Application of an Equation Representing Reciprocal Fracture Strength versus Square Root of Fracture Origin Size to the Estimation of Fracture Toughness of Hard or Brittle Materials

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SYNOPSIS
In the past study on the transverse-rupture strength ($\sigma_0$) of WC-10mass%Co hardmetal, the authors had derived an equation of $\sigma_0^{-1} = \sigma_0^{-1} + K a^{1/2}$ ($\sigma_0$ the external stress which operated on the fracture origin at the moment of fracture, $\sigma_0$; the intrinsic fracture strength, $K$; $2\sigma_0^{-1} \rho^{-1/2}$, $\rho$; the effective tip radius of microstructural defect which acted as the fracture origin, $a$; the half of the major-axis length of the fracture origin), and also had clarified that this equation holds on various kinds of hard materials. Furthermore, the slope value ($K$) of the $\sigma_0^{-1} - a^{1/2}$ regression line was suggested to have a correlation with the fracture toughness ($K_Ic$) in a similar way as the slope of $\sigma_0 - S_m^{1/2}$ regression line ($S_m$; total macroscopic fracture surface area of fragments). In this study, this suggestion was investigated on four kinds of hard or brittle materials such as high speed steel, WC-10mass%Co hardmetal, Si$_3$N$_4$ ceramics and Mn-Zn ferrite. It was found that the $K$ has surely a strong correlation with $K_Ic$, and $K_Ic$ can be expressed by an unified equation of $K_{IC} \approx 0.7K$.

KEY WORDS
fracture origin, fracture toughness, estimation, transverse-rupture strength, hard materials

1 Introduction
In the study on the fracture stress ($\sigma$) of various kinds of glass and ceramics, it has been reported that the equation of $\sigma r^{-1/2} = A$ ($r$ is the fracture mirror radius and $A$ is called "mirror constant") holds and the relation between $A$ and the fracture toughness ($K_Ic$) can be expressed by the equation of $A = YK_{IC}(2c/r) - 1/2$. Where, $Y$ is a shape factor and $c$ is the critical crack length, respectively.

However, in the present authors' previous studies on the transverse-rupture strength (expressed by $\sigma_m$ instead of $\sigma$) of WC-10mass%Co hardmetal, $\sigma_m$ had a more close correlation with the size ($2a$) of fracture origin than $2r$. Here, $\sigma_m$ is the external stress which operated on the fracture origin at the moment of fracture. Then, the authors have theoretically derived the following equation expressing the relation between $\sigma_m$ and $a$, referring to the derivation process of Griffith-Orowan equation.

$$\sigma_m^{-1} = \sigma_0^{-1} + 2\sigma_0^{-1} \rho^{-1/2} a^{1/2}$$

(1)

Where, $\sigma_0$ is the intrinsic fracture strength, i.e., the $\sigma_0$ at $a=0$ where $a$ is the half of major-axis length of the fracture origin, and $\rho$ is the effective tip radius of the fracture origin. The $\sigma_0$ is generally smaller than $\sigma_m$ due to the followings: (1) the external stress in three-point bending test piece is not uniform, and (2) the location of the fracture origin is generally apart from both the tensile surface and span center of the test piece, the tensile stress at which is $\sigma_m$. The fracture origin in the hardmetal is usually one microstructural defect such as pore and coarse WC grain, while it is a sharp crack in Griffith-Orowan equation. In the derivation of the equation (1), the factor "1" in the stress concentration factor of $1 + 2(a/\rho)^{1/2}$ at the tip of microstructural defect was not omitted, differing from in the derivation of Griffith-Orowan equation. Because, the value of $2(a/\rho)^{1/2}$ in the stress concentration factor was not so much larger than "1" in the range of the observed value of $a$, due to the extremely larger value (about 10 $\mu$m) of $\rho$ in the case of the microstructural defect differing from that (atomic-order size) of the sharp crack. Actually, equation (1) have been clarified to fit very well the experimental relation between $\sigma_m^{-1}$ and $a^{1/2}$ on various hard materials such as TiC and Ti(C,N) base cerments, Si$_3$N$_4$ ceramics and Mn-Zn ferrite as well as WC-Co base hardmetals with various contents of Co, TiC and TaC: the extrapolated value of the $\sigma_m^{-1} - a^{1/2}$ regression line to the vertical axis of $\sigma_m^{-1}$, is not zero but a positive value (this corresponds to
When the above equation is expressed in the form of
\[ \sigma_{\theta}^{-1} = \sigma_0^{-1} - K a^{1/2} \] (K; 2\( \sigma_0^{-1} \rho^{-1/2} \)), the slope of the \( \sigma_\theta^{-1} - a^{1/2} \) regression line which is experimentally obtained corresponds to K in the equation. The K value of WC-Co hardmetal was found to decrease with increasing Co content (\( K_{IC} \) increases with increasing Co content)\(^8\), and the value of WC-10mass%Co hardmetal with high fracture toughness (\( K_{IC} \)) (13 MPa•m\(^{1/2} \)) is much smaller than Mn-Zn ferrite with low \( K_{IC} \) (1.3 MPa•m\(^{1/2} \)). Namely, the K value decreases with increasing \( K_{IC} \). Furthermore, the unit of K is \( 1/(\text{MPa} \cdot \text{m}^{1/2}) \) which is the inverse of the unit of \( K_{IC} \), i.e., MPa•m\(^{1/2} \). These suggest that the inverse value of K has a strong correlation with \( K_{IC} \). This suggestion is also supported by the modified equation of \( K_{IC} \) i.e., \( Y^{-1} \sigma_\theta^{-1} = K_{IC}^{-1} a^{1/2} \), which indicates that a linear correlation exists between \( Y^{-1} \sigma_\theta^{-1} \) and \( a^{1/2} \) in the same way as equation (1) \( (Y^{-1} \sigma_\theta^{-1} \) corresponds nearly to \( \sigma_\theta^{-1} \) in the equation of \( \sigma_\theta^{-1} = \sigma_0^{-1} + K a^{1/2} \).

