Landslips on the Ground Water Surface and its Mechanism-II

—Vane tests in sand and its interpretation—

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地下水面でのすべりとその発生機構—II

——砂におけるベーンテストとその解釈——

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<要旨>

モデル砂層内において地下水面を平行に上昇させる時、地下水面近辺で生じるすべりがくり返し観察されたが、このすべりはモール・クーロン式では説明できないと思われ、かつパイピング現象によっても説明できない⁵⁾。

この水面でのすべりの原因を調べる場合、砂層内のセン断力分布は比較的単純であり、① すべり深さ（ほとんど2～4cm）に対し斜面長が150cmあり、十分長い斜面と考えられ、② 側面のマサツは砂層の幅を変えても常に水面でのすべりが生じたことから、すべりの発生機構を変えるものではないと考えられる——ことからセン断応力には疑う余地は少なく、研究の目標はセン断強度分布がモール・クーロン式から推定されるものと異なる分布をしているのではないかという点に絞られる。モデル砂層内のセン断強度分布を調べることは一般に認められているどの試験法でも困難であるが、強いて求めればベーンテストがもっとも適していると考えられる。

砂質土に対してベーンテストを用いる場合、粘土に対する試験と異なり、“排水条件”でセン断が行われるためセン断面上の有効応力がセン断中に変化する。したがってセン断強度の算定が困難であり、砂質土に対するベーンテストは一般には行われていない。

しかし上記の強い必要性から砂質土に対するベーンテストの解釈を試み、A. Farrentl が1960年に試みた研究¹⁾をもとに Hill の塑性学²⁾に述べられている一般塑性材料内でのすべり線の曲りとその線上でのセン断応力、垂直応力の関係を用いて本文の(10)式の関係を導入した。そしてこの式を現在利用できる限られたデータで検討したところ、定性的にも定量的にもほぼ適正と考えられる値を得た。

Synopsis

To look for the interpretation of the slips on the ground water surface which were observed in the model experiments and reported in the preceding paper⁵⁾, the variation of shear strengths in the model sand layers is strongly wanted, however it is difficult to measure the variation in the model sand layers by means of sample tests. Therefore, the application of vane tests to this purpose was examined on the basis of Farrent's equation which interprets vane tests in soils having friction, and the following equation (Eq. 10) was deduced.

$$S_v = \frac{T}{\frac{2}{3}\pi r^3 + 2hr^2 \cot\phi (e^{\pi \tan\phi} - 1)}$$

1. Preface

Both of slips on the boundary to the stable layer and slips in the homogeneous layer have been observed in natural sandy slopes. The first type of slips is explained by the Mohr-Coulomb law, but the second one can not be interpreted by the law⁵⁾. Therefore, S. Tanaka has given the slips in the homogeneous layer the interpretation⁷⁾⁸⁾ that the piping phenomenon causes the

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outflow of sand along the ground water surface and decreases the shear resistance, then slips take place on the ground water surface in the homogeneous layer. It seemed to the authors from their experiences that some of slips in homogeneous sandy layers can be caused by the piping phenomenon, but some of them can not be explained by it. Therefore, the authors performed the model experiments and confirmed that slips take place on the ground water surface parallel to slope without the piping phenomenon, as stated in the preceding paper⁵⁾. From these results, it is clear that this type of slips can be explained by neither the Mohr-Coulomb law nor the piping phenomenon. What is the mechanism of this type of slips? The variation of shear stress in the sand layer is comparatively clear, and quite clear if the slope is enough long and side frictions are negligible. In the case of the model experiments, the slope can be regarded to be enough long since the depths of slips are 1~7cm (almost 2~4cm) and the slope length is 150cm. The side frictions are estimated to have an influence on the depth of slip surface since the deeper slips took place in the wider sand layers⁵⁾, but no influence on the mechanism of slips since slips always took place on the ground water surface regardless of the width. Hence, the variation of shear stress will be out of probe for the mechanism of these slips. It should be the target of our search whether the variation of shear strength will be different from that presumed by the Mohr-Coulomb law and it will be such a variation as the safety factor takes its smallest value at the ground water surface or not.

To measure the variation of shear strength in the model sand layer is nearly impossible by means of sample tests. It must be measured by some in-situ shear tests of small scale. It is difficult to imagine any other in-situ shear test than vane test for this purpose. Vane tests in sand has not been authorized, therefore, we needed to examine the application of vane tests to sand within the available informations at the present level.

