimental Setup and Procedures
Figure 2 shows a schematic diagram of the experimental apparatus. The circular test pipe, made of brass, has an inner diameter of \( D = 78 \) mm and a length of 5000 mm. The working fluid was air. The produced volumetric flow rate \( Q \) was controlled with a hot-wire anemometer by integrating the radial distribution of the measured axial velocity component. The cross-sectional mean velocity \( u_m \) was then calculated by \( Q /[ ( \pi /4) D^2] \).

The effect of the compressibility can be evaluated by the pressure wave propagation in the pipe. The pressure wave is generated by the butterfly valve set at the pipe exit, and the time delay for the pressure wave traveling from the inlet is estimated as \( 0.015 \) s with the velocity of sound of \( 340 \) m/s. As mentioned below (see e.g., Fig. 6), the characteristic time for the present experiment is about \( 4.0 \) s which is sufficiently larger than the time delay due to the compressibility. Therefore, the effect of the compressibility
can be negligible in the present experiment.

As mentioned in the introduction, the arbitrary controlled accelerating flow was generated with a butterfly valve driven by a stepper motor. The stepper motor followed pre-programmed rotational speed to produce three kinds of target input histories: Run 1, 2 and 3 together with the already known input history Run 0 (see Table 1 below). Transition to turbulence was ascertained by monitoring the root-mean-square (RMS) level of the axial velocity signal.

The output signal of the hot-wire anemometer was recorded on a personal computer at a sampling frequency of 2000 Hz. The start-time of measurement (t = 0) was determined by using a trigger function for the A/D converter which was connected to the butterfly valve.

The velocity was measured in the cross-section at x = 4000 mm from the inlet of the pipe (see Fig. 2). An I-probe was traversed in the vertical direction at twenty radial positions between r/R = 0.00 and 0.99 where R is the radius of the pipe (R = D/2) and r is the distance from the centerline of the pipe in the cross-section.

The RMS value of the axial turbulence component is calculated from

$$\Delta u_x = \sqrt{\frac{1}{N-1} \sum_{j=1}^{i+25} \left( u_j - u_{sfa,i} \right)^2}$$  (1)

with

$$u_{sfa,i} = \frac{1}{N} \sum_{j=1}^{i+25} u_j,$$  (2)

where $u_{sfa,i}$ is the averaged value, in a short-time (averaging period is 50/2000 s), of the axial velocity component $u$, and N the number of measurement points with respect to time (in this experiment we take N = 51 points). The long-time average is used for investigating steady turbulent flows, where the period of the average is sufficiently long, whereas the short-time average is used for investigating unsteady turbulent flows. In this study, the period was taken much longer than the characteristic time such as the burst period relating to the turbulence generation but much shorter than that for the unsteady nature of the flow such as the pulsation period. One can refer to the previous papers [1, 2] for the detailed calculation procedure.

3. Analysis

3.1. Problem statement and the axisymmetric Navier–Stokes equations

For a fully developed pipe flow, the axisymmetric Navier–Stokes equation in the x-direction is described as

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right)$$  (3)

with a set of initial and boundary conditions

$$u(R; t) = 0 \quad \text{and} \quad u(0; t) < \infty.$$  (4)

Here, u is the velocity and p the pressure. The dimensionless variables are selected as follows:

$$u' = R u / \nu, \quad x' = x / R, \quad t' = t \nu / R^2,$$

$$p' = p R^2 / (\rho \nu^2), \quad r' = r / R.$$  (5)

In a pipe flow problem the velocity u is usually averaged over the cross-section:

$$\frac{\partial u'_m}{\partial t'} + \frac{1}{r'} \frac{\partial p'}{\partial r'} + 2 \tau'_{w} = 0,$$  (6)

in which $u'_m$ is the dimensionless mean velocity averaged over the cross-section, i.e., $u'_m = \frac{1}{r} \int_0^r r' u'dr'$, and $\tau'_{w}$ is the dimensionless shear stress on the wall $\tau'_{w} = \frac{(-\partial u'/\partial r')r_{w}}{u'_m}.$