If the above suggestion is correct, the \( K_{IC} \) of a material can be estimated from the K value of the \( \sigma_\theta^{-1} - a^{1/2} \) regression line. Namely, a new estimation method of \( K_{IC} \) can be developed, following the authors' previously-proposed new method\(^8\); \( K_{IC} \) is estimated from the experimental relation between the \( \sigma_\theta \) and the root of the total macroscopic fracture surface area \( S_{m} \) of fragments of test piece, which is based on the semi-theoretical equation of \( \sigma_\theta = \psi K_{IC} S_{m}^{1/2} \) (\( \psi \); shape factor). Additionally, the \( \sigma_\theta \) value relating to the attainable strength (\( \sigma_\theta \) value at 2a of the mean grain size) of the materials can also be estimated by extrapolating this \( \sigma_\theta^{-1} - a^{1/2} \) regression line to the \( \sigma_\theta^{-1} \) vertical-axis\(^3\).

In this study, it was investigated whether this suggestion is correct or not for four kinds of hard or brittle materials such as high speed steel, WC-10mass%Co hardmetal, Si\(_3\)N\(_4\) ceramics and Mn-Zn ferrite, whose \( K_{IC} \) values are in the range of 21 ~ 1.3 MPa•m\(^{1/2} \).

### 2 Specimens and Experimental Procedure

As the specimens, the following four kinds of hard or brittle materials were used: high speed steel (SKH2 defined in JIS G4403: abbreviated as HSS, hereafter), WC-10mass%Co hardmetal (WC-10Co), Si\(_3\)N\(_4\) ceramics (Si\(_3\)N\(_4\)) and Mn-Zn ferrite (Ferrite), which were used in the previous studies on the another new estimation method of \( K_{IC} \)\(^8\,11\,19\). The variation range of \( \sigma_{\theta} \), mean \( \sigma_{\theta} \), and \( K_{IC} \) value measured by SEPB method\(^8\,11\,19\) for these specimens were shown in Table 1. The size and number of test piece for these specimens were 4 x 8 x 25 mm\(^3\) and 7, 4 ~ 8 ~ 25 mm\(^3\) and 5, 3 x 4 x 36 mm\(^3\) and 18, and 4 x 8 x 36 mm\(^3\) and 7, respectively. The span (l) in the transverse-rupture test was 20 mm for HSS, WC-10Co and Ferrite, according to CIS 026 for hardmetal, and 30 mm for Si\(_3\)N\(_4\), according to JIS R1601 for ceramics, respectively.

The region near the center of the white spot\(^2,3\) (which is the same as the fracture mirror\(^1\) in the fracture surfaces of fragments were observed with SEM for each test piece to investigate the kind, major-axis length (2a) and location of the fracture origin. Here, the location was expressed as the distances (\( \Delta t \), \( \Delta l \)) from the tensile surface and span center. The external stress (\( \sigma_{\theta} \)) which operated on the fracture origin at the moment of fracture for each test piece was calculated by use of the equation of \( \sigma_{\theta} = \sigma_{\theta \theta}(1-2\Delta l/t) \) (1 ~ 2\( \Delta l/l \)\(^2\)), where t is the thickness of test piece.