2. Vane tests in sand and its interpretation.

When a vane is rotated in soils, total stresses are increased by the applied force from the vane. If the soil is quite cohesive one of $\phi=0$ or the test is performed under undrained condition, the increase of total stresses by the vane has no influence on the measured shear strength. Vane tests have been conventionally used in soft clays and there the undrained condition is almost kept. However, if vane tests will be used in sand like our case, the condition of shear is "drained" and the effective stresses in sand are increased by the applied force. Since sand is a frictional material, the shear resistance exerted around the vane must be greater than that in the natural state. It is A. Farrent¹⁾ who proposed the interpretation of vane tests under the drained condition for the first time. He led the equation to calculate the shear strength in the natural state from the measured torque by a vane and the angle of internal friction with reference to "Plasticity in Engineering" by Van Iterson⁴⁾. However, his description of the assumptions and the process to lead the equation is not enough, then it is not very easy to understand it. Therefore, the equation was re-led with reference to the theory of plane strain of a general plastic material in "The Mathematical Theory of Plasticity" by R. Hill²⁾ as mentioned below.

The assumptions to lead the equation are stated at first.

Assumption 1)

It is assumed that vane tests in sand are carried out under complete drained condition, no pore pressure takes place during shear.

Assumption 2)

Shear on the vertical plane of vane is assumed to be in a state of plane strain. Shear on the horizontal planes of vane is assumed to be the same in the drained condition with that in the undrained condition (Normal stresses on the horizontal planes do not change during shear).

Assumption 3)

When a vane is rotated in sand, a curve of the maximum shear strain rate (a slip line, or a shear surface) is formed in the shape to connect four tips of the vane. (Slip lines which do not pass through the tips of the vane are also formed, but they are out of interest here.) It is assumed that the shape of the slip line to pass through the tips of the vane can be approximated to a circle which the center of the vane is its origin.

Assumption 4)

Because of drained condition, volume changes take place during shear. Volume changes during vane tests may be different from those during actual landslips. However, the difference is assumed to be negligible.

Assumption 5)

Normal stress on the slip plane varies depending on the relative position to the vane blades. At the front side of the rotating vane blade the normal stress is the maximum, whereas it is the minimum at the back side. The normal stress at the back side is assumed to be equal to that in the natural state.

According to "The Mathematical Theory of Plasticity"²⁾ the following relations exist in a state of plane strain of a general plastic material. In Fig. 1,

$$dR - 2Qd\theta_\beta = 0 \quad \text{along } \alpha\text{-curve}$$

$$dR + 2Qd\theta_\alpha = 0 \quad \text{along } \beta\text{-curve}$$

here

$\theta_\alpha, \theta_\beta$: anti-clockwise orientation an α -curve, a β -curve
from the fixed direction

α, β : the characteristics (slip lines)

P, R : normal stress

Q : shear stress

As we can regard the angle of internal friction constant in the sand layer around the vane,

$$d\theta_\beta = d\theta_\alpha = d\theta$$

$$\therefore dR = \pm 2Qd\theta \dots\dots\dots(1)$$

θ is the difference of orientation between two points on a curve. Applying this equation to our problem, in Fig. 2,

$$d\sigma = \pm 2\tau d\theta \dots\dots\dots(2)$$

Differentiating the Mohr-Coulomb law $\tau = c + \sigma \tan \phi$,

$$\frac{d\tau}{d\theta} = \frac{d\sigma}{d\theta} \tan \phi \dots\dots\dots(3)$$

Substituting (3) to (2)

$$\begin{aligned} \frac{d\tau}{d\theta} &= \pm 2\tau \tan \phi \\ \frac{d\tau}{\tau} &= \pm 2 \tan \phi \cdot d\theta \dots\dots\dots(4) \end{aligned}$$

Integrating this equation from 0 to θ ,

$$\begin{aligned} \int_0^\theta \frac{d\tau}{\tau} &= \pm \int_0^\theta 2 \tan \phi \cdot d\theta \\ \frac{\tau_\theta}{\tau_0} &= e^{\pm 2\theta \tan \phi} \dots\dots\dots(5) \end{aligned}$$

When a vane is rotated clockwise, the sign is positive, and negative for the anticlockwise rotation. A vane is usually penetrated vertically, then τ_θ, τ_0 in Eq. 5 are shear stresses on vertical planes. Therefore, we will express them as $\tau_{v\theta}, \tau_{v0}$ from now on to distinguish shear stresses on

Fig. 1 Stress components along slip lines (characteristics)

horizontal planes.

