EXPERIMENTAL STUDY ON THE INTERACTION BETWEEN BURGERS VORTEX AND A SOLID PARTICLE USING 2D PIV MEASUREMENT

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ABSTRACT
In recent years, many researchers study on characteristic concentrations of particles in turbulent flows. However, it is difficult to observe experimentally interactions between the particles and turbulent vortices because of strong non-linearity of turbulence. In the present study, we focus on the elementary step of an interaction between single vortex tube and a solid particle in a solid-liquid two-phase flow. Burgers vortex is chosen as single vortex tube. Burgers vortex is the simplest model of turbulence.

KEY WORDS
Burgers Vortex, Turbulence Modulation, Particle-Laden Turbulent Flows, Particle-Turbulence Interaction, 2D PIV

INTRODUCTION
The motion of heavy particles through a turbulent flow field is one of the most interesting research topics in fluid mechanics. Prediction and control of turbulent flows laden with heavy particles is practical engineering interest since these flows occur in many technologically important areas, such as flows in energy convection devices and issues in air pollution dispersion. These fields have non-linear interactions (particles-vortices, particles-particles, vortices-vortices) that take place between motions of vastly different length and time scales. Since these interactions have the three-dimensional unsteady chaotic nature, many researchers mainly study on the statistical properties of the turbulence [1], the dynamics and kinematics of particle motion.

In numerical studies of turbulence modulation, the particles are often modeled as point forces [2, 3, 4]. In these studies, the details of flow at the micro scale around each particle are ignored. This model becomes increasingly accurate for particles of size much smaller than the Kolmogorov scale. However, there are many situations where the particle size is the order of Kolmogorov scale or larger, and the particle Reynolds number is larger than unity. The effect of finite particle size, particle wake, and vortex shedding are important in this range [5].

Moreover, many researches on turbulent flow without particles clarified that tube-like coherent structures play an important role of the intermittency and energy dissipation in the range from the Taylor microscale to the integral scale of turbulence [6]. Burgers vortex is the simplest model of tube-like coherent structures [7], and an axisymmetric vortex represents one of the few known exact solutions to the full Navier-Stokes equations [8, 9]. The turbulence is modeled by an ensemble of strained vortices without spiral vortex structures [10], i.e. the Burgers vortices, and a statistical analysis is carried out to extract the properties of homogeneous isotropic turbulence from the model vortices including the external straining field [11, 12].

In the present study, we focus on the elementary step of an interaction between single Burgers vortex tube and a heavy particle. Particular attention will be paid to understanding how heavy particle influences Burgers vortex, and how the vortex is strengthened. Experimental results presented here are the first of their kind of particle-Burgers vortex interaction.

In future works, we create a mathematical model of the elementary step of an interaction between single Burgers vortex tube. And this model is built into Kambe's turbulence model [12]. We finally create the model of the interaction between turbulence and particles.

BURGERS VORTEX
The vortex tube consists of a pure swirl flow superposed on an irrotational base strain flow. The fluid is incompressible. In the cylindrical coordinate the solution can be written as follows.

\[ u(r, \theta, z) = (-a z, u_0, 2a z) \]

where,

\[ u_0(r) = \frac{1}{\pi} \frac{1}{r} \exp(-r^2/\lambda_0^2) \]

\[ \lambda_0 = \frac{1}{\sqrt{2\pi}} \]

\[ \Gamma \]

\[ \tau \]

\[ \alpha \]

\[ \epsilon_{\alpha} = 4\pi(3\alpha^2 + \epsilon_{\alpha}) \]

where \( \epsilon_{\alpha} = -\frac{1}{2}(u_0(r), -u_0(r)/r) \). The combined field \( u(r, \theta, z) \) is characterized with negative skewness at points satisfying \( \alpha < \epsilon_{\alpha} \), while neither circulating component \( (\alpha = 0) \) nor pure straining component \( (u_0(r) = 0) \) is in this field.

A phenomenological consideration is given here in order to make clear the theoretical background of the Burgers vortex model in turbulence. It is postulated that an average separation distance between neighboring vortices is of the
order of the Taylor-microscale $\lambda$. Denoting the root-mean-square (rms) value of turbulence velocity by $u_{rms}$, the Taylor Reynolds number is defined as $Re_t=u_{rms}/v$.

