FULL-FIELD DETERMINATION OF PRINCIPAL-STRESS DIRECTION FROM COLOR PHOTOELASTIC FRINGES OBTAINED WITH A SEMICIRCULAR POLARISCOPE

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ABSTRACT
A method was developed for determining the directions of the principal-stresses over an entire model from color photoelastic fringes obtained in a semicircular polariscope. The polariscope was composed of a circular polarizer and an ordinary linear polarizer. A model was illuminted with circularly polarized light and the light transmitted from the model was analyzed with only the rotation of analyzer so that phase-stepped color photoelastic fringes were obtained and the map of the principal-stress directions was later deduced from them. The effectiveness of the method was investigated with color photoelastic fringes of a circular disk under diametric compression. Results showed that the method was effective to determine the principal-stress directions with reasonable accuracy.

KEY WORDS
Color photoelasticity, Principal-stress directions, Isoclinic parameter, Semicircular polariscope

INTRODUCTION
In order to completely analyze the stress induced in the structural member, the magnitude and directions of the stresses at point of interest are necessary. There are several methods such as experimental and numerical methods used to determine these quantities. One of the experimental methods being widely used to determine such quantities is the photoelasticity. Its key feature is that it can visually represent the induced stress in form of a fringe pattern. There are two types of fringe pattern: isochromatic and isoclinic fringe patterns. The former provides isochromatic parameter δ or stress difference (σ₁-σ₂) that directly relates to the magnitude of stress whereas the latter gives the isoclinic parameter φ that directly connects to the stress directions. In photoelastic fringe analysis, it is necessary to properly assign such parameters to their fringes. Recently, the effective method, digital photoelasticity method, is used as a tool for full-field assigning those two parameters with a substantial accuracy [1].

For φ, it cannot be, however, accurately assigned due to the effect of δ, i.e., when δ has generally a half- or an integer-order fringe. To overcome this problem, several methods employing incident light of different wavelengths were proposed and the reasonable accuracy of φ can be possibly gained [2-7]. Since, in such proposed methods, the interference filters were used to obtain those different wavelengths in most cases, the act of filter exchange was needed; therefore, it is considered to be a tedious task and also time consuming process. Further, of the methods employing the plane polariscope [8-10], both the polarizer and analyzer have to be rotated through some of the phase steps. Then, it limits the application of the methods only to static problems.

In this work with the attempt of developing the method for solving dynamic problem kept in mind, the method based on the color phase shift technique is developed to determine φ with the color photoelastic fringe images obtained from a mixed or semicircular polariscope. Note that for the work presented here, in the context of dynamic problem, the author attempt to develop system that can simultaneously acquire the photoelastic data needed to produce full-filed map of φ at only one time. To make the method as simple as possible, the exchange of filters during capturing fringe images is unused and only the rotation of the analyzer is needed. Furthermore, in the view of reduction of the analyzer rotation, it is possible to combine a multi-spectrum separation optical system [11] into this developed system in or simultaneously capture fringe images. Based on the concept of the application to the dynamic problem, authors have no claims for the accuracy of the method because it still requires further improvement.

DETERMINATION OF ISOCLINIC PARAMETER
The arrangement of the optical elements shown in Fig. 1 is the semicircular polariscope system that consists of circular polarized light system (polarizer and quarter wave plate). As for the circular light, it is clockwise circular polarized light when looking from the camera toward the light source.

After the specimen is properly kept in the system and loaded by a force, the general equation of the irradiance I with generic orientations m of the transmission axes by the analyzer coming out of a digital camera is given by

\[ I_{m,\lambda} = \frac{1}{\lambda} \int_{\lambda_{\text{low}}}^{\lambda_{\text{upp}}} \left[ \frac{F_{\lambda} \pi^2}{2} [1 + \sin \delta_{m} \sin 2(\theta_{m} - \phi)] + I_{\lambda,\lambda} \right] d\lambda \] (1)