In the case of Fig. 2, the rotation is clockwise and τ_{v0} is the shear stress in the back sides of a rotating vane blade which is equal to the shear stress exerted in the natural state from the assumption 5). The torque caused by shear stress acting to the vertical plane of a vane T_v is expressed by

$$T_v = 4 \int_0^{\pi/2} \tau_{v0} r^2 \cdot h \cdot d\theta = 4hr^2 \cdot \tau_{v0} \int_0^{\pi/2} e^{2\theta \tan \phi} d\theta \\ = 2hr^2 \tau_{v0} \cdot \cot \phi (e^{\pi \tan \phi} - 1) \dots \dots \dots (6)$$

here r : radius of vane

h : length of vane

When following the conventional way (and also the Farrent's equation), the torque caused by shear stresses acting to the horizontal planes of a vane T_H is expressed by

$$T_H = \frac{4}{3} \pi r^3 \cdot \tau_H \dots \dots \dots (7)$$

here τ_H is a shear stress on the horizontal planes of a vane. However, shear resistance on the upper surface of a vane does not seem to work effectively because sand in this part is cut by four blades and four empty zones are formed, they must cause stress relief and the decrease of shear strength of sand under drained condition. In addition to the above fact, the assumption of uniform stress on the whole area of horizontal planes of a vane usually overestimates the shear resistance, because the peak strength can not be exerted simultaneously on the whole area of horizontal planes of a vane. Therefore, it will be a better approximation in vane tests of sand to neglect the shear resistance on the upper surface of a vane. Hence, we use the following equation instead of Eq. 7.

$$T_H = \frac{2}{3} \pi r^3 \cdot \tau_H \dots \dots \dots (8)$$

The total torque T measured by a vane is the sum of T_v and T_H . If we will carry out tests in use of different vanes in the ratio r/h , we might distinguish τ_v from τ_H . However, this is not the purpose of this paper. It is enough to know qualitatively how shear strength varies within a homogeneous sand layer including the ground water surface. Therefore, we approximate τ_H with τ_{v0} (shear strength on vertical planes in the natural state) in this paper. Here we will check the magnitude of error due to this approximation. When the standard vane of $h=4r$ is used and $\tau_{v0} = \frac{1}{2} \tau_H$ (K_0 -state, $c=0$) and the angle of internal friction $\phi=30^\circ$, the error is 2.6%. In the case of the vane of $h=32.0\text{mm}$, $r=11.6\text{mm}$ which was used in the model sand layers, it is 3.9% at the same condition of $\tau_{v0} = \frac{1}{2} \tau_H$ and $\phi=30^\circ$. This value is out of importance, hence the approximation of τ_H with τ_{v0} is passable. Then we can obtain the following relation.

$$T = \frac{2}{3} \pi r^3 \tau_{v0} + 2hr^2 \tau_{v0} \cdot \cot \phi (e^{\pi \tan \phi} - 1) \dots \dots \dots (9)$$

τ_{v0} is the shear stress on a slip plane at the back side of a vane and it is regarded to be equal to the shear stress at failure in the natural state (assumption 5). Shear stress at failure, that is shear strength, is usually expressed as S or τ_f then it will be better to replace τ_{v0} with S_v for the general use hereafter.

$$S_v = \frac{T}{\frac{2}{3} \pi r^3 + 2hr^2 \cot \phi (e^{\pi \tan \phi} - 1)} \dots \dots \dots (10)$$

Fig. 2 A slip line and the stresses around a vane

S_v : shear strength on the vertical plane

3. The application of vane tests to the measurement of the variation of shear strength in model sand layers.

Now we need to know the variation of shear strength in a model sand layer including the ground water surface to research the cause of slips on the ground water surface. The foregoing section shows that the torque measured by the vane test is a function of the angle of internal friction as well as shear strength. If the angle of internal friction is constant or approximately constant, we can estimate the variation of shear strength from vane tests. However, unless the angle of internal friction is even approximately constant, we can not estimate the variation of shear strength in model layers at all. In this section we will examine whether it is possible or not to regard the angle of internal friction as a constant in the model layers.

The model experiments of slips have been done in use of the Toyoura standard sand as stated in the foregoing paper, and vane tests have been done in the model layers of the Toyoura standard sand. Therefore, the grain shape, the quality and the specific gravity and the grain distribution can be regarded to be constant within the model layers. The measurement of shear strength by vane tests was performed from the surface to the depth of 16cm, which is enough deep in comparison with the slip surface (1~7cm) and enough small in comparison with the length (150cm) and the width (160cm) of the model layer to avoid the influence of side frictions.

The examples of void ratio in this depth of the sand layers are shown in Table 1. This table shows that the void ratio is slightly higher near the surface of the ground, however the difference of it is 0.02~0.04. Fig. 3 is the relation between the initial void ratio and $\tan\phi$ of the Toyoura sand by Inoue H.. It shows that the difference of void ratio ± 0.02 means the difference of $\tan\phi \pm 0.01$ (1% of the full value) in triaxial tests and ± 0.03 (4% of the full value) in direct shear tests. Whether the angle of internal friction which is effective for Eq. 10 is that measured by triaxial tests or that by direct shear tests is not clear, but $\tan\phi$ in the model layers can be regarded to be constant with the error of 1% (triaxial shear)~4% (direct shear).

