This paper presents the determinant of the covariance matrix of the estimates of effects based on a fractional $2^m$ factorial ($2^m$-FF) design $T$ of resolution $V$, where $T$ is constructed by adding some restricted assemblies to an orthogonal array (O-array) or by removing some restricted ones from an O-array of index unity. In the class of $2^m$-FF designs considered here, we also present D-optimal designs of resolution $V$ for each $m$ with $4 \leq m \leq 6$ and for a range of practical values of $N$, where $N$ denotes the total number of assemblies. In addition, these results are compared with the corresponding D-optimal designs of resolution $V$ in the class of balanced fractional $2^m$ factorial designs.

1. Introduction

As a generalization of an orthogonal array (O-array), the concept of a balanced array (B-array) as will be defined in the next section was first introduced by Chakravarti [1]. We now define a balanced fractional $2^m$ factorial ($2^m$-BFF) design, which is in close connection with a B-array (e.g., [18]), as follows: A fractional factorial (FF) design is said to be balanced if the covariance matrix of the estimates is invariant under any permutation on $m$ factors. An explicit expression of the characteristic polynomial of the information matrix of a $2^m$-BFF design of resolution $2^l+1$ was obtained by Yamamoto, Shirakura and Kuwada [19] by utilizing the decomposition of a triangular multidimensional partially balanced (TMDPB) association algebra into its $(l+1)$ two-sided ideals. This polynomial includes the results obtained by Srivastava and Chopra [15]. In the class of $2^m$-BFF designs, optimal designs of resolution V or VII with respect to the trace (A-optimal) and/or the determinant (D-optimal) criteria were obtained by Srivastava and/or Chopra [3-8, 16, 17], and Shirakura [11, 12].

A BFF design has very useful properties mentioned above. It, however, belongs to a subclass of the class of all FF designs. One, therefore, has a question such that whether A-, D- or other optimal designs in the class of BFF designs are also optimal in the one of all FF designs or not. For the A-optimal case, one answer has been given by Kuwada [9]. In this paper, we consider a class of $2^m$-FF designs such that a design is constructed by adding some restricted assemblies to an O-array or by removing some restricted ones from an O-array of index unity. Note that a design in the class of FF designs considered here does not always belong to the class of BFF designs, and vice versa. In each case, the determinant of the covariance matrix of the estimates of effects is described. In addition, for
each $m$ with $4 \leq m \leq 6$ and for a range of practical values of $N$ (the total number of assemblies), $D$-optimal designs of resolution $V$ in the class of $2^m$-FF designs considered here are presented and compared with the corresponding $D$-optimal designs of resolution $V$ in the one of $2^m$-BFF designs.

2. Preliminaries

Consider a $2^m$-FF design $T$ of resolution $V$ with $N$ assemblies. Then $T$ can also be expressed as a $(0,1)$ matrix of size $N \times m$. As usual, the linear model based on $T$ is given by $y(T) = ET\theta$, where $y(T)$ is the $N \times 1$ vector of observations, $E_T$ is called the design matrix whose elements consist of 1 and -1, and $\theta' = (\{\theta(0); \{\theta(1); \{\theta(2)\}))$ is the $1 \times v$ vector of effects. Here the $N$ observations in $y(T)$ are independent random variables with homogeneous variance $\sigma^2$. By solving the normal equation for estimating $\theta$, the best linear unbiased estimate of $\theta$ is given by $\hat{\theta} = V_T E_T y(T)$, if $E_T' E_T$ is nonsingular, where $V_T = (E_T' E_T)^{-1}$. The covariance matrix of $\hat{\theta}$ is given by $\sigma^2 V_T$.

For reader's convenience, we define a $B$-array of strength $t$ here.

Definition. A $(0,1)$ matrix $T$ of size $N \times m$ is called a balanced array of strength $t$, size $N$, $m$ constraints, 2 levels and index set $\{\mu_0, \mu_1, \cdots, \mu_t\}$, written $BA(N, m, 2, \{\mu_0, \mu_1, \cdots, \mu_t\})$ for brevity, if every submatrix $T_{k_1 \cdots k_l}$ composed of the $k_1$-th, $\cdots$, $k_l$-th columns of $T$ is such that every vector with weight $i$ occurs exactly $\mu_i$ times $(i = 0, 1, \cdots, t)$ as a row of $T_{k_1 \cdots k_l}$. Here the weight of a $(0,1)$ vector means the number of ones in the vector.

