Noninvasive Evaluation of Fractured Bone
Using a Speckle Interferometry

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Abstract
This paper proposes a noninvasive method for evaluating the state of a fractured bone using time-series analysis based on speckle interferometry. The fine speckle fringes generated at the fractured section are detected using a gradient-based method and a new evaluation index is contributed for monitoring the state of the fractured bone. A performance test using a test phantom and numerical analysis using a particle method reveal that there is a detectable difference in the proposed evaluation index between cases with and without a cutting plane in an internal object.

Key words
Speckle Interferometry, Noninvasive Evaluation, Fractured Bone, Gradient-based Method, Particle Method

1. Introduction
In general, patients expected to be diagnosed by an expert doctor who has considerable experience. In order to compensate for the shortage of doctors, automatic diagnosis systems are required that provide diagnosis equivalent to that of an experienced doctor. One example of a task that could potentially be performed by such a system is the assessment of the state of healing of a fractured bone. When doctors perform open-heart surgery, they cut the sternum, which is the bone that protects the heart. It is difficult to evaluate the state of healing of an artificial fracture at the sternum from surgery using the conventional noninvasive methods [1-3]. The shape of the damaged bone can be checked from X-ray radiographs; however, this is not the suitable for quantitatively evaluating the state of healing. Nomura et al. proposed an impulse response method for evaluating the state of healing of such a bone [4]. In this method, an impulsive force is applied to one end of the bone, and the oscillatory waveform is monitored at the other end. The state of healing of the fractured bone is based on the resonance frequency determined by frequency analysis of the oscillatory wave. However, this method is designed to assess appendicular bones rather than bones in the trunk, such as the sternum.

The authors aim to develop a simple and noninvasive technique for evaluating the state of healing of a fractured bone. The authors evaluate the state of healing of a fractured bone using the out-of-plane displacement of the skin surface generated by weak oscillations inside the human body, such as the heartbeat. This paper introduces a new estimation index to evaluate the state of internal fractured part and it also confirms the effectiveness of the index by performance test using a test phantom and numerical analysis based on a particle method.

2. Measurement Procedure
2.1 Speckle interferometry
In general, if a coherent laser beam is diffused by the coarse surface of an object, the phase of the diffused light is randomly changed resulting in speckle patterns that are generated by the interference of light diffused from different points on the surface. These speckle patterns contain a lot of information about the object, including its displacement, and strain, and vibrations in the object. These parameters can be measured by applying optical information processing techniques to observed speckle patterns.

The out-of-plane displacement of a target object can be measured using speckle interferometry [5, 6]. Figure 1 shows a Twyman-Green type interferometer. A half-silvered mirror splits the light from a laser into two beams. These two beams illuminate the target object and a reference board. The speckle patterns on the object and the reference board are overlapped so that they interfere with each other as a result of the phase differences between them. If the target object moves in the out-of-plane direction, the light intensity of speckle pattern will change in accordance with the amount of the out-of-plane displacement.

In this paper, the authors assume that the out-of-plane displacement of an object’s surface depends on the internal condition of the object (e.g., a fractured bone in a human body) and the authors employ speckle interferometry to non-invasively evaluate the state of healing of a fractured bone.

Figure 2 shows two speckle patterns captured using a speckle interferometer at two different times (t and t+Δt) separated by a short time interval, and the resulting fringes for a thin metal plate. These fringes in Fig.2 (c) are obtained by taking the absolute difference between the speckle patterns in Figs.2 (a) and (b). The fringes in the difference image are effectively a contour map of the out-of-plane displacement. In the case shown in Fig.2, the thin metal plate is fixed at its lower end while the other end can move freely and oscillates slightly in the out-of-plane direction.
Thus, the out-of-plane displacement of the plate varies linearly with the vertical position.

Let us consider the situation in which the bone is simply fractured so that the skin surface is displaced in the direction perpendicular to the surface (as shown in the lower figure of Fig.3). In regions far from the fracture, the out-of-plane displacement does not vary spatially, whereas it changes steeply in the region near the fracture. Therefore finer fringes appear in the difference image (see Fig. 2 (c)), from regions near the fracture than in that from regions far from the fracture (see in the upper figure in Fig.3).

2.2 Gradient-based method

Finer fringes observed around the internal fractured part as shown in Section 2.1 rapidly shift on the difference image, and the finer the more rapid. Hence it is expected that the position of the fractured part is detected by finding fast-moving area in speckle fringe pattern. This paper employs a gradient-based method \[7, 8\] for evaluating moving-speed of finer fringes. Optical flows obtained using the gradient-based method are the in-plane shift of the grayscale distribution in a time-series of images. In the gradient-based method, the grayscale distribution \(f\) is assumed to shift while preserving the initial form of the distribution. For this case, we have

\[
f(x, y, t) = f(x + \Delta x, y + \Delta y, t + \Delta t),
\]

where \(f\) is an arbitrary function for the grayscale value, \(x\) and \(y\) are the spatial coordinates, and \(t\) is the time. Applying a Taylor expansion to the RHS of Eq. (1) and then neglecting higher -order terms, the following equation is obtained

\[
\frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial t} = 0.
\]

