Wave Maker in a Circulating Water Channel *1

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An apparatus of exciting a wave-suppressor plate usually equipped at outlet top of a circulating water channel (CWC) in heave and pitch motion is devised for making a progressive wave system with only one wave length on the water surface of a uniform flow in the CWC in order to conduct ship model performance test in waves. As well known an oscillating body in a uniform flow makes generally four progressive wave systems or two systems, the wave number being \(\alpha_1\) (following wave) and \(\alpha_2\) (head wave), in usual test conditions for model ships. At first a numerical simulation is performed by applying the two-dimensional linear theory of an oscillating planing plate to confirm possibility of making \(\alpha_1\)-wave disappear and of generating \(\alpha_2\)-wave only and to obtain \(\alpha_1\)-wave-free condition of combination of heaving and pitching of the wave-suppressor-plate. Next, experiments are conducted in a small CWC after developing a method of separating \(\alpha_1\)- and \(\alpha_2\)-components from the measured envelop of beating waves generated by the oscillating suppressor-plate. Through some iterations of a process of getting \(\alpha_1\)-wave-free condition of oscillation combination of the wave-suppressor-plate from the two experiments of different combinations, wave fields consisting of only \(\alpha_2\)-wave are obtained for three frequencies (3.4 and 5Hz) and two amplitudes at three velocities (0.5 \(\sim\)1.0m/s). Though these \(\alpha_1\)-wave-free results do not correspond very well with the simulations mainly due to the non-linearity of the real motion of the wave-suppressor-plate, it is concluded that more accumulation of experimental data makes this type of wave maker generate any progressive wave system with single wave length in CWC.

Keywords: Wave Maker, Waves Generated by an Oscillating Object in a Uniform Flow, Theory of Oscillating Planing Plate, Wave-Suppressor-Plate

1. Introduction

Resistance of a ship advancing in a calm sea is usually estimated by an extrapolation of the data of model test using a towing tank and many resistance tests have been also performed in a circulating water channel (CWC), recently the performance being improved. However, in the case of measuring wave load or resistance increase of a ship in waves, we are limited to rectangular tank or towing tank with wave maker and CWC has not yet been used. This is due to the fact that it is difficult until now to make a progressive sinusoidal wave system in any period with only one wave length on the water surface of a uniform flow in a CWC. In general when setting an oscillating body in a uniform flow, four wave systems having wave lengths differ with each other generated around the body as shown in Fig.1 and usually we cannot obtain a progressive wave system with only one wave length.2)

\[
\begin{align*}
\alpha_2 & \quad \alpha_1 & \quad \beta_1 & \quad \beta_2 \\
\end{align*}
\]

Fig.1 Waves generated by an oscillating object in a uniform flow.

In the following we shall call this wave system with only one wave length as single wave system.

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the present paper it is examined whether the single wave system can be realized by the control of oscillation mode (combination of heaving and pitching) of the wave-suppressor-plate usually equipped at outlet top of a CWC to make so-called steady waves degenerate\(^3\).

Kashiwagi et al. in their private memorandum\(^3\) first carried out an concrete proposition on the method of making a single wave system in a CWC. They concluded that a single progressive wave system is possible by controlling the oscillating mode of a two-dimensional submerged elliptic cylinder in heave and pitch and got the conditions of oscillation mode both theoretically and numerically. Recently Bessho\(^3\) proposed a method of making a single wave system by setting two hydro-foils in tandem in a CWC and by controlling the pitching motion of each foil. The above two proposals are both the methods of making a single wave system by means of submerged bodies. It is not so easy to make a fine wave system by oscillating the submerged bodies because an apparatus piercing the water surface makes waves by itself or because it is not so desirable from the point of view of constructions and water proof to penetrate the rotational axes through the side wall of a CWC. Prior to this paper Sekino et al.\(^4\) proposed a method of making the wave-suppressor-plate oscillate to generate a single progressive wave system. They constructed a linear theory on an oscillating planing plate in a uniform flow and the generated waves and showed the possibility of generation of a single progressive wave system through the numerical simulations. They compared the waves experimentally generated by heaving the wave-suppressor-plate with the theory but they did not realize the single wave system experimentally.