A linear relations was confirmed to hold between \( \sigma_{\theta}^{-1} \) and \( a^{1/2} \) for HSS in a similar way to the other three kinds of hard or brittle materials\(^2,3,7,9\), as described later. Then, the slope value (K) of regression line for each specimen was estimated for these four kinds of specimens. Next, the ratios of \( K^{-1} \) to \( K_{IC} \) i.e., \( K^{-1}/K_{IC} \) for each specimen was calculated and these ratios were compared with each other in order to know whether these ratios for four kinds of specimens are nearly equal to each other, i.e., whether \( K_{IC} \) can be expressed with an unified equation.

### Table 1

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Range of ( \sigma_{\theta} ) (GPa)</th>
<th>Mean ( \sigma_{\theta} ) (GPa)</th>
<th>Kinds of fracture origin</th>
<th>Range of 2a (( \mu m ))</th>
<th>Measured ( K_{IC} ) (SEPB) (MPa • m(^{1/2} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSS</td>
<td>3.3 ~ 3.8</td>
<td>3.6</td>
<td>Concave or convex region</td>
<td>58 ~ 125</td>
<td>21</td>
</tr>
<tr>
<td>WC-10Co</td>
<td>1.5 ~ 3.0</td>
<td>2.4</td>
<td>Pore</td>
<td>34 ~ 198</td>
<td>13</td>
</tr>
<tr>
<td>Si(_3)N(_4)</td>
<td>1.0 ~ 1.5</td>
<td>1.3</td>
<td>Agglomerate of additives (WC)</td>
<td>10 ~ 36</td>
<td>3.1</td>
</tr>
<tr>
<td>Ferrite</td>
<td>0.16 ~ 0.22</td>
<td>0.18</td>
<td>Agglomerate of large grains</td>
<td>80 ~ 240</td>
<td>1.3</td>
</tr>
</tbody>
</table>
3 Experimental Results and Discussion

The SEM observation on the region near the tensile surface of fracture surface with low and high magnifications revealed that the fracture of all HSS test pieces occurred from one region as the fracture origin in a similar manner as WC-10Co\(^9\), Si\(_3\)N\(_4\)\(^7\) and Ferrite\(^6\). An example of white spot (which is the same as fracture mirror) in HSS was shown in Fig. 1 (a). Two examples of fracture origin observed nearly at the center of the white spot of two test pieces with \(\sigma_u\) of 3.8 and 3.5 GPa are shown in (a') and (b), respectively. Each fracture origin is indicated with four arrows. The judgement that the region surrounded by the

![SEM micrographs of a white spot or fracture mirror (a) and fracture origins (a', b) for HSS specimen with \(\sigma_u\) of 3.8 and 3.5 GPa.](image)

The (a'), (b) and (b') show an agglomerate of coarse carbide at the boundary of the fracture origin, inside the fracture origin and in the compressive side of the test piece with \(\sigma_u\) of 3.5 GPa, respectively. The direction from the upper periphery of each photograph to the lower one is parallel to that of the compressive side to the tensile side of test piece. Pores (black regions) are observed in coarse carbide grains. (a): \(\times 50 \times 0.8\), (a') and (b): \(\times 500 \times 0.8\), (a''), (b') and (b''): \(\times 5000 \times 0.8\).
arrows is the fracture origin was based on the following two facts: (1) it is a concave or convex region, differing from the periphery, and (2) radial undulations, which are characteristics for the brittle crack propagation, start from its boundary. The kinds of fracture origin and the range of $2a$ for four kinds of specimens were shown in the above Table 1. They were concave or convex region and $58 \sim 125 \mu m$, pore and $34 \sim 198 \mu m$, agglomerate of additives (WC) and $10 \sim 36 \mu m$, and agglomerate of large grains and $80 \sim 240 \mu m$, respectively.

Figs. 1(a'), (b') and (b'') show an agglomerates of coarse carbide grains (probably MC-type, M of which are mainly Fe and W) for the reference of fractography. The (a') was at the boundary of the fracture origin in (a'), (b') is inside the fracture origin in (b), and (b'') is in the compressive side of test piece with 3.5 GPa, respectively. The fracture mode of coarse carbide grains in these three kinds of regions are commonly trans- or intergranular, which depended on their grain size, and there were no clear differences among three regions.

Fig. 2 (a) and (a') show each relation between $\sigma_r^{-1}$ and $a^{1/2}$ or $2a$ for four kinds of specimens. The (a') is a vertically elongated figure of a part in (a). It is clear that a linear relation holds between $\sigma_r^{-1}$ and $a^{1/2}$ for HSS in a similar way as for the other three kinds of specimens. The slope value ($K$) of the regression line and the measured $K_{IC}$ value for each specimen are described near each line in (a) or (a').