Here, we define the Laplace transform $\tilde{W}$ of a function $\mathcal{W}(x', t')$ for $s > 0$ as

$$\tilde{\mathcal{W}}(x'; s) = \mathcal{L} \{ \mathcal{W}(x', t') \} = \int_0^{\infty} \mathcal{W}(x', t') \exp(-st')dt'.$$

Accordingly, taking the Laplace transform of Eqs.(3) and (4), one readily obtain the governing equation in dimensionless form as

$$\frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'} \frac{\partial u'}{\partial r'} - s u' + u'(0) = \frac{-1}{r'} \frac{\partial p'}{\partial r'}.$$  (7)

The above differential equation is easily solved using the last two boundary conditions of Eq.(4):

$$\tilde{u'} = \frac{1}{s} \left\{ \left( -\frac{\partial p'}{\partial r'} \right) + u'(0) \right\} \left[ 1 - J_0 \left( i \sqrt{s} r' \right) / J_0 \left( i \sqrt{s} \right) \right].$$  (8)

where $i = \sqrt{-1}$ and $J_m$ is the mth-order usual Bessel function of the first kind. Furthermore, averaging Eq.(8) over the cross-section gives

$$\tilde{u'} = \frac{1}{s} \left( J_0 \left( i \sqrt{s} r' \right) / J_0 \left( i \sqrt{s} \right) \right) \tilde{u'}_m.$$  (9)

Substituting Eq.(9) into Eq.(8), we finally have the exact solution to Eq.(3)

$$\tilde{u'} = \frac{1}{s} \left( J_0 \left( i \sqrt{s} r' \right) / J_0 \left( i \sqrt{s} \right) \right) \tilde{u'}_m.$$  (10)
3.2. Exact solution to accelerating pipe flow having combined linear accelerations

We deal with an accelerating flow whose cross-sectional mean velocity is given by

\[ u_m(t') = \begin{cases} 
\alpha'_1 t' + u_m(0) & \text{for } 0 \leq t' \leq t'_1, \\
0 & \text{for } t'_1 \leq t' \leq t'_2, \\
\alpha'_2 (t' - t'_2) & \text{for } t'_2 \leq t' \leq t'_3, \\
\alpha'_m, st & \text{for } t' > t'_3, 
\end{cases} \]  

(11)

Note that the coefficients \( \alpha'_1 \) and \( \alpha'_2 \) are respectively a real number in negative and positive, and \( \alpha'_1 t'_1 + u_m(0) = 0 \) and \( \alpha'_2 (t'_3 - t'_2) = u_m, st \). The solution of the problem can be obtained by substituting the Laplace transformed \( u_m(t') \) into Eq.(10) and then taking the inverse Laplace transform.

The Laplace transform of the given mean velocity (11), i.e., \( \bar{u}_m = \int_0^t [u(t') + u_m(0)] \exp(-s t') dt' + \int_{t'_1}^{t'_2} \alpha'_1 \exp(-s t') dt' + \int_{t'_2}^{t'_3} \alpha'_2 \exp(-s t') dt' \), is readily calculated using the integration by parts:

\[ \bar{u}_m = \frac{u_m(0)}{s} + \alpha'_1 \left( \frac{1}{s^2} - \frac{e^{-s t'_1}}{s^2} \right) + \alpha'_2 \left( \frac{e^{-s t'_2} - e^{-s t'_3}}{s^2} \right). \]

(12)

Substituting Eq.(12) into Eq.(10), we have

\[ \bar{u} = \left[ \frac{J_0(i \sqrt{s' t'})}{J_1(i \sqrt{s})} \right] \left[ \frac{u_m(0)}{s} \right] + \alpha'_1 \left( \frac{1}{s^2} - \frac{e^{-s t'_1}}{s^2} \right) + \alpha'_2 \left( \frac{e^{-s t'_2} - e^{-s t'_3}}{s^2} \right). \]

(13)