2. Two-Dimensional Linear Theory of Oscillating Planing Plate

According to the prior papers\(^4,6\) we shall state in brief the linearized theory of two-dimensional planing plate oscillating in a uniform flow. The frame of reference is taken as Fig. 2, \(y = 0\) being static water surface, \(z = x + iy\) being complex coordinate variable, \(t\) being time and \(\eta = \eta_0(x; t)\) being elevation of plate and water surface. Let the pressure be \(p_0(x; t)\) on the plate and atmospheric pressure be zero. And \(g\) stands for gravitational acceleration, \(\rho\) density of water.

\[
F(x; t) = -Uz + f_\omega(x; t), \quad (1)
\]

\[
f_\omega(x; t) = \phi_\omega(x; t) + i \psi_\omega(x; t) = Re_\omega \left[ f(z)e^{-jwt} \right],
\]

where \(j\) stands for imaginary unit other than \(i\) and they are not interfered with each other. Velocity potential, stream function, pressure distribution and wave height are also assumed sinusoidal in circular frequency \(\omega\) and they have the same representations as the above \(f_\omega(z; t)\). Let the characteristic length be \(L\) and we define the reduced frequency \(\alpha\), the reduced wave number \(\gamma\) and their ratio \(\Omega\) as the following:

\[
\alpha = \frac{\omega L}{U}, \quad \gamma = \frac{gL}{U^2}, \quad \Omega = \frac{\alpha}{\gamma} = \frac{\omega U}{g}.
\]

then we can write the linearized condition of free surface as:

\[
\left( j\alpha + \frac{\partial}{\partial x} \right)^2 \mu \left( j\alpha + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right) \phi(z) \quad (2)
\]

\[
= \left( j\alpha + \frac{\partial}{\partial x} \right) p(z) \quad \text{on} \quad y = 0,
\]

where \(\mu\) denotes an artificial frictional coefficient. We define a source singularity located at point \(\zeta = \xi + i \eta\) satisfying the linearized free surface condition and having the harmonically pulsating strength as:

\[W_Q(z, \zeta; t) = W_Q(z, \zeta)e^{-jwt}.
\]

Now we obtain the following boundary integral equation\(^3\):

\[
\gamma \eta(z) = -p(z) + \frac{1}{2\pi Re} \int_{-\infty}^{\infty} p(\xi) \frac{\partial W_Q}{\partial \eta}(z, \xi) d\xi
\]

\[-Re \Delta \eta \left( j\alpha - \frac{\partial}{\partial \xi} \right) W_Q(z, x_{fp}). \quad (3)
\]

Fig.2 Coordinate system for an oscillating planing plate.
The above equation is regarded as an integral equation with respect to the pressure distribution \( p(x) \) under the given oscillation mode \( y = \eta(x) \) of the planing plate. The pressure must satisfy Kutta condition:

\[
p(x_{AP}) = 0.
\]

Once the pressure is solved the water surface elevation around the planing plate can be calculated from the same equation by instituting the pressure.

In the usual experimental region (\( \Omega > 1/4 \)) of Froude number greater than 0.2 and wave length ratio to ship length less than 3.0, two systems of wave numbers \( \beta_1 \) and \( \beta_2 \) disappear\(^{(2,4)} \) and we need only consider two systems of wave numbers \( \alpha_1 \) and \( \alpha_2 \), roots of the following quadratic equation:

\[
k^2 - (\gamma + 2\alpha)k + \alpha^2 = 0.
\]

The \( \alpha_2 \)-wave system generated in a CWC has the properties equivalent to the head wave in a towing tank. When making a wave maker of the towing tank oscillate in frequency \( \omega \) the encounter frequency \( \omega_e \) of model ship advancing at velocity \( U \) is given by:

\[
\omega_e = \omega + \omega^2 U/g.
\]

Now let as:

\[
\lambda = \lambda/\lambda_0, \quad \lambda = 2\pi g/\omega^2, \quad \lambda_0 = 2\pi U^2/g,
\]

then the non-dimensional encounter frequency becomes the following and it coincides with that of \( \alpha_2 \)-wave\(^{(5)} \):

\[
\Omega_e = \omega U/g
\]

\[
= 1/\Lambda + 1/\sqrt{\Lambda}.
\]

This relation is shown by solid line in Fig.3. The encounter frequency for following wave generated in a towing tank is written as follows:

\[
\Omega_e = |1/\Lambda - 1/\sqrt{\Lambda}|.
\]

These values are identical with those of \( \alpha_1, \beta_1 \) and \( \beta_2 \) waves in a CWC and the relation is shown by broken lines in Fig.3.

