Accuracy of Whole-Field Measurement of Photoelasticity Considering Variation of Incident Light Intensity and Background Intensity

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The whole-field measurement of photoelasticity using linearly polarized light is performed from a method measuring the phase of the object wave by rotating of a quarter-wave plate and a linear polarizer. The paper studies the accuracy of the method by considering the variation of the incident light intensity and the background intensity. The magnitude of variation of the light intensities is expressed by using random numbers. The simulation for the relative phase retardation $\rho$ and the directions of principal stresses $\alpha$ in the circular disk under diametral compression load is carried out. In the results of simulation using the intensity variation with random numbers, $\rho$ and $\alpha$ are found to be accurate by using this measuring method.

Key words: Automated Photoelasticity, Whole-Field Measurement, Phase Stepping, Noise

1. Introduction

Photoelasticity is one of the most widely used optical methods for stress analysis. It can measure the directions of principal stresses and the difference of principal stresses, which is related to the relative phase retardation, in a model as visual patterns. However, the traditional stress analysis procedures are tedious and time consuming. Therefore, many authors have investigated automated measurement systems of the photoelasticity to carry out more accurate analysis. In the automatic measurements using a computer combined with a TV camera, the phase stepping method has been used widely for whole-field stress analysis. The method is based on changing the absolute phase of the object wave and measuring the light intensity. It is, in general, achieved by rotating a quarter-wave plate and a linear polarizer.

The accuracy of some of the phase stepping methods was surveyed for a comparative study of the light intensities recorded.

Umezaki et al. have studied with the numerical simulation the accuracy of the phase stepping method by the plane polariscope only. It was investigated for the magnitude of noise and set of noise signs in which the noise caused by the variation in the incident light intensity and the background intensity. The results showed that the phase stepping method was very sensitive to the noise.

The author presented the JCAQP (judicious choice of azimuthal settings of a quarter-wave plate and a linear polarizer) method to obtain the relative phase retardation and the directions of principal stresses by using incident linearly polarized light, which is a type of the phase stepping method.

This paper presents a study for the accuracy of the JCAQP method using incident linearly polarized light. It investigates the influence of errors of the incident light intensity and the background intensity. The magnitude of these noises is expressed from a practical point of view by using random numbers.

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2. JCAQP Method Using Linearly Polarized Light

The optical arrangement is shown in Figure 1. The incident linearly polarized light of azimuth $\alpha_1$ at a wavelength $\lambda$, impinges a model and passes through a quarter-wave plate $Q_2$ and a linear polarizer $P_2$. The light intensity $I(\alpha_1, \beta_2, \alpha_2)$ emerging from $P_2$ can be represented as eq (1) by the following azimuth settings (azimuth values in degrees) of $Q_2$ and $P_2$:

$$I(\alpha_1, \beta_2, \alpha_2) = \frac{1}{2} I_{\alpha_1} \left[ (1 + \sin \delta \rho) \cos 2(\beta_2 - \alpha_2) \cos 2(\alpha_2 - \alpha_1) + \sin 2(\alpha_2 - \alpha_1) \cos 2(\beta_2 - \alpha_2) \sin 2(\alpha_2 - \alpha_1) \right] + I_N$$

where,

- $\alpha_1$: azimuth of the incident linearly polarized light,
- $\beta_2$ and $\alpha_2$: azimuth of the fast axis of $Q_2$ and azimuth of the transmission axis of $P_2$,
- $\delta \rho$: phase difference error of $Q_2$ to $\lambda$,
- $I_{\alpha_1}$: incident linearly polarized light intensity at azimuth $\alpha_1$,
- $I_N$: background intensity.