where \( \Delta \lambda = \lambda_{\text{upp}} - \lambda_{\text{low}} \), \( \lambda_{\text{upp}} \) and \( \lambda_{\text{low}} \) are the upper and lower limits of the spectrum of light source, \( F_{\lambda} \) is the spectral response of the camera filters, \( a_{\lambda} \) is the amplitude of the light coming out of the polarizer, \( \delta_{m} \) is the relative retardation for a given primary wavelength \( \lambda \) (= R, G, B) of a white light source, \( \phi \) is the isoclinic angle or the angle of σ₁ with respect to the x-axis, \( \theta_{m} \) is the phase shift angle, \( h_{\lambda,\lambda} \) is the background irradiance. Note that \( \delta_{m} \) relates to \( (\sigma_{1} - \sigma_{2}) \) in plane-stress state by

\[ \frac{\delta_{m}}{2\pi} = N_{\lambda} \frac{C_{\lambda}}{\lambda}(\sigma_{1} - \sigma_{2}) = \frac{h}{f_{\lambda,\lambda}}(\sigma_{1} - \sigma_{2}) \] (2)

where \( N_{\lambda} \) is the relative fringe order, \( C_{\lambda} \) is the stress-optic coefficient, \( f_{\lambda,\lambda} \) is the well-known material stress fringe value obtained by calibration and \( h \) is the model thickness.

Equation (1) can be simply expressed as

\[ I_{m,\lambda} = I_{\text{mod},\lambda} \sin 2(\theta_{m} - \phi) + I_{\text{eff},\lambda} \] (3)

in which
\[ I_{\text{mod},\lambda} = \frac{1}{\Delta \lambda} \int_{\lambda_{\text{imin}}}^{\lambda_{\text{imax}}} \left( \frac{F_A a_2^2}{2} \sin \delta_\lambda \right) d\lambda, \]  
\[ I_{\text{eff},\lambda} = \frac{1}{\Delta \lambda} \int_{\lambda_{\text{imin}}}^{\lambda_{\text{imax}}} \left( \frac{F_A a_2^2 + F_B a_2^2}{2} \right) d\lambda. \]  
(Note that the change of Eq. (1) to Eq. (3) using Eqs. (4) and (5) is valid due to the fact that \( F_A, a_2, I_{\text{h}}, \) and \( I_{\text{eff}} \) are only the function of \( \lambda \) not the phase shift angle \( \delta_\lambda \), whereas the term \( \sin 2(\delta_\lambda - \phi) \) is independent of \( \lambda \). It should be noted further that though there is one quarter wave plate in the polariscoppe, its influence of mismatch error is not an issue here. Some works considering this influence can be found elsewhere [12-13]. Further, as seen in Eq. (1) the background irradiance \( I_{\text{h},\lambda} \) is the function of \( \lambda \); however, the works done by Ajovalasit et al. [14-15] made this irradiance the constant quantity. That is, Eq. (5), in another form, can be written as \[ I_{\text{eff},\lambda} = I_h + \frac{1}{\Delta \lambda} \int_{\lambda_{\text{imin}}}^{\lambda_{\text{imax}}} \left( \frac{F_A a_2^2}{2} \right) d\lambda. \]  
(Note that the background irradiance in Eq. (1) should also be revised accordingly. This difference is, however, unaffected the final expression for the isoclinic parameter as seen later.)\)