To obtain the inverse Laplace transform of Eq.(13), the residue at the singular points of Eq.(13) is needed to calculate. When the zeros of \( J_2(i \sqrt{s}) \) are defined as \( s_n \) (note that \( i \sqrt{s_n} := y_n \neq 0 \) for \( n \geq 1 \)), \( J_2(i \sqrt{s}) \) behaves near \( s = s_n \) like \( J_2(i \sqrt{s}) = [(d/d s) J_2(i \sqrt{s})]_{s=s_n} \) in which the derivative of \( J_2(i \sqrt{s}) \) is calculated using the well-known formula of \( (d/d s) J_2(y) = J_1(y) - (2/y) J_2(y) \). Then, the required residue, \( \text{Res, at } s = s_n \) is respectively calculated as, for \( m = 1 \) and 2,

\[ G_m(t'_1, t'_2) := \text{Res, at } s = s_n \left[ J_0(i \sqrt{s't'}) - J_0(i \sqrt{s}) \right] \frac{e^{-s t'}}{s^m} \]

\[ = (-1)^m \frac{4 \pi}{y_n} \frac{e^{-s y_n t'}}{y_n^2 J_0(y_n)} \left[ J_0(y_n) - J_0(y_n, r') \right]. \]

(14)

Here, the zeros \( y_n \) are obtained from the formula 9.5.12 of Abramowitz & Stegun [4]. The unsteady solution to Eq.(13) recovers from the Laplace space using the inverse Laplace transform defined by

\[ \mathcal{W}(t') = \mathcal{L}^{-1} \left[ \mathcal{W}(s) \right] = \frac{1}{2 \pi i} \int_{c-i \infty}^{c+i \infty} \mathcal{W}(s) \exp(s t') ds. \]

To calculate the integral, one takes the contour \( L_1 \) which consists of the real axis from 0 to \( -\infty \). Changing the variable \( s = e^{i \theta} \), the integral range of \( \mathcal{L}^{-1}[u'(s)] \) is divided into \( \int_0^c \mathcal{W}(-i \pi) (-i \pi) ds + \int_\pi^{c+i \infty} \mathcal{W}(s) ds + \sum_n \text{Res, at } s = s_n \) in which the first two terms cancel each other. We then consider the third integral \( \int_\pi^{c+i \infty} \mathcal{W}(s) ds \), defined as \( W_m(t'_1, t'_2) \) with the order of pole \( m \) at the origin. Then, the third integrals of \( W_1 \) and \( W_2 \) are calculated as [3]

\[ W_1(t'_1, t'_2) = 2(1 - r'^2), \]

(15)

\[ W_2(t'_1, t'_2) = 2(1 - r'^2) t' - \frac{1}{24} (1 - r'^2)(1 - 3 r'^2). \]

(16)

Finally, taking into account that the well-known formula \( \mathcal{L}[H(t' - t') f(t' - t')] = \exp(-t'_1 s) f(s) \), and substituting Eqs.(14), (15) and (16) into Eq.(13), the exact solution \( u'(r', t') \) is obtained as

\[ u'(r', t') = u'_m(0) Z_1(t'_1, t'_2) + \alpha'_1 Z_2(t'_1, t'_2) - \alpha'_1 H(t'_1 - t') Z_2(t'_2, t'_1) + \alpha'_2 H(t'_1 - t'_2) Z_2(t'_1, t'_1) - \alpha'_2 H(t'_1 - t'_2) Z_2(t'_1, t'_1) \]

(17)

with

\[ Z_m(t'_1, t'_2) := \int W_m(t'_1, t'_2) \sum_{n=1}^{\infty} G_n(t'_1, t'_2). \]

(18)

Note that \( W_m \) and \( G_n \) are given in Eqs.(14) and (15)–(16) for a specific order.

4. Results and Discussion

4.1. Cross-sectional mean velocity \( u'_m \) compared with the laminar analytical solution

Figure 3 shows the cross-sectional mean velocities for Runs 0, 1, 2 and 3 with the hot-wire anemometer, together with the laminar analytical solution of Eq.(17), in which the infinite series involved is computed within \( n = 200 \). As shown in Fig. 3, the present system successfully controls the present target flows. In addition, the analytical solution derived in this study is found to be valid before the appearance of the turbulence.
4.2. Velocity distributions at around \( t' = t_2' \)