For purposes of illustration, we now present a $B$-array of strength 4.

Example 1. Below, we present a $BA(22, 6, 2, 4)(1, 2, 1, 1, 3)$, $T$. Notice that the array $T$ can also be considered as a $2^6$-FF design with 22 assemblies. Here the rows and the columns of $T$ correspond to factors and assemblies, respectively.

\[
T' = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1
\end{bmatrix}.
\]

By use of the algebraic structure of the TMDPB association scheme, the matrix $V_T$ of a $2^m$-BFF design $T^*$ of resolution $V$ derived from a $BA(N, m, 2, 4)(\mu^*, \mu^*, \mu^*, \mu^*)$ is isomorphic to the matrices $K_s^{-1}$ of order $(3 - \beta)$ with multiplicities $\binom{m}{\beta} - \binom{m}{\beta - 1}$, say $\phi_s$, for $\beta = 0, 1, 2$, where

\[
K_s = ||\kappa_i|| \quad \text{and} \quad \kappa_i^i = \kappa_i^i.
\]

Here
\[ i_{i} = \sum_{j=0}^{i} \sum_{p=0}^{j-1} (-1)^{p} \binom{i}{p} \binom{4-i}{j-i+p} n_{i}^{j}, \]

\[ z_{j_{a}^{+}} = \sum_{a=0}^{4} (-1)^{a} \left( \begin{array}{cc} u \! - \! b & v \! - \! b \\ b & b \end{array} \right) \left( \begin{array}{cc} m \! - \! u \! - \! \beta \! + \! b & \frac{1}{2} \left( v \! - \! u \right) \\ v \! - \! u & b \end{array} \right), \]

and

\[ \left( \begin{array}{c} p \\ q \end{array} \right) = 0 \text{ if and only if } q < 0 \text{ or } p < q \]

(see [18, 19]). Thus the determinant of \( V_{T} \) is given by

\[ |V_{T}| = \prod_{j=0}^{2} |K_{j_{a}^{+}}|^{1/4}, \]

where \( |A| \) denotes the determinant of a matrix \( A \).

Let \( T_{0} \) be an \( O \)-array of strength \( \lambda \), size \( N_{0} \), \( m \) constraints, 2 levels and index \( \lambda \), written \( OA(N_{0}, m, 2, \lambda) \) for brevity. Throughout this paper, we use the notation \( T_{0} \) as an \( OA(N_{0}, m, 2, \lambda) \). Clearly, \( N_{0} = 2^{m} \). Then for \( \nu \leq 2^{m} \), \( (E_{d}E_{d})^{-1} = (1/N_{0})I \) (\( = V_{0} \), say), where \( E_{d} \) is the \( N_{0} \times \nu \) design matrix of \( T_{0} \), and \( I_{p} \) denotes the unit matrix of order \( p \). Let \( T \) be a 2\( ^{m} \)-FF design with \( \nu \) assemblies such that \( T' = [T'_{0} : T'_{1}] \), where \( T_{1} \) is a 2\( ^{m} \)-FF design with \( N_{1} \) assemblies and \( N = N_{0} + N_{1} \). Then we have

\[ V_{T} = V_{0} - V_{0}E_{d}(I_{N_{1}} + E_{d}V_{0}E_{d})^{-1}E_{d}V_{0}, \]

where \( \nu \leq 2^{m} \) and \( E_{d} \) is the \( N_{0} \times \nu \) design matrix of \( T_{1} \) (see, e.g., [10]). Thus the determinant of \( V_{T} \) is given by

\[ |V_{T}| = |V_{0}||I_{N_{1}} + E_{d}V_{0}E_{d}|^{-1} = (N_{0})^{\nu-p} |N_{0}I_{N_{1}} + E_{d}E_{d}|^{-1}, \]