Applying four phase shift angles such that \( \theta_m = (m-1)\pi/4 \) for \( m = 1, 2, 3 \) and 4, Eq. (3), for each wavelength, can be expressed as listed in Table 1. Then, summing up at each step with Eq. (7) \[ I_{\text{mod},\lambda} = I_{\text{mod},\lambda}^R + I_{\text{mod},\lambda}^G + I_{\text{mod},\lambda}^B, \] yields \[ I_{\text{mod},\lambda} = I_{\text{mod},\lambda}^R + I_{\text{mod},\lambda}^G + I_{\text{mod},\lambda}^B = \sqrt{(I_{\text{mod},\lambda}^R - I_{\text{mod},\lambda}^G)^2 + (I_{\text{mod},\lambda}^G - I_{\text{mod},\lambda}^B)^2}, \]  
\[ I_{\text{mod},\lambda} = I_{\text{mod},\lambda}^R + I_{\text{mod},\lambda}^G + I_{\text{mod},\lambda}^B. \]  
in which \[ \begin{align*} I_{\text{mod},\lambda} &= I_{\text{mod},\lambda}^R + I_{\text{mod},\lambda}^G + I_{\text{mod},\lambda}^B = \sqrt{(I_{\text{mod},\lambda}^R - I_{\text{mod},\lambda}^G)^2 + (I_{\text{mod},\lambda}^G - I_{\text{mod},\lambda}^B)^2}, \end{align*} \]  
\[ \begin{align*} I_{\text{eff},\lambda} &= I_{\text{eff},\lambda}^R + I_{\text{eff},\lambda}^G + I_{\text{eff},\lambda}^B. \end{align*} \]  
Combining Eqs. (8) to (11), yields the expression for \( \phi \) as \[ \phi = \frac{1}{2} \arctan \left( \frac{I_{\text{mod}}^G - I_{\text{mod}}^B}{I_{\text{mod}}^G - I_{\text{mod}}^R} \right) = \frac{1}{2} \arctan \left( \frac{I_{\text{mod}}}{I_{\text{mod}}} \right) \]  
for \( I_{\text{mod}}^G = 0 \) \[ \phi = \frac{1}{2} \arctan \left( \frac{I_{\text{mod}}^G - I_{\text{mod}}^B}{I_{\text{mod}}^G - I_{\text{mod}}^R} \right) = \frac{1}{2} \arctan \left( \frac{I_{\text{mod}}}{I_{\text{mod}}} \right) \]  
for \( I_{\text{mod}}^G = 0 \). Note that the arctangent operator used here provides only the principal results in the interval \( -\pi/2 \) to \( +\pi/2 \). In addition, in the calculation process, before substituting the results from Eq. (7) into Eq. (14) the summation should be normalized by a factor such that the summation does not exceed the maximum gray level value used [16].

The reason for this is to reduce the effect of the isochromatics and the variation of light occurring with different phase shift \( \theta \). As seen in Eq. (14), \( I_{\text{mod},\lambda} \) is cancelled out; hence, no matter what the definition of \( I_{\text{mod},\lambda} \) is (dependent or independent of \( \lambda \)), it is also deleted from Eq. (14). However, that \( I_{\text{mod},\lambda} \) is dependent of \( \lambda \) as done here reflects more reality than the works of Ajovalasit et al. [14-15].

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It is seen that \( \delta \) has a great effect on the computation of \( \phi \). That is, when \( \delta = \pi \) or \( N \) \( = \pi / 2 \) \((n = 0, 1, 2, \ldots)\) at some points, the obtained irradiance is approximately a half of the intensity coming out of the polarizer. When this happened (\( \text{in} = \text{out} = 0 \)), \( \phi \) is indeterminate at those points and not continuous over the entire domain of the model.

To solve this problem, a correction method is introduced here. That is, when \( |\text{in}| \geq T \) and \( |\text{out}| \geq T \) where \( |\text{in}| \) is a predefined threshold value, the value of \( \phi \) as obtained from Eq. (14) is unchanged. However, when \( |\text{in}| < T \) and \( |\text{out}| < T \), the values of \( \phi \) for each \( R, G, \) and \( B \) wavelength are computed and, then, assigned to be final isoclinic value according to some conditions as shown in Table 2 for which the parameters for comparison can be given by the following equation.

\[ I_{\text{mod},\lambda} = \sqrt{(I_{\text{mod},\lambda} - I_{\text{in}})^2 + (I_{\text{mod},\lambda} - I_{\text{out}})^2}. \]

It should be remarked that Eq. (13) is the parameters on the right side of Eq. (12). 

![Fig. 1 Mixed or semicircular polariscoppe with the circular disk under compression being kept inside.](image-url)
RESULTS AND DISCUSSION

To verify the ability of the method, it is first applied to the simulated photoelastic fringe patterns and then its application to the experimentally generated photoelastic fringe patterns is demonstrated.