In the homogeneous isotropic turbulence, $\varepsilon_{\infty}$ and the Kolmogorov dissipation scale $\eta_k$ are given by the following expressions [14].

$$
\varepsilon_{\infty} = 15\nu(u_{rms}/\lambda)^2, \quad \eta_k = (\nu/\varepsilon_{\infty})^{1/4}.
$$

Assuming that the strain rate $\alpha$ of the externally superposed flow in Eq. (1) is of the order of $u_{rms}/\lambda$ (that is, $2\alpha = u_{rms}/\lambda$), the radius of the Burgers vortex is given as [12]

$$
l_b = (2\nu/\alpha)^{1/2} = (4\nu/\alpha)^{1/2} = (240\nu^3/\varepsilon_{\infty})^{1/4} \approx 3.9\eta_k.
$$

Namely, the Burgers radius is of the order of the Kolmogorov length. Moreover, this shows that the radius $l_b$ is scaled in the same way as the dissipation scale $\eta_k$. Hydromechanical manifestation of the Kolmogorov scale is considered as the Burgers radius in the present study. Using the definitions of $l_b$ and $Re_t$, together with $2\alpha = u_{rms}/\lambda$, one obtains

$$
\lambda/l_b = 1/2 Re_t^{1/2}.
$$

**EXPERIMENTAL SETUP AND CONDITIONS**

Figure 1 shows the experimental setup designed to observe directly the interaction between Burgers vortex and a single particle.

![Experimental setup for direct measurement of the interaction between Burgers vortex tube and a single particle](image)

The interaction is measured directly from the observation window ($150 \times 150$ mm$^2$) installed in the bottom side of the tank using PIV measurement (Camera: Photron 1024×1024 pixels, 250 frame/sec, Laser sheet: CW YAG 3A sheet thickness of approximately 1 mm. Tracer particle: 50 μm ORGASOL, specific gravity:1.03). The rotating disk (60 mm in diameter) attached to motor (1500 rpm) generates vortex tube at the topside of the tank. The water flow goes to the bottom side from the topside of the tank ($150 \times 150 \times 1000$ mm$^3$) by four water circulation pumps installed at four sides of the tank.

We used the particle (2.95±0.10 mm in diameter, specific gravity: 3.69, material: alumina ceramics). The particle freely falls from the upper part of the tank, and the particle passes through the test cross-section perpendicular to the rotational axis of the vortex.

Then the particle passes through the laser sheet of area sensor (KEYENCE LV-H52) and a cross section of vortex tube sliced by laser light sheet. When the area sensor detects the particle, camera starts to capture images of the interaction between the single vortex tube and the particle (magnified image in Fig. 1). Reynolds number is 630 (water tank width: 150mm, main flow: 31.5 mm/sec, water temperature: 20°C).

![Figure 2 Comparison between experimental data and exact solutions](image)

In Figure 2 the experimental data obtained from PIV measurement is compared with the exact solution Eqs. (2) and (3). Here, $\omega^*(r/l_b)$ and $u^*(r/l_b)$ are defined as the non-dimensionalized distributions of the vorticity and the circumferential velocity by their maximum values $\omega_{max}$ and $u_{max}$, respectively. Since these results conclude that the experimental data fits well with these solutions, the experimental setup generates Burgers vortex.

In order to non-dimensionalize experimental parameters, we introduced the following parameters [15].

$$
\eta_k = (\nu^3/\varepsilon_{\infty})^{1/4}, \quad u_k = (\varepsilon_{\infty})^{1/4}, \quad \tau_k = (\nu^3/\varepsilon_{\infty})^{1/2}.
$$

These parameters are called Kolmogorov scale, Kolmogorov velocity and Kolmogorov time, respectively. Here, parameters have the relation of $Re_t = u_k/\tau_k = 1$.

We obtain the Burgers scale $l_b = 10$ mm from circumferential velocity measured in experiments. Substituting $l_b$ value into Eq. (6), Kolmogorov parameters (scale, velocity and time) are $\eta_k = 2.0$ mm, $u_k = 0.5$ mm/sec and $\tau_k = 4.2$ sec, respectively.

The particle Reynolds number $Re_p$ considered in this study is 1150 ($Re_p = d_p/\nu_{rms} - u_i/\nu, \nu_{rms}$:particle terminal velocity, $u_i$:time averaged velocity in $z$ direction) and the ratio of the particle diameter $d_p$ to Kolmogorov scale $d_p/\eta_k$ is 1.5. The vortex Reynolds number 1/4 of Burgers vortex is 79. $\eta_k$ is represented by $Re$ based on the large-scale eddies, $Re = UL/\nu$, as follows

$$
\eta_k = L Re^{3/4}.
$$
In a typical wind tunnel experiment we might have $Re = 4000$ ($L=1000$ mm $U=40$ mm/sec). Here, $L$ and $U$ are the representative length and velocity, respectively.