for the case of adding assemblies. On the other hand, for \( T' = [T'_{0} : T'_{1}] \), it holds that

\[ V_{T} = V_{0} + V_{0}E_{d}(I_{N_{1}} - E_{d}V_{0}E_{d})^{-1}E_{d}V_{0}, \]

provided \( (I_{N_{1}} - E_{d}V_{0}E_{d}) \) is positive definite, where \( N = N_{0} - N_{1} \geq \nu \). Thus it can be easily obtained that for the case of removing assemblies,

\[ |V_{T}| = |V_{0}||I_{N_{1}} - E_{d}V_{0}E_{d}|^{-1} = (N_{0})^{\nu-p} |N_{0}I_{N_{1}} - E_{d}E_{d}|^{-1}, \]

provided \( (I_{N_{1}} - E_{d}V_{0}E_{d}) \) is positive definite.

3. Determinant of \( V_{T} \)

At the beginning, we consider a 2\( ^{m} \)-FF design \( T \) of adding \( T_{1} \) with \( N_{1} \) assemblies to an \( O \)-array \( T_{0} \).

**Theorem 1.** For \( T' = [T'_{0} : T'_{1}] \) and \( N_{1} = 1 \), it holds that

\[ |V_{T}| = (N_{0})^{\nu-p} (N_{0} + \nu)^{-1}, \]

where \( T_{0} \) and \( T_{1} \) are an \( OA(N_{0}, m, 2, \lambda) \) and an arbitrary assembly, respectively.
Proof. Since $E_i$ is the $1 \times v$ design matrix of $T_i$ whose elements consist of 1 and $-1$, $E_iE_i' = v$. Hence, from (2.2), it can be easily proved.

Note that a design given in Theorem 1 is also D-optimal over all $2^m$-FF designs (see Cheng [2]).

For the case in which $N_i \geq 2$, let $T_i$ be a $(0,1)$ matrix of size $N_i \times m$ whose transpose $T_i'$ is a $BA(m, N_i, 2, 2, \{\mu_0, \mu_1, \mu_2\})$. Then we establish the following:

**Theorem 2.** Let $T$ be a $2^m$-FF design with $N$ assemblies of adding $T_i$ with $N_i$ ones to an $O$-array $T_o$ with $N_o$ ones, where $T_i'$ is a $BA(m, N_i, 2, 2, \{\mu_0, \mu_1, \mu_2\}), N = N_o + N_i$ and $N_i \geq 2$. Then

$$|V_T| = (N_o)^{-\mu_0(N_i + v + (N_i - 1)\mu_1 - 1)}.$$

where $\mu_0 = 8\mu_1 - 4(m + 1)\mu_1 + \nu$ is a function of $\mu_1$ alone for fixed value of $m$.

Proof. Since $T_i'$ is a $BA(m, N_i, 2, 2, \{\mu_0, \mu_1, \mu_2\})$, we have

$$m = \mu_0 + 2\mu_1 + \mu_2$$

and

$$m = \begin{pmatrix} m \\ 2 \end{pmatrix} = \begin{pmatrix} \mu_0 \\ 2 \end{pmatrix} + \begin{pmatrix} 2\mu_1 \\ 2 \end{pmatrix} + 2\mu_1(\mu_0 + \mu_2) + \mu_0 \mu_2.$$

Thus it holds that

$$N_o I_{N_1} + E_i E_i' = (N_o + \nu)A_{N_1} + \mu(G_{N_1} - I_{N_1})$$

$$= (N_o + \nu + (N_i - 1)\mu_1)A \dagger + (N_o + \nu - \mu)A \dagger,$$

where $G_{N}$ denotes the matrix of order $p$ all of whose elements are unity, $A \dagger = (1/N)G_{N_1}$ and $A \dagger = I_{N_1} - (1/N)G_{N_1}$. Since $A \dagger$ and $A \dagger$ are symmetric, idempotent and mutually orthogonal matrices with rank $(A \dagger) = 1$ and rank $(A \dagger) = N_i - 1$, it holds that

$$|N_o I_{N_1} + E_i E_i'| = (N_o + \nu + (N_i - 1)\mu_1)^{-1}(N_o + \nu - \mu)^{N_i - 1}.$$

Therefore, from (2.2), we can obtain (3.2).