As we measure the shear strength above and below the ground water surface, we will check the influence of saturation on the angle of internal friction. Fig. 4 is the result of direct shear tests of 30cm in diameter, where the normal stress is so small as vane tests in the model layers, and the void ratio is also similar to that of the model sand

Table 1 Examples of void ratio in model sand layers

Depth	Void ratio		
0 ~ 5 cm	0.82	0.82	0.85
5 ~ 10cm	0.80	0.79	0.83
10 ~ 15cm	0.80	0.78	0.83

Fig. 3 The relation between the angle of internal friction and void ratio (after Inoue H.)

○: $S_r=42.7\%$, $\tan\phi=0.74$
●: $S_r=100\%$, $\tan\phi=0.73$

Fig. 4 The change of shear strength by saturation

layers for vane tests. Shear strength decreases by saturation because of disappearance of capillary suction, but the angle of internal friction remains nearly constant. ($\tan \phi = 0.73 \sim 0.74$)

The above-stated examinations of the influence of void ratio and saturation on the angle of internal friction indicate that the angle of internal friction is practically constant in the layers of the Toyoura standard sand (void ratio 0.78~0.85) which were used in the vane tests as well as the model experiments of slips. Therefore, it is possible to say that the in-situ shear strength in the sand layers can be estimated by vane tests.

Finally we will check the precision of the equation 10 led in Section 2 in available data. To compare the shear strength measured by vane tests with that measured by other shear tests, the value of lateral stress is necessitated. Though it is usually difficult to know the correct value of the lateral stress in soil layers, it can be calculated by Jaky's equation if the sand layer is in Ko-state (normally consolidated state with no lateral strain). Hence, a sand layer of Ko-state was formed by pouring dry sand in the water filled in a shallow tub ($150 \times 75 \times 15 \text{ cm}$). And vane tests were carried out in the sand layers. Fig. 5 is the result of it. Plots of circles (o) are the shear strengths calculated by Eq. 10 from vane tests. A real line is the variation of shear strengths calculated by Fig. 3 and Jaky's equation as follows

$\tan \phi = 0.73$ in $e = 0.86$ from Fig. 3.

$K_0 = 1 - \sin \phi = 0.41$ from Jaky's equation.

$$S = \sigma \tan \phi = \gamma' z K_0 \tan \phi = 0.27z \dots\dots\dots (11)$$

here γ' : effective unit weight in water (0.9 g/cm^3)

S : shear strength

($c=0$ because of the full submergence)

The line of Eq. 11 well agrees with the shear strengths calculated by Eq. 10. On the other hand plots of crosses (+) in Fig. 5 are the shear strengths calculated by the conventional equation (12),

$$S = \frac{T}{\frac{2}{3} \pi r^2 (3h + 2r)} \dots\dots\dots (12)$$

The values and the dotted line to represent them are much higher than the theoretically expected values of Eq. 11. This result indicates that Eq. 10 is a right correction of the conventional equation for vane tests in sand, and it gave reasonable shear strengths in Fig. 5, though it will be not enough as the verification of Eq. 10.

4. Summary

The main problem on the application of vane tests to sand is that the effective stress on the shear plane is changed by the force transmitted from the vane blades because of the drained condition. (A minor problem on the application of vane tests to sand is that the void ratio at failure is usually different from the initial void ratio under the drained condition, but the void

Fig. 5 Comparison of the shear strengths measured by vane tests with the theoretical values

ratio at failure can not be measured in vane tests. However, since slips themselves are estimated to take place under the drained condition in sand layers and the influence of void ratio is not so great as shown in Fig. 3, it will be unnecessary to be so sensitive for this problem.)

The equation proposed by Farrent and the equation rededuced here are the corrections for this main problem. The equation deduced here is considered to be fundamentally right from the process to lead it and the examination of it in Fig. 5 indicates that it is numerically appropriate, though it will not be said that it has been verified enough. The corrective equation of vane tests includes the angle of internal friction in it. Therefore, when attempting the measurement of the variation of shear strength in the sand layer by vane tests, the variation of the angle of internal friction is also needed. However, the measurement of void ratio in the model sand layers and the relation between void ratio and the angle of internal friction by H. Inoue and the examination of the influence of saturation on the angle of internal friction have demonstrated that the tangent of the angle of internal friction ($\tan\phi$) can be regarded to be constant with 1~4% error in the model layers of the Toyoura standard sand (void ratio 0.78~0.85).

Now it will be possible to say that the necessary means to measure the variation of in-situ shear strength in the model sand layer has been established.

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