From the above discussion we may say that our purpose is to make the \( \alpha_1 \)-wave disappear by controlling oscillation mode of the wave-suppressor-plate.

Here the already mentioned linearized theory is to be noted based on the assumptions that the wavesuppressor-plate is flat and does not disturb the water surface when it is rest and that the motion is infinitesimally small. The following calculations correspond with the experimental states described in the next section (Fig.8). The wave-suppressor-plate is linked to the outlet top of the CWC with a flexible sheet (approximated by a cubic curve) which moves smoothly as the flow does not separate from the sheet and the upstream from the outlet is assumed flat in enough long distance. When making the wave-suppressor-plate pitch the calculated pressure distributions non-dimensionalized by the maximum value over the plate are shown in Fig.4, x-coordinate being non-dimensionalized by length of the wave-suppressor-plate, the solid line in phase component and the broken out of phase, and the wave-profiles downstream the plate are shown in Fig.5.
where $H_k^-(\alpha_1)$ stands for complex amplitude of free $\alpha_1$-wave generated by oscillation mode $k$. Therefore we obtain $\alpha_1$-wave free condition as follows:

$$h_0H_0^-(\alpha_1) + h_1H_1^-(\alpha_1) = 0.$$  

(7)

The simulated wave profiles satisfying the above $\alpha_1$-wave free condition are shown in Fig.6. It is found that $\alpha_1$-wave with shorter wave length disappears and beat of amplitude also degenerates. The pressure distribution in this case is shown in Fig.7.

From the simulations as the above it is confirmed theoretically and numerically that we can get a single progressive wave system of $\alpha_2$-wave in a CWC. As for the experiment the condition(7) will become a good guidance to reach the real $\alpha_1$-wave free state by using the measured wave-making performances.

3. Experimental Results and The Comparison with The Theory

The experiment is conducted by using personal small CWC. The observation part is 0.6m length, 0.3m breadth and 0.2m water depth, and the maximum velocity is 1.0m/s. The apparatus of exciting the wave-suppressor-plate is designed and made as shown in Fig.8. It has two crank mechanisms, the amplitudes of which are adjustable to 50mm and the pulleys of which are linked with each other by a timing belt, the phase angle between them being set arbitrarily. The upstream pulley is driven by a servo-motor up to 5Hz. The wave-suppressor-plate made of aluminium is able to be excited in combined oscillation of heave and pitch by two rods of these cranks. In order to prevent flow separation upstream the wave-suppressor-plate a sheet of vinyl
chloride board (0.45mm thickness) is stretched from the outlet top lever tensioned by springs, over the wave-suppressor-plate and to the tip overhang from the plate (the flat part 110mm length). Experiments are performed in three cases of velocity (0.496, 0.736 and 0.928m/s) and three cases of frequency (3, 4 and 5Hz).

Since the wave system generated by the oscillating wave-suppressor-plate is composed of $\alpha_1$- and $\alpha_2$-waves, the amplitudes and the phase differences of each wave must be separated. Sekino et al.\textsuperscript{4)}, taking note of beat of amplitude envelop of the composed wave, developed a method of obtaining the amplitudes and phase differences of $\alpha_1$- and $\alpha_2$-waves from the locations, amplitudes and phase differences at the maximum and minimum point of the envelope.

Fig.9 shows amplitude ratios (black circles), and phase differences (stars) at each location where the elevations of the water surface and the wave-suppressor-plate are measured and analyzed by Fourier series expansion method. This envelop of amplitude is assumed to beat like the following form:

$$A_1 + A_2 \cos(k_B z + \epsilon),$$

and the values of $A_1$, $A_2$ and $\epsilon$ are calculated by method of least squares as shown by a broken line in the figure. After getting the coordinates of amplitude being maximum and minimum and the amplitudes at those locations from the above equation and the phase differences (pluses) from the linear interpolation, the amplitudes and phase differences of $\alpha_1$-
Fig. 12 Amplitude ratios and phase differences for a combine motion at nearly α₁-wave free  
(U=0.736m/s, f=4.0Hz).