Figure 1 shows the experimental setup, consisting of a white light source, polarizer whose optical axis is in vertical, quarter wave plate whose slow axis is at 45° with the horizontal axis, analyzer whose optical axis is in horizontal, digital camera (SLR D70, NIKON Corp.) and personal computer for fringe analysis. Specimen used for analysis was made of an epoxy resin plate as a circular disk (diameter 30 mm, thickness 6 mm, photoelastic sensitivity $a_{\rho=\rho_0} = 0.1$ mm/N). Note that $f_{e,\rho} = 1/a_{\rho}$. When performing the experiment, the specimen was subjected to vertically diametral compressive load $P = 156$ N. For numerical analysis, all parameter described above were used to generate the simulated fringe pattern according to the well known analytical solution [17].

Numerical Results

Figure 2 displays the simulated color photoelastic fringe images associated with the angular rotation of the analyzer for four different phase steps. With images in Fig. 2 and Eq. (14), the full-field isoclinic-angle map expressed in the interval $-\pi/4$ to $+\pi/4$ is qualitatively depicted in Fig. 3(a). In the map, one can clearly see the lines of wrong isoclinic values. For visual clarity of such lines, Fig. 3(b) shows the magnification of the top part of the map in Fig. 3(a) in order to easily observe the wrong isoclinic values near the abrupt phase jump.

The theoretical and simulated profiles of $\phi$ values along a horizontal line across an upper half of the disk radius $r$ at the ratio $y/r = 0.5$, where $y$ is the vertical axis measured upwards from the center of the specimen, are shown in Fig. 4. Note that the scale bar in Fig. 3(a) is linear. The theoretical profiles can be generated using [17]

\[ \tan \phi = \frac{2xy}{r^2 - y^2 + x^2} \]  

in which $x$ and $y$ are the spatial coordinates measured from the disk center positively to the right and top, respectively. It is seen in Fig. 4 that the simulated profile agrees well with that of theory for the most part except some places at which small abrupt jumps occurred. These small abrupt jumps occurred at points for which $\Delta_1 = m\pi$ or $N_2 = n\pi/2$ ($n = 0, 1, 2,...$) and they are on those erroneous lines (Fig. 3(a)).

Figure 5 qualitatively depicts the full-field isoclinic-angle map after doing the correction by those conditions shown in Table 2. Note that $\phi$ which stands for the corrected isoclinic angle values is now used instead of $\phi$. It is observed that after the correction process with $T = 0.5$ the erroneous lines have been greatly reduced or at best deleted (cf. Fig. 3(a)). Note that for theory, $T$ value can be changed down to 0.1 because the erroneous lines are very thin (about one or two pixels wide). This can be clearly observed in Fig. 6 which displays the profile of $\phi$ along the same line of the profile in Fig. 4.

As previously mentioned, the reason why the erroneous lines happened even though this isoclinic-angle map (Fig. 3(a)) was obtained from the simulated images (Fig. 2) is that at those points $\Delta_1 = m\pi$ or $N_2 = n\pi/2$ ($n = 0, 1, 2,...$). The profile of the G irradians along the horizontally diametral line of the fringe images (Fig. 2) which is plotted as shown

Fig. 2 Simulated color photoelastic fringe images corresponding to four configurations of a semicircular polariscope: (a) $\theta_1 = 0$, (b) $\theta_2 = \pi/4$, (c) $\theta_1 = \pi/2$ and (d) $\theta_1 = 3\pi/4$.

Fig. 3 Isoclinic-angle map expressed in the range of $-\pi/4$ to $+\pi/4$: (a) full-field map and (b) enlarged top portion of (a).

Fig. 4 Distribution of $\phi$ values along the horizontal line of ratio $y/r = 0.5$ of the full-field isoclinic-angle map as shown in Fig. 3(a). Note that $r$ is the disk radius and $y$ is measured upwards from the disk center.
in Fig. 7 confirms these conditions. That is, those erroneous lines occurred when the four irradiance profiles crossed each other. It should be remarked here that this is also true for other wavelengths. Hence, this means that the irradiances obtained from Eqs. (7) to (10) possessed the same value and then the result given by Eq. (12) turns to zero which strongly violates the condition shown in Eq. (14). As a consequence, the isoclinic angle values obtained were unreliable, i.e., \( \phi \) values were forced towards \(-\pi/4\) or \(+\pi/4\). However, the use of conditions shown in Table 2 reduced those errors. The reason is that the locations where the irradiances crossed each other are different for different wavelengths. Therefore, one of \( \phi \) determined at one wavelength can be used.