Table 2 shows each $K$ value and $K^{-1}/K_{IC}$ ratio for four kinds of specimens. The unit of $K$ values is converted from GPa$^{-1}$·mm$^{-1/2}$ to the inverse of MPa·m$^{1/2}$ which is an usual unit of $K_{IC}$. The $K^{-1}/K_{IC}$ ratios for HSS, WC-10Co, Si$_3$N$_4$, and Ferrite were $1.44 \pm 0.11$, $1.37 \pm 0.07$, $1.40 \pm 0.03$ and $1.43 \pm 0.01$, respectively. These ratios are nearly equal with each other as expected, and the mean value of these was 1.4. Namely, it was found that the relation between $K$ and $K_{IC}$ for these four kinds of specimens can be commonly expressed with an unified equation of $K_{IC} \equiv 0.7K^{-1}$.

The above result that the equation of $K_{IC} \equiv 0.7K^{-1}$ holds irrespective of the kinds of specimens or materials was considered as follows. Fig. 3 (a) and (a)' shows the $\sigma_r^{-1} - a^{1/2}$ regression lines shown in Fig. 2 (a) and the $Y^{-1} \sigma_0^{-1} - a^{1/2}$ regression lines connecting the data points and the coordinate-origin, whose slope is $K_{IC}^{-1}$, for four kinds of specimens. (a') is a vertically elongated figure of a part in (a). The slopes of these two kinds of lines for each specimen are different from each other even at large $2a$ where the "1" in the stress concentration factor of $1 + 2(a/p)^{1/2}$ can be neglected because the $2(a/p)^{1/2}$ value becomes extremely larger than "1". This is considered to be because the $\rho$ value of the microstructural defect in the $\sigma_r$ test is always larger than that of the sharp crack in $K_{IC}$ test. Therefore, the two results that the $K^{-1}/K_{IC}$ ratio is larger than the unit, i.e., "1" and that the $K^{-1}/K_{IC}$ ratio is a constant value of 1.4 irrespective of the kinds of materials indicate the following: (1) the $K^{-1}/K_{IC}$ ratio is related with the ratio of the $\rho$ value in $\sigma_m$ test to the $\rho$ value in $K_{IC}$ test, and (2) this ratio is nearly the same irrespective of kinds of materials.

Thus, it was experimentally confirmed that the slope value ($K$) of equation $\sigma_r^{-1} = \sigma_0^{-1} + Ka^{1/2}$ is applicable to the estimation of $K_{IC}$ of hard or brittle materials. Namely, the

| $K$ values and $K^{-1}/K_{IC}$ ratios for four kind of brittle or hard materials. |
|------------------|------------------|------------------|
| $K$(MPa$^{-1}$·m$^{1/2}$) | $K^{-1}/K_{IC}$   |
|------------------|------------------|------------------|
| HSS              | 0.033±0.004      | 1.44±0.11        |
| WC-10Co          | 0.056±0.002      | 1.37±0.07        |
| Si$_3$N$_4$      | 0.230±0.005      | 1.40±0.03        |
| Ferrite          | 0.537±0.017      | 1.43±0.01        |
Application of an Equation Representing Reciprocal Fracture Strength versus Square Root of Fracture ...

Fig. 3: $\sigma^2 \cdot a^{1/2}$ regression lines in $\sigma_\alpha$ test and $Y^2 \cdot \sigma^2 \cdot a^{1/2}$ lines in $K_{IC}$ test for four kinds of specimens. (a') is a vertically elongated figure of a part in (a). Data points in $K_{IC}$ test are indicated by marks attached with "1". The value of $2a$ in $Y^2 \cdot \sigma^2 \cdot a^{1/2}$ line is the twice of the depth of rectangular precrack with the width of about 4 mm. The solid lines are drawn, based on the measured values. The unit of $2a$ in the horizontal axis is "mm", differing from the unit "um" in Fig. 2.

$K_{IC}$ value of a new material was suggested to be estimated by use of both the $K$ value of $\sigma^2 \cdot a^{1/2}$ regression line of the material and the equation of $K_{IC} \approx 0.7K^{-1}$. 

(3) Therefore, the $K_{IC}$ value of a new hard or brittle material was suggested to be estimated by use of the $K$ value of $\sigma^2 \cdot a^{1/2}$ regression line of the material and the equation of $K_{IC} \approx 0.7K^{-1}$.

References