![Fig.1 Schematic diagram of an automated whole-field measurement system using linearly polarized light: $P_1$ and $P_2$, linear polarizers; $Q_1$ and $Q_2$, quarter-wave plates; D, diffuser](image-url)
The light intensity $I(\alpha_1, \beta_2, \alpha_2)$ is taken at the 6 kinds of azimuth settings of $Q_2$ and $P_2$ by using the incident linearly polarized light of azimuths $0^\circ$ and $45^\circ$. Then, the relative phase retardation $\psi$ and the principal stress directions $\psi$ at $\lambda_i$ are obtained as follows:

$$\psi = 0.5\tan^{-1}\left\{\frac{I(0,0,0) - I(0,90,90)}{I(45,45,45) - I(45,135,135)}\right\}^{0.5}$$

$$\psi = 0.5\tan^{-1}\left\{\frac{\sin^2\left(\frac{\psi}{2}\right) \sin^2\psi}{\sin^2\left(\frac{\psi}{2}\right) \cos^2\psi}\right\}^{0.5}$$

where,

$$I(\alpha_1,0,0) = (I(\alpha_1,0,0) - I(\alpha_1,90,90))^2 + (I(\alpha_1,45,45) - I(\alpha_1,135,135))^2$$

The relative phase retardation and the principal stress directions in the circular disk are given by eqs (8),(9),

$$\psi = (\alpha)(\sigma_x + \sigma_y)^2 + 4\tau_{xy}^2)^{0.5}$$

$$\psi = 0.5\tan^{-1}\left\{\frac{2\tau_{xy}}{(\sigma_x + \sigma_y)}\right\}^{0.5}$$

where $\alpha$ is the photoelastic sensitivity of the model at $\lambda_i$. The common available CCD cameras have 512×512 pixels and 0-255 gray level (8 bits). The values of the incident light intensity $I_{in}$ and background intensity $I_{bn}$ are decided at 140 and 40 in consideration of values recorded from the CCD camera, respectively. The intensity variation is caused by the intensity variation of a light source, rotating the optical components with dust, the dust in the room and other factors. In the intensity variation of light source in actual cases, it is ordinarily under ±3% over 20 hours and about 10% in other light sources. The variation of intensities from the image is given at 0±2%, 0±5%, 0±10%, 0±20%. These intensities variation is considered to arise in random and these treatments were calculated by using uniformly distributed random numbers. The $\psi$ and $\psi$ from the variation of $I_{in}$ and $I_{bn}$ are investigated along the horizontal line at $y=3R/4$ in the circular disk. This position is in a general state of stress not a specific stress.

When the variation of the intensities does not arise, the distribution of the intensities $I(0,0,0)_{th}$ and $I(0,90,90)_{th}$ along the position are shown in Figure 3(a),(b). Also the theoretical values $\psi_{th}$ and $\psi_{th}$ in this method are shown in Fig. 4(a),(b).

Table 1 The values of the circular disk model for the numerical simulation

<table>
<thead>
<tr>
<th>Load</th>
<th>P 127.5 N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>2R</td>
</tr>
<tr>
<td>Thickness</td>
<td>6.0 mm</td>
</tr>
<tr>
<td>Incident light intensity $I_{in}$</td>
<td>160</td>
</tr>
<tr>
<td>Background intensity $I_{bn}$</td>
<td>40</td>
</tr>
</tbody>
</table>

4. Numerical Simulation

The values of the circular disk model for the numerical simulation are indicated in Table 1. The diameter (40.0 mm) of the circular disk for the simulation corresponds to 240 pixels. The values of the circular disk model for the numerical simulation are indicated in Table 1. The diameter (40.0 mm) of the circular disk for the simulation corresponds to 240 pixels.
Fig. 4 (a) and (b) show the theoretical values $\rho_m$ and $\psi_m$ for $\rho$ and $\psi$ along the horizontal line at $y=3R/4$ in the circular disk, respectively.

Fig. 5 (a) and (b) show the distributions for the light intensities $I(0,0,0)$ and $I(0,90,90)$ from the variation of intensities ($\Delta I_{a1}$, $\Delta I_{b1}$) at (0±10)%, respectively.