For a $(0,1)$ matrix $T_i'$ being a $BA(m, N_i, 2, 2, \{\mu_0, \mu_1, \mu_2\}),$

$$T_iT_i' = \mu_0 I_{N_1} + \mu_2 G_{N_1} = (\mu_1 + N_1 \mu_2)A \dagger + \mu_1 A \dagger.$$

When $m < N_i$, the matrix $T_iT_i'$ is singular. Thus $\mu_i = 0$ (hence $\mu = \nu$) for $m < N_i$.

**Corollary.** Let $T$ be a $2^m$-FF design of Theorem 2. Then, for $m < N_i$,

$$|V_T| = (N_o)^{-\mu_0(N_o + N_i \nu)}.$$

Next, let $T$ be a $2^m$-FF design of removing $T_i$ from an $O$-array $T_o$ of index unity, where $T_i$ is a $2^m$-FF design with $N_i$ assemblies contained in $T_i$.

**Theorem 3.** For $T_i'[= T': T_i]$, if $N = N_o - 1 \geq \nu$ and $N_i = 1$, it holds that

$$|V_T| = (N_o)^{-\mu_0(N_o - \nu)}.$$

where $T_o$ and $T_i$ are an $OA(N_o, m, 2, 1)1$ and an arbitrary assembly contained in $T_i$, respectively.

Proof. Since $E_iE_i' = v$, it can be easily proved.

Note that a design given in Theorem 3 is also D-optimal over all $2^m$-FF designs.
Since $T$ is a $2^n$-FF design of removing $T_i$ from an $O$-array $T_i$ of index unity, there does not exist a $B$-array $T'_i$ with $\mu_i=0$ for $N_i \geq 2$. On the other hand, $\mu_i=0$ for $m < N_i$. Hence in the remainder of this section, we consider only the case in which $1 \leq \mu_i \leq [m/2]$, $2 \leq N_i \leq m$ and $4 \leq m$, where $[x]$ denotes the greatest integer not exceeding $x$.

**Theorem 4.** Let $T$ be a $2^n$-FF design with $N$ assemblies of removing $T_i$ with $N_i$ ones from an $O$-array $T_i$ of index unity with $N_0$ ones, where $T'_i$ is a $A(m, N_i, 2, 2)\{\mu_0, \mu_1, \mu_2\}$ contained in $T_i$ and $N_i \geq 2$. Then it holds that

$$\begin{align*}
|V_T| &= \mu_0 + \mu_1 \mu_2, \\
N_0 - \nu &= (N_0 - \nu + \mu)^{\mu_1} (N_0 - \nu - (N_i - 1) \mu)^{\mu_2 - 1},
\end{align*}$$

provided $N_0 - \nu - (N_i - 1) \mu > 0$ and $\nu \leq N$, where $\mu$ is defined in Theorem 1.

**Proof.** Since $T'_i$ is a $B(m, N_i, 2, 2)\{\mu_0, \mu_1, \mu_2\}$, it holds that

$$
|N_0 I_{x_i} - E_{x_i}| = (N_0 - \nu)(N_0 - \nu - (N_i - 1) \mu)^{\mu_2 - 1}.
$$

Therefore, from (2.3), we can obtain (3.5), provided $N_0 - \nu - (N_i - 1) \mu > 0$ and $\nu \leq N$.

4. D-optimal designs of resolution V in the class of $2^n$-FF designs for $m=4, 5$ and 6

We now consider a $2^n$-FF design $T$ of resolution $V$ of adding $T_i$ to an $O$-array $T_i$ or of removing $T$, from $T_i$ of index unity, where $T'_i$ is a $A(m, N_i, 2, 2)\{\mu_0, \mu_1, \mu_2\}$. As seen in Section 3, $|V_T|$ is determined only by $\mu$ (hence the index $\mu_i$ of a $B$-array $T'_i$) for fixed values of $m$ and $N$. Note that for given values of $m, N_i (\geq 2)$ and $\mu_i$, a $B$-array $T'_i$ is not always unique, since index $\mu_0$ (or $\mu_2$) is free.