Fig. 13 Estimated free wave profiles at α₁-wave free  
(U=0.736m/s, f=4.0Hz).

and α₂-waves are able to be calculated respectively by those values.

In Fig. 10 for heaving and in Fig. 11 for pitching, wave amplitude ratios (A₁, circles) to the trailing edge 
angle and wave phase differences (Δφ, triangles) from the trailing edge phase angle obtained by the 
experiments are shown to the base of oscillation frequencies. They are compared with the theoretical amplitudes (solid lines) and phase differences (broken lines).

The amplitudes coincide qualitatively with the theoretical values except at low velocity (U = 0.496m/s) 
but the agreement of the phase differences is not good.

Accordingly when pursuing the α₁-wave free state 
the theoretical results must be only a reference and 
the experimental values are mainly to be used.

The main reason of the difference between theory and experiment is due to that the actual motion of the 
wave-suppressor-plate of our apparatus is nonlinear 
and different considerably from the one assumed in the linear theory.

The oscillation of the wave-suppressor-plate can be 
approximated by a summation of heave and pitch motions. Therefore the respective components (amplitude ratio and phase difference) of α₁- and α₂-waves 
for the pure heave motion of the plate and the respective 
ones for the pure pitch motion can be obtained if 
two different state experiments are performed. From 
these wave-making performances we may obtain the 
α₁-wave free condition from eq.(7). However, the actual 
motion of the wave-suppressor-plate is not linear 
as mentioned above and this process to the α₁-wave 
free state is to be iterated. Fig. 12 shows the result at 
early α₁-wave free obtained through some iterations. 
The profiles of the free wave of this case are estimated 
and shown in Fig. 13.

The states of α₁-wave free are experimentally obtained 
for some cases of flow velocities and oscillation 
frequencies and they are tabulated in Table 1. A₀ 
and A₁ stand for the amplitudes of the aft and fore 
cranks respectively and Δε the phase difference 
between them. H₂ denotes complex amplitude of α₂-
wave and \( \lambda_2 \) is \( \alpha_2 \)-wave length. In all the cases of the table the amplitude proportions (\( A_1/A_2 \)) of \( \alpha_1 \)-wave to \( \alpha_2 \)-wave are hold less than 7.5 to several percents, the amplitude/length ratios being minimum 1/105 to maximum 1/24.

The wave making performances and the \( \alpha_1 \)-wave free conditions may not be linear. The experimental results for larger wave amplitude/length ratios are shown in Table 2. The wave amplitude/length ratios are minimum 1/58 to maximum 1/22 and the amplitude proportions of \( \alpha_1 \)-wave to \( \alpha_2 \)-wave are less than several to 8 percents. The experiments for 5Hz are not performed because of the non-natural motion of the vinyl chloride board in these cases.

The amplitude ratios \( A_f/A_e \) and \( A_{ao}/A_e \) of the fore and aft cranks to the trailing edge and the phase differences \( \Delta \) are shown and compared with the theoretical values in Fig.14 to 16. The experimental values meet a qualitative agreement with the theoretical values except at the lowest velocity. The wave amplitude ratios to the trailing edge of the wave-suppressor-plate are shown in Fig.17 and compared with the theoretical values. We may not yet complete the cross curves from these experimental results. The nonlinearity due to the variance of wave height/length ratio is not so large but is not completely disregarded. Accordingly further accumulation of the experimental data is necessary in order to generate any single wave system at any flow velocity.
4. Conclusions and Acknowledgement

An apparatus which excites a wave-suppressor-plate in oscillation to generate waves in a flow of a small CWC was made, various experiments were conducted to obtain a progressive wave system with only one wave length and the following results are obtained.

1. The proposed type of wave maker showed a good performance of making progressive wave systems in a CWC.

2. The single wave systems with two amplitude ratios were generated on the water surface of the CWC in the three flow velocities and at three oscillation frequencies.

3. By the further accumulation of experimental data it is possible to generate any single progressive wave system at any flow velocity.

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References