Equation (2) is the basic expression for the gradient-based method. Since the optical flows have two components in vector notation, \((u, v) = \left(\frac{dx}{dt}, \frac{dy}{dt}\right)\), the unknowns of the two components are determined by assuming that the displacement components are constant in a small area around the point of interest. If the residual in Eq. (2) is defined in the small area as

\[
E_r = \sum_{m} \left( \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial t} \right)^2,
\]

where \(M_a\) is the number of calculation points, the unknowns can be obtained by the least-square method in such a way that the residual \(E_r\) is minimized. The calculation points are arranged at five-pixel intervals in the speckle image and the small area is taken to be a 7×7pixel square centered on each calculation point.

2.3 Evaluation index

This section describes the proposed index for evaluating the state of healing of a fractured bone. These finer fringes can be well detected using the gradient-based method described in Section 2.2. First, the present method described above was applied to artificial fringe images without noise to check the feasibility of the method in detecting fast-moving fringes. Figure 4 shows the correlation between the fringe size and optical flows. Figures 4 (a) and (b) show a pair of difference images numerically generated by translating two-dimensional Gaussian distributions in the \(x\)-direction. There are finer fringes near the central regions of these images and the marked displacement vectors are also near the central regions. Hence, the position of fracture can be accurately determined by evaluating the finer fringes using the gradient-based method.

The authors have employed the following index for detecting the position of the fracture from speckle images:

\[
H = \sum_{m_i=1}^{M} \frac{1}{2} \left( \frac{dx}{dt} \right)_{m_i}^2 + \left( \frac{dy}{dt} \right)_{m_i}^2,
\]
where \( M_b \) is the number of measurement points in the target area. \( H \) is the integrated value of the positive displacements in the \( x \) direction in the target area since there is a cutting plane in the bone along the \( y \)-direction. The reason why only positive displacement is used in the definition of \( H \) is that the index \( H \) is not affected by noisy and disturbed wave reflected at an end of target. Figure 5 depicts the method used to search for the location of a fracture in a bone. The evaluation index \( H \) is computed by shifting the target area to find the position where the index takes the maximal peak. As the width of the area is larger, the effect of smoothing is too strong to detect characteristic signals in \( H \). In this study width of the target area sets at 10pixel, corresponding to approximately 1.0mm. In order to compute \( dx/dt \) in Eq. (4), three derivative terms \( (\partial f/\partial x, \partial f/\partial y, \partial f/\partial t) \) must be calculated from the gray level distributions of a time-series of difference images. In this paper, the temporal derivative term is discretized using the backward or central difference approximation and the spatial derivative terms are calculated at the central point.

3. Experiments

3.1 Experimental setup

The experimental setup is shown in Fig. 6. The light emitted from a solid-state laser (YVO\(_4\), L) is expanded by a spatial filter (SF). The laser wavelength and power are 532nm and 20mW, respectively. Optical system is a Twyman-Green type interferometer. A test phantom (TP) has a three-layer structure, in which an acrylic plate is sandwiched between two urethane gel sheets as shown in Fig.7. An acrylic plate (150×60×20mm) is used as a material for a bone part. Its bending strength is 1.50×10\(^8\) Pa, while that for real bone is 1.17×10\(^8\)Pa. A urethane gel sheet is used as a material which simulates subcutaneous structure, such as skin and muscle. Gel with a tension strength of 3.40×10\(^5\)Pa is formed into the shape of a sheet. In order to simulate a fracture, the acrylic plate has a cutting plane in its center. Rice flour is applied to the gel sheet surface in order to obtain good speckle patterns.
is set at the position of 45mm from the left end of the test phantom as shown in Fig.7 (a), and it is given a small impact by a magnetic actuator (A). The thrust force and stroke of the actuator are 0.83N at 6V and 5mm, respectively. The distance between the actuator and the impact rod is 4mm. The test phantom is observed using a high-speed camera (C) from the vertical direction. Frame rate is 3900frame/sec. Each frame contains 340×160pixel corresponding to approximately 34×16mm (see Fig.7 (a)). The cutting plane is located 160 pixels form the left side of the image. The actuator is controlled by a pulse generated by a function generator. This pulse is used as a trigger signal.

3.2 Results

Figures 8 (a) and (b) are examples of difference images obtained by the speckle interferometry and displacement vectors obtained by the gradient-based method for objects without and with a cutting plane, respectively. In the case of the object without a cutting plane, the displacement vectors are short, (see in Fig.8 (a)). By contrast, the difference image in Fig.8 (b) has a fine, dark fringe parallel to the cutting plane and large displacement vectors are generated along this dark fringe.