4.1 Results and Discussions

The simulation for $\rho$ and $\psi$ is performed with the incident light intensity variation $\Delta a_1$, the background intensity variation $\Delta b_1$, and the incident light intensity and background intensity variations ($\Delta a_1$, $\Delta b_1$). Figures 5(a), (b) show the distributions for the light intensities $I(0,0,0)$ and $I(0,90,90)$ from the variation of intensities ($\Delta a_1$, $\Delta b_1$) at (0±10)%. Comparing Fig. 5(a), (b) with Fig. 3(a), (b), the intensities $I(0,0,0)$ and $I(0,90,90)$ show a remarkable variation due to uniformly distributed random numbers, and the other intensity $I_{a_1}$, $I_{b_1}$ changes similarly.

The results for the simulated $\rho$ and the error ($\rho - \rho_{th}$) from the intensity variation at (0±10)% are shown in Figs. 6, 7. Figures 6(a), (b), (c) and 7(a), (b), (c) show the distributions for $\Delta a_1$, $\Delta b_1$.  

Fig. 6 Simulated $\rho$ for the intensity variation at (0±10)%. (a), (b), and (c) show the distributions for $\Delta a_1$, $\Delta b_1$, and ($\Delta a_1$, $\Delta b_1$) only, respectively.

Fig. 7 Error ($\rho - \rho_{th}$) for the intensity variation at (0±10%). (a), (b), and (c) show the distributions for $\Delta a_1$, $\Delta b_1$, and ($\Delta a_1$, $\Delta b_1$) only, respectively.

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and \((\Delta I_{a1}, \Delta I_{b1})\) only, respectively. Similarly, the distributions for the simulated \(\psi\) and the error \((\psi - \psi_{th})\) at \((0 \pm 10)\%\) of the intensity variation are shown in Figs. 8(a),(b),(c) and 9(a),(b),(c).

In the calculation for \(\rho_{i}\) in eq (2) due to the light variations, \(\rho_{i} = \pi\) when \([[(0,0,0) - (0,0,90)] / I_{0} + [(45,45,45) - (45,135,135)] / I_{45}] < 0\), and \(\rho_{i} = 0\) when \([[(0,0,0) - (0,0,90)] / I_{0} + [(45,45,45) - (45,135,135)] / I_{45}] > 1\), then, for \(\psi\) in eq (3), \(\psi = \pi/4\) when \([[(45,45,45) - (45,135,135)] / I_{45}] = 0\), and \(\psi = 0\) when \([[(0,0,0) - (0,0,90)] / I_{0} + [(45,45,45) - (45,135,135)] / I_{45}] < 0\).

Therefore, the values of the error \((\rho - \rho_{th})\) tend to be the negative values for the vicinity of \(\rho_{i} = \pi\) and the positive values for \(\rho_{i} = 0\). On the other hand, the values of the error \((\psi - \psi_{th})\) tend to be the negative values for the vicinity of \(\psi = \pi/4\) and \(\rho_{i} = 0\).

These effects that the mean value \(m\) for the error does not become zero are seen in Figs. 6-9 and Table 2. However, the accuracy of the measurement for \(\rho_{i}\) and \(\psi\) is considered fully from the simulation results. And, the standard deviation \(\sigma\) for the error is more important than the mean value \(m\).

The \(\sigma\) and the \(m\) for the error of the simulated \(\rho_{i}\) and \(\psi\) were calculated to treat the amount of these errors. The simulation results for \(\rho_{i}\) and \(\psi\) are denoted in Table 2, in which the standard deviation \(S\) and the mean value \(M\) show the mean values

Table 2 The simulation results for \(\rho_{i}\) and \(\psi\). Standard deviation \(S(\%)\) and mean value \(M(\%)\) of error \((\rho - \rho_{th})\) and error \((\psi - \psi_{th})\) for intensity variation \((0 \pm 2)\%, (0 \pm 5)\%, (0 \pm 10)\%\) and \((0 \pm 20)\%\) are denoted. Error \((\psi - \psi_{th})\) shows values calculated for the whole positions of \(\rho_{i} = 2\pi N_{1} + \{(0 - 360) / 180\} \pi\) to the positions of \(\rho_{i} = 2\pi N_{1} + \{(90 - 270) / 180\} \pi\).