**Example 2.** Let $T'_i$ be a $B(4, 2, 2, 2)$ with $\mu_1=1$. Then $T_i$ of size $2 \times 4$ is given by

$$T_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}.$$

The existence conditions for $B$-arrays (see [11, 14]) show that there do not exist the following $B$-arrays $T'_i$: (i) of $\mu_1=2$ for $m=4$ and $N_i=3, 4$, (ii) of $\mu_i=2$ for $m=5$ and $N_i=3, 4, 5$, (iii) of $\mu_i=2$ for $m=6$ and $N_i=5, 6$, and (iv) of $\mu_i=3$ for $m=6$ and $N_i=3, 4, 5, 6$.

D-optimal designs of resolution $V$ in the class of $2^n$-FF designs considered here can be obtained by the formulas (3.1) through (3.5). They are, respectively, given in Tables 1, 2 and 3 for $m=4$ and $12 \leq N \leq 27$, $m=5$ and $18 \leq N \leq 32$, and $m=6$ and $27 \leq N \leq 40$ together with the values of $\mu_i$ and $|V_T|$. In Tables 1, 2 and 3, D-optimal designs for $m=4$ and $17 \leq N \leq 27$, $m=5$ and $17 \leq N \leq 26$, and $m=6$ and $33 \leq N \leq 40$ are constructed by adding $N_i$ assemblies to $O(2^4, 4, 2, 4)$, $O(2^4, 5, 2, 4)$, and $O(2^4, 6, 2, 5)$, respectively. And in Tables 1, 2 and 3, D-optimal designs for $m=4$ and
Table 1.

| N   | $\mu_1$ | $|V_T|$   | $\mu^*$ | $|V_{T^*}|$ |
|-----|---------|----------|---------|-----------|
| 12  | 1       | 0.72760E-11 | 10111  | 0.72760E-11 |
| 13  | 1       | 0.20788E-11 | 20111  | 0.36957E-11 |
| 14  | 1       | 0.60633E-12 | 01110  | 0.60949E-12 |
| 15  | **      | 0.18190E-12 | 11110  | 0.18190E-12 |
| 16  | OA      | 0.56843E-13 | 11111  | 0.56843E-13 |
| 17  | **      | 0.33685E-13 | 21111  | 0.33685E-13 |
| 18  | 1       | 0.19989E-13 | 21112  | 0.20211E-13 |
| 19  | 1       | 0.11879E-13 | 31112  | 0.14436E-13 |
| 20  | 1       | 0.70709E-14 | 12111  | 0.70709E-14 |
| 21  | 0       | 0.12810E-13 | 21121  | 0.42162E-14 |
| 22  | 0       | 0.11091E-13 | 11211  | 0.25912E-14 |
| 23  | 0       | 0.77795E-14 | 21211  | 0.15487E-14 |
| 24  | 0       | 0.87451E-14 | 12121  | 0.93569E-15 |
| 25  | 0       | 0.79086E-14 | 22121  | 0.59096E-15 |
| 26  | 0       | 0.72182E-14 | 22122  | 0.38278E-15 |
| 27  | 0       | 0.66386E-14 | 22122  | 0.23283E-15 |

$\delta$ ; $|V_T|$ is strictly smaller than $|V_{T^*}|$.

** ; Arbitrary assembly.

OA; O-array.

Table 2.