![Fig.8 Comparison of displacement vectors for objects with and without a cutting plane in an internal acrylic plate](image)

(a) Without cutting plane

(b) With cutting plane

Figure 9 compares the evaluation index of objects with and without a cutting plane. It depicts the time-evolution in different target areas. Target area number represents the position of target area in x-coordinate, as show in Fig.5, and frame number does the number of time steps in time-series analysis. For the case with a cutting plane, it can be seen from Fig.9(b), that there occurs some characteristic peaks on the evaluation index H around the region in frame number 50-100 and target area 20-35, while there is no characteristic peak for the case without a cutting plane, as shown in Fig.9(a).

![Fig.9 Detection of cutting position in an acrylic plate using the proposed evaluation index](image)

4. Numerical Analysis

In a previous paper [9], the authors performed the numerical analysis of an elastic body using a particle method for the purpose of investigating the performance of the above-mentioned method and to simply examine the influence of modeling, parameters of elastic body on the vibration state of its surface. The particle method was first proposed by Koshizuka [10-13].

A three-layer elastic body corresponding to the test phantom used in the experiment is expressed as shown in Fig. 10 using a particle method. The dimensions of the target object are 150×30mm. 60×12 particles are uniformly arranged at intervals of 2.5mm for the initial particle positions. The dimension of the impact rod is 10×10mm corresponding 4×4 particles. The impact rod is 0.25mm away from the target object at initial time, and it impacts the target object at the velocity of 250mm/sec. In this study, a computational time step is 1.0×10^{-7}sec and total number
of the time steps is $2.0 \times 10^4$. Table 1 shows the material parameters of the target object and the impact rod.

Table 1 Modeling parameters

<table>
<thead>
<tr>
<th></th>
<th>Young’s modulus</th>
<th>Poisson’s ratio</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>External layer</td>
<td>$1.0 \times 10^6$[Pa]</td>
<td>0.46</td>
<td>$1.0 \times 10^3$[kg/m$^3$]</td>
</tr>
<tr>
<td>Internal layer</td>
<td>$3.0 \times 10^9$[Pa]</td>
<td>0.36</td>
<td>$1.2 \times 10^3$[kg/m$^3$]</td>
</tr>
<tr>
<td>Impact rod</td>
<td>$3.0 \times 10^9$[Pa]</td>
<td>0.36</td>
<td>$1.2 \times 10^3$[kg/m$^3$]</td>
</tr>
</tbody>
</table>

The target objects are assumed to be elastic objects with and without a cutting plane at the center of the middle layer. If the particles are separated into two groups by the cutting plane, the interaction between the two groups can be neglected. The temporal gradient of the out-of-plane displacement is given by:

$$b_z(t, x) = \frac{a_z(t, x) - a_z(t - \Delta t, x)}{\Delta t},$$  \hspace{1cm} (5)

where $a_z$ is the out-of-plane displacement on the surface of the elastic body. The spatial gradient of $b_z$ is given by:

$$c_z(t, x + \Delta x) = \frac{b_z(t, x + \Delta x) - b_z(t, x)}{\Delta x}. \hspace{1cm} (6)$$

Hence, the variance of $c_z$ can be calculated using

$$\sigma^2 = \frac{1}{N_t} \sum_{n=0}^{N_t} (c_z - \bar{c}_z)^2, \hspace{1cm} (7)$$

where $N_t$ is the total number of computational time steps and $\bar{c}_z$ is the average of $c_z$. The vertical and horizontal axes in Fig.11 indicate the variance $\sigma^2$ and the particle position, respectively. The variance, defined in Eq. (7), will differ for a three-layer elastic body with and without a cutting plane. The spatial gradient of the out-of-plane displacement on the surface of the elastic body will change if the variance $\sigma^2$ increases.

From Fig.11, it is found that the variance has a peak at the position of particle, $x=75$mm, corresponding to the give position of a cutting plane, for the case with a cut in the other hand, there is no peak on the distribution of the variance for the case without a cut, as shown in the experimental results in Section 3.2. Thus, it can be said that the experimental results and numerical analysis results qualitatively agree with each other.

5. Concluding Remarks

This paper proposed a noninvasively measurement method for evaluating the state of a fractured bone using a time-series analysis based on speckle interferometry. The out-of-plane displacement of a target object was obtained by using speckle interferometry. The fringes observed in speckle interferometry were evaluated by using a gradient-based method. The performance test using a test phantom demonstrated that there was a detectable difference in the proposed evaluation index $H$ between objects with and without a cutting plane. The authors conducted the numerical analysis of an elastic body using a particle method. There was a difference in the variance $\sigma^2$ for three-layer elastic bodies with and without a cutting plane. The numerical results were in qualitative agreement with the experimental results.

Nomenclature

- $f$: an arbitrary function for gray level
- $M_a$: the number of calculation points
- $M_b$: the number of measurement points in the target area
- $H$: evaluation index, pixel/sec
- $a_z$: out-of-plane displacement on the surface of an elastic body, mm
- $b_z$: temporal gradient of $a_z$, mm/sec
- $c_z$: spatial gradient of $b_z$, 1/sec
- $\sigma^2$: variance of $c_z$, (1/sec)$^2$
- $N_t$: total number of computational time steps

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References