The laminar analytical solution of Eq.(17) is plotted in Fig. 4 in which the left of Fig. 4 shows the axial velocity \( u' \) for Run 1 at various locations of \( r/R \) at around \( t' = t_2' \) together with \( u'_{sta} \), and the right of Fig. 4 shows the axial velocity \( u' \) for all Runs at \( t' = t_2' \). Details for each run refers to Table 1 below. In Fig. 4, the backflow is observed near the cylinder surface in all Runs although its strength depends on the Runs, i.e., the strong backflow appears in Run 1. We here notice that the present experiment using the I-type hot-wire anemometer cannot exactly measure the velocity in the backflow region. In fact, the centerline velocity experimentally measured is found to yield a little difference against the laminar analytical solution at around \( t' = t_1' \) and \( t_2' \), as seen in Fig. 5. To avoid this difficulty, we intend not to handle the situation at around \( t' = t_2' \) that the backflow can occur near the cylinder surface, hereafter.

4.3. Axial velocity component, together with \( 1/n \)-th power law, and transition to turbulence

The present experimental results are shown in Table 1. Here, \( u'_{sta} \) means the transition Reynolds number at which the turbulence appears. Figure 6 shows the axial centerline velocity signals \( u \) for Runs 0, 1, 2 and 3. As observed in Fig. 6, the turbulence before \( t' \leq 0 \) decays during \( 0 \leq t' \leq t_1' \) (deceleration period) and the turbulence appears again between \( t' = t_2' \) and \( t_3' \) (acceleration period). In particular, within the present experimental conditions, the constant acceleration of Run 1 seems not to suppress transition to turbulence because the turbulence appears at the smaller value of \( t_{sta} \) than that in other Runs as seen in Table 1. As mentioned above, the accuracy of the measurement for Run 1 at around \( t' = t_2' \) is thought to be low because the strong backflow appears near the cylinder surface (see Fig. 4). Further investigation is required to discuss the transition to turbulence for Run 1.

Figure 7 shows comparisons of the axial velocity \( u' \), after \( t' = t_2' \), in Runs 0, 1, 2 and 3 between the experimental results and the laminar analytical solution of Eq.(17). In the computation of the analytical solution, we sufficiently adopt the first two hundreds of \( y_n \) to calculate the infinite series in Eq.(14). The preceding paper [3] demonstrated that the adoption of \( y_n \) within \( n = 100 \) is sufficient for the computation of the infinite series for \( t' \lessgtr 0.1 \). As shown in Fig. 7, the present experimental results are in good agreement with the laminar analytical solution before \( t' = t_2' + 0.016 \).

After \( t' > t_2' + 0.016 \), the axial velocity experimentally measured is compared with the \( 1/n \)-th power law [5]:

\[
u_{sta}^l = u_{sta,el}^l(1 - r')^{1/n},
\]

where \( u_{sta,el}^l \) is the dimensionless axial velocity averaged in a short time, and \( u_{sta,el}^l \) is the dimensionless axial velocity on the centerline of the pipe, i.e.,

\[
u_{sta,el}^l = \frac{[(n + 1)(2n + 1)]}{2n^2}u_{sta}^l.
\]

In addition, \( n \) is given by \( n = 2 \log(Re/10) \) in which \( Re \) is the Reynolds number equals to \( 2u_{m}^l \). As shown in Fig. 8, the velocity distributions experimentally measured are found to be approximated in the \( 1/n \)-th power law after transition to turbulence.

5. Conclusions

Effect of initial unsteadiness on transition to turbulence in constant acceleration pipe flow has been investigated experimentally together with the laminar analytical solution. The acceleration pattern was a combination of a constant deceleration after the steady regime and then a constant acceleration. Three kinds of the zero velocity period \( t_z \) between the constant deceleration and acceleration are selected as \( t_z = 0.0s, 0.5s \) and 5.0s, and the results are compared with that for a constant acceleration from rest. The main results are as follows:

---
The zero velocity period $t_0$ could not distinctly affect transition to turbulence except for $t_0 = 0.0 \text{s}$ within the present experimental conditions.

The laminar solution obtained for the accelerating pipe flow was found to be valid for the present target flow until transition to turbulence and, after the turbulence appears, the axial velocity was approximated with the $1/n$-th power law for a steady flow.

References