| N   | $\mu_1$ | $|V_T|$   | $\mu^*$ | $|V_{T^*}|$ |
|-----|---------|----------|---------|-----------|
| 16  | OA      | 0.54210E-19 | 11111  | 0.54210E-19 |
| 17  | **      | 0.27105E-19 | 21111  | 0.27105E-19 |
| 18  | 1 or 2  | 0.13553E-19 | 21112  | 0.14046E-19 |
| 19  | 1       | 0.67763E-20 | 31112  | 0.94794E-20 |
| 20  | 1       | 0.33881E-20 | 12111  | 0.48187E-20 |
| 21  | 1       | 0.16941E-20 | 22111  | 0.16941E-20 |
| 22  | 0       | 0.77443E-20 | 22122  | 0.84703E-21 |
| 23  | 0       | 0.67763E-20 | 32112  | 0.53673E-21 |
| 24  | 0       | 0.60233E-20 | 32113  | 0.36322E-21 |
| 25  | 0       | 0.54210E-20 | 22121  | 0.19494E-21 |
| 26  | 0       | 0.49282E-20 | 12211  | 0.52940E-22 |
| 27  | 1       | 0.26470E-22 | 22211  | 0.36470E-22 |
| 28  | 1       | 0.13235E-22 | 22212  | 0.14296E-22 |
| 29  | 1       | 0.66174E-23 | 32212  | 0.96528E-23 |
| 30  | 1 or 2  | 0.33087E-23 | 12221  | 0.38501E-23 |
| 31  | **      | 0.16544E-23 | 22221  | 0.16544E-23 |
| 32  | OA      | 0.82718E-24 | 22222  | 0.82718E-24 |

$\delta$ ; $|V_T|$ is strictly smaller than $|V_{T^*}|$.

** ; Arbitrary assembly.

OA; O-array.
Table 3.

| $N$  | $\mu_1$ | $|V_T|$  | $\mu^*$  | $|V_{T^*}|$ |
|------|---------|---------|---------|-----------|
| 27   | 1       | 0.62330 $E - 30$ | 22211  | 0.17729 $E - 30$ |
| 28   | 2       | 0.98608 $E - 31$ | 22212  | 0.88647 $E - 31$ |
| 29   | 2       | 0.28174 $E - 31$ | 32212  | 0.55060 $E - 31$ |
| 30   | 1 or 2  | 0.82173 $E - 32$ | 32213  | 0.36860 $E - 31$ |
| 31   | **      | 0.24652 $E - 32$ | 22221  | 0.24652 $E - 32$ |
| 32   | OA      | 0.77037 $E - 33$ | 22222  | 0.77037 $E - 33$ |
| 33   | **      | 0.45652 $E - 33$ | 32222  | 0.45652 $E - 33$ |
| 34   | 1 or 2  | 0.27099 $E - 33$ | 32223  | 0.28014 $E - 33$ |
| 35   | 1       | 0.16096 $E - 33$ | 42223  | 0.20206 $E - 33$ |
| 36   | 1       | 0.95750 $E - 34$ | 42224  | 0.14674 $E - 33$ |
| 37   | 1       | 0.57022 $E - 34$ | 33222  | 0.56145 $E - 34$ |
| 38   | 1       | 0.33994 $E - 34$ | 33223  | 0.31502 $E - 34$ |
| 39   | 0       | 0.13254 $E - 33$ | 43223  | 0.20386 $E - 34$ |
| 40   | 0       | 0.11852 $E - 33$ | 43224  | 0.14456 $E - 34$ |

$J; |V_T|$ is strictly smaller than $|V_{T^*}|$.

$**;$ Arbitrary assembly.

$OA; O$-array.

$12 \leq N \leq 15$, $m=5$ and $27 \leq N \leq 31$, and $m=6$ and $27 \leq N \leq 31$ are also constructed by removing $N_1$ assemblies from $OA(2^4, 4, 2, 4)_1$, $OA(2^5, 5, 2, 5)_1$ and $OA(2^6, 6, 2, 5)_1$, respectively. On the other hand, by use of the formula (2.1), D-optimal designs $T^*$ of resolution $V$ in the class of $2^m$-BFF designs, which are derived from $BAs(N, m, 2, 4)$, $\mu^*, \mu^*_1, \mu^*_2, \mu^*_3, \mu^*_4$, can be obtained. They are also given in Tables 1, 2 and 3 for same values of $m$ and $N$ mentioned above. In each Table, all D-optimal designs in the class of $2^m$-FF designs are also A-optimal (see [9]), and D-optimal designs in the class of $2^m$-BFF designs except for $m=5$ and $N=20, 25$ are also A-optimal (see [16]).

Note that in each Table, $\mu^*=(\mu^*_1, \mu^*_2, \mu^*_3, \mu^*_4)$ and $n_i E_1 n_i = n_i 10^{-ni}$.

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