<table>
<thead>
<tr>
<th>(\rho_{i} - \rho_{th})</th>
<th>((0 \pm 2)%)</th>
<th>((0 \pm 5)%)</th>
<th>((0 \pm 10)%)</th>
<th>((0 \pm 20)%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\Delta I_{a1}, \Delta I_{b1}))</td>
<td>(\Delta I_{a1})</td>
<td>(\Delta I_{b1})</td>
<td>(\Delta I_{a1} + \Delta I_{b1})</td>
<td>(\Delta I_{a1} + \Delta I_{b1})</td>
</tr>
<tr>
<td>(\rho = 2\pi N_{1} + {(0 - 360) / 180} \pi)</td>
<td>(M = 0.3)</td>
<td>(0.3)</td>
<td>(0.3)</td>
<td>(0.3)</td>
</tr>
<tr>
<td>(\rho = 2\pi N_{1} + {(0 - 359) / 180} \pi)</td>
<td>(M = 0.3)</td>
<td>(0.3)</td>
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<td>(0.3)</td>
</tr>
<tr>
<td>(\rho = 2\pi N_{1} + {(0 - 358) / 180} \pi)</td>
<td>(M = 0.3)</td>
<td>(0.3)</td>
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</tr>
<tr>
<td>(\rho = 2\pi N_{1} + {(0 - 357) / 180} \pi)</td>
<td>(M = 0.3)</td>
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<td>(M = 0.3)</td>
<td>(0.3)</td>
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<td>(0.3)</td>
</tr>
</tbody>
</table>

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(S=Σσ/n, M=Σm/n, here n=7) calculated from seven different series of random numbers. Since ψ is undefined at the positions of ρ=2πN in the simulation results the ψ is more out of the theoretical value in the vicinity of position ρ=2πN as it is clear from the Figs. 4-9. The S and M of the error (ψ-ψth) in Table 2 were also calculated at the positions except the positions of ρ=2πN.

In the simulation results, large values of the error (ρ-ρth) are seen in the vicinity of ρ=3√, and the S of the error is affected on Δμ at more than Δσ. The S of the error from Δμ at is about 1.8 times the S of Δσ in intensity variation at (0±5)%. The error of ρ in the intensity variation (Δμ, Δσ) at (0±5)° is 2.4° at S and 0.1° at M. In the results of simulation considering the intensity variation with random numbers, ρ and ψ are found to be accurate by using the JCAQP method. And ψ is able to obtain more accuracy than ρ, when ψ is measured on the positions of ρ away from the positions of ρ=2πN.

5. Conclusions

The accuracy of the measurement of JCAQP method was studied, in which the relative phase retardation and the principal stress direction in a model are measured by judicious choice of azimuthal settings of a quarter-wave plate and a linear polarizer. The simulation for ψ and μ was performed along the horizontal line at y=3R/4 in the circular disk under diametral compression load.

In the simulation results from the JCAQP method, the S of the error (ρ-ρth) is affected on Δμ at more than Δσ. The S of the error from Δμ at is about 1.8 times the S of Δσ in intensity variation at (0±5)%. The error of ρ in the light variation (Δμ, Δσ) at (0±5)% is 2.4° at S and 0.1° at M.

The S of the error (ψ-ψth) from Δμ is also larger than the S of Δσ and the error for the intensity variation (Δμ, Δσ) is influenced almost by Δσ. The accuracy of the measurement of μ is more out of the theoretical value, even when the position of ρ=2πN is removed. ψ should be measured on the position of ρ further away from the position of ρ=2πN as the intensity variation is larger. In the intensity variation (Δμ, Δσ) at (0±5)%, the error of ψ is S=0.4° and M=0.1° for the positions of ρ=2πN+(70-290)/180°π. In the results of simulation considering the intensity variation with random numbers, ρ and ψ are found to be accurate by using the JCAQP method. And ψ is able to obtain more accuracy than ρ, when ψ is measured on the positions of ρ away from the positions of ρ=2πN.

References