RESULTS AND DISCUSSIONS

Burgers vortex has the balance between the vorticity distribution $\omega_{ij}$ and the divergence distribution $D_{ij}$ from Eq. (1). We define the interaction parameter $B.V.D.$ (Balance between Vorticity and Divergence) in order to evaluate the effect of a heavy particle as follows.

$$B.V.D. = | \omega_{ij} | - | D_{ij} |$$  \hspace{1cm} (10)

Vorticity $\omega_{ij}$ becomes the dominant parameter in case of $B.V.D. > 0$, and divergence $D_{ij}$ becomes the dominant parameter in case of $B.V.D. < 0$.

In order to clarify the time correlation of vorticity and divergence, the maximum value and the minimum value of $B.V.D.$ sensitive to the time response are observed in Fig.3.

![Figure 3 Time series of B.V.S.max and B.V.S.min](image)

Figure 3 shows that vorticity and divergence have a negative cross correlation (-0.62) in the range from $t/\tau_K=1.0 \times 10^{-2}$ to $t/\tau_K=1.5 \times 10^{-2}$. From captured images, it is confirmed that the particle passing through the test section is observed at $t/\tau_K=1.1 \times 10^{-2}$.

$B.V.D._{\text{max}}$ and $B.V.D._{\text{min}}$ are representative values of vorticity and divergence, respectively.

In Figure 4, $B.V.D.$ planes with velocity vectors are shown at each time. The process of the interaction between the Burgers vortex and the particle is as follows:

a) Steady vortex
b) Diffuse flow is induced by front side of particle. Vortex peak is shifted, and vortex area is reduced.
c) The suction flow is induced by particle wake. Vortex is enhanced.
d) Recovering vortex area
e) Steady vortex.

Figure 3 and 4 show the vorticity and the divergence are enhanced by particle wake simultaneously.

![Figure 4 B.V.D. plane](image)
Figure 5: The influence of the distance between vortex center and passing point of the particle on the vorticity increase

Figure 5 indicates the influence of the distance between vortex center and passing point of the particle on the increase rate of the maximum vorticity value. The increase rate is defined as \( \frac{\omega_{\text{max}} - \omega_{\text{ave}}}{\omega_{\text{ave}}} \) (\( \omega_{\text{ave}} \): time averaged value of the maximum vorticity value without particle passing).

The vortex is divided into forced vortex (\( d\omega(r)/dr > 0 \)) and free vortex (\( d\omega(r)/dr < 0 \)) at \( r/l_b = 1.12 \) obtained from the velocity distribution \( \omega(r) \) in Eq. 2. It is found that the particle passing at forced vortex area \( (r/l_b < 0.7) \) increases vorticity more than particles passing free vortex area \( (r/l_b > 1.12) \).

CONCLUSIONS
The present research reports the elementary step of an interaction between single Burgers vortex tube and a heavy particle. The main conclusions obtained are as follows:

1. The experimental setup generated Burgers vortex.
2. Vorticity and divergence are enhanced by particle wake simultaneously.
3. The particle passing at forced vortex area increases vorticity more than particles passing at free vortex area.

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NOMENCLATURE
\( d_p \) particle diameter, m
\( e_{g0} \) negative skewness, 1/m
\( \Gamma \) circulation, m²/sec
\( l_b \) Burgers vortex scale, m
\( a \) strain rate of the background flow field, 1/sec
\( v \) kinematic viscosity, m²/sec
\( \omega(r) \) circumferential velocity, m/sec
\( \omega_0 \) steady vorticity, 1/sec
\( \omega_{\text{ave}} \) local rate of energy dissipation, m²/sec
\( e_{g0} \) skewness, 1/sec
\( u_{rms} \) root-mean-square value of turbulence velocity, m/sec
\( \text{Re} \) Taylor Reynolds number, -
\( \eta_k \) Kolmogorov dissipation scale, m
\( u_k \) Kolmogorov velocity, m/sec
\( t_k \) Kolmogorov time, sec
\( \text{Re}_k \) Kolmogorov Reynolds number, -
\( u_{tens} \) particle terminal velocity, m/sec
\( \epsilon_i \) time averaged velocity in z direction, m/sec
\( d_p \) particle diameter, m
\( \text{Re} \) Reynolds number, -
\( L \) representative length, m
\( U \) representative velocity, m/sec
\( \omega_j \) vorticity distribution, 1/sec
\( D_j \) divergence distribution, 1/sec
\( B.V.D. \) balance between vorticity and divergence, 1/sec
\( B.V.D. \) maximum value of \( B.V.D. \) , 1/sec
\( B.V.D. \) minimum value of \( B.V.D. \) , 1/sec
\( \omega_{\text{max}} \) maximum value of vorticity, 1/sec
\( \omega_{\text{ave}} \) time averaged maximum value of vorticity, 1/sec

SUBSCRIPTS
b Burgers vortex
K Kolmogorov

REFERENCES