**Experimental Results**

The experimentally generated fringe images (analogous to Fig. 2) are digitally captured and shown in Fig. 8. By the same manner as done in the case of the simulated images, both pre- and post-correction isoclinic angle-maps are qualitatively depicted in Fig. 9. Note that the predefined \( T \) value was set to be 0.75 and it is user-dependent parameter.

One can see the great effect of \( \delta \) (Fig. 9(a)); however, after performing the correction, such effect has been greatly reduced except at and near the boundary of the specimen (see Fig. 9(b)). This is because the values of \( \delta \) for all wavelengths used are zero (physical reality) and the residual stress occurred when the specimen was being prepared. It should be remarked here that for \( T < 0.75 \), the isoclinic-angle maps obtained were almost similar to Fig. 9(a) while \( T > 0.75 \) the map still evidently contained the erroneous lines (images are not shown here). The profiles of \( \phi \) and \( \delta \) being compared with that of theory are quantitatively plotted in Fig. 10(a) and (b), respectively. Close inspection of Fig. 9(a) and Fig. 10(a) reveals that the experimental profile shows a great deviation from theory not only when \( \delta = n\pi \) (\( n = 0, 1, 2, \ldots \)) but also when \( \delta = \pi/4 \). It is evident in Fig. 10(b) that the agreement between the theoretical and experimental profiles is moderate after making correction and the strong deviation is still clearly seen near the edge of the circular disk. Although the work presented here, as the first stage, has not yet been applied to the dynamic problem, a minimum requirement of the operator interaction, i.e., only the analyzer rotation, expresses its character.
Patterson and Wang [18] proposed the simultaneous observation system for automated photoelasticity with a novel instrument that can be simultaneously grabbed four fringe images; however, their methods were based on the circular polariscope in which rotation of both the analyzer and second quarter wave plate is necessary. This is also the same as the work done by Yoneyama et al. [19] That is, they employed the circular polariscope and the developed system required the rotation of analyzer and second quarter wave plate. In contrast, if the presented method were integrated into such automated system, it would be less complicated (without second quarter wave plate) and might be faster for the data acquisition. However, the presented method is only applied to determine δ whereas the method of Yoneyama et al. [19] can determine both δ and θ under both static and increasing load. A simulated comparison of several real-time phase shifting techniques can be found in the work of Zhenkun et al. [20].

Recently, Yoneyama et al. [21] presented a simultaneous acquisition system with a pixilated micro-retarder array for analysis photoelasticity fringe pattern. The concept of presented work also directs to the same objective as that of Yoneyama et al. [21]. However, the work of Yoneyama et al. [21] furnished experimental results of the isoclinic-angle map with unsolved isochromatic-isoclinic interaction. As a result, the isoclinic-angle map still has those erroneous lines (see Fig. 9(a)). On the other hand, the work presented here has solved this problem with reasonable accuracy using the color phase-shifting method.

CONCLUSION
As a first stage, the method to automatically evaluate the principal-stress directions or the isoclinic parameter from photoelastic fringes obtained employing a mixed or semicircular polariscope has been demonstrated with the problem of the circular disk subjected to a vertical compressive load through the disk center. Based on the attempt in developing the simultaneous acquisition system that can be applied to the dynamic problem, the developed method expresses itself the ability to calculate the principal-stress directions over the entire domain of the circular disk model.

As obviously seen in Fig. 9(a), the influence of δ clearly isolated or divided the whole isoclinic-angle map into several discontinuous regions even though Fig. 9(b); therefore, in the context of the continuous isoclinic-angle values, smoothing technique is necessary. Furthermore, for the completely physical meaning of the isoclinic-angle map, the extension of the phase interval of the isoclinic-angler values from −π/4 to +π/4 to −π/2 to +π/2 is preferable. With this, the use of phase unwrapping method [22] recently proposed could extend the analysis to the problem having the isotropic point(s) or singular point(s). Therefore, future work is based on these aspects.

REFERENCES


