Relation between Helmholtz-Kohlrausch Effect, Purity Discrimination, and Go Function

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ABSTRACT

Using the nonlinear color-appearance model, predicted are the four kinds of color-perception phenomena, which are the Go function studied by Evans, Munsell Values for different hues, purity discrimination, and the Helmholtz-Kohlrausch effect. The analysis confirmed that all the four phenomena are caused by the same common factor, the chromatic strength of spectral color stimulus reported by Evans in 1967.

1. Introduction

The Helmholtz-Kohlrausch effect (sometimes called brightness/luminance ratio effect for illuminant colors or lightness/luminance-factor ratio effect for object colors),^{1,2)} purity discrimination^{3,4)} and zero-gray point Go by Evans⁵⁾ are completely different color perceptions each other. However, the experimental results reported so far¹⁻⁵⁾ suggest that mutually very close relations exist between the three phenomena. On the lightness/luminance-factor (L/Y) ratio effect, its experimental results¹⁻⁷⁾ were well predicted^{8,9)} within all the chromaticity gamut including spectral color stimuli by using the nonlinear color-appearance model^{10,11)} and the method for deriving the perceived lightness of chromatic colors⁸⁾ by the present authors.

The present paper shows that the other two phenomena are also predicted by using the nonlinear color-appearance model with a sufficiently good accuracy. In the study process, the present authors recognized the importance of Evans' claim that his Go function is a very fundamental phenomena in color-appearance study. Further,

based on this Go function the reason was made clear that Munsell Values of saturated colors are high in 5Y Hue and low for 5R, 5p, and 5B Hues.

In addition, the present authors also reconfirmed the other claim by Evans that the above various phenomena are illustrated commonly by the change of chromatic strength of spectral color by changing its wavelength^{5,12)}.

2. Go Function

Evans reported the following experimental results on color appearance of related colors^{5,12)}. Each of spectral color stimuli was presented to the aperture with 1° visual angle at the center of white background with 10° visual field. The stimuli studied consist of spectral colors and colors on the red-purple locus. The background was illuminated by light source with 7,143K, and its luminance was kept at 100mL (≈ 320 cd/m²).

The experiment was to ask each subject for assessing the color appearance of the test color stimulus presented at the center by raising its luminance from zero value. Its color appearance was perceived as an object color mode.

When the luminance of the test stimulus was set at zero, it was perceived as black. By increasing its luminance from zero, the blackness in its color appearance decreased, and its chroma and grayness perception appeared. Further, by increasing the luminance of the test stimulus, Evans determined its specific luminance value G_0 , at which just the grayness vanished and only the chroma perception existed in the color appearance of the test stimulus. Below the G_0 luminance, the test stimulus showed an object color mode of appearance, and the lightness perception was recognized in it.

An additional increase of the luminance of the test stimulus beyond the G_0 value, the test stimulus was perceived as a fluorescent color. Evans named this state fluorence. The G_0 luminance value of a spectral stimulus gave a critical point between the object color mode and the fluorence in its color appearance. Evans named this critical luminance value the zero-gray point corresponding to the zero grayness in the color appearance of the test stimulus, and specified the G_0 value by eq. (1).

$$G_0 = \log (L_0/L_t) \quad (1)$$

where L_0 is the luminance of the background and L_t is that of the test stimulus at the point of G_0 .

The experimental results by Evans himself⁽⁵⁾ is shown in **Fig. 1** by a dotted line from 400 nm to 700 nm. The results show the minimum G_0 value near 570 nm to 580 nm. This means the value L_t is high for yellow hue and low for red, purple, and blue hues at the G_0 point for a constant background luminance L_0 .

Now, we will predict this G_0 point by using the nonlinear color-appearance model. The computed condi-

Fig. 1 Comparison between G_0 and $\langle G_0 \rangle$. Dashed line corresponds to G_0 , and real line to $\langle G_0 \rangle$.

tions are as follows; the white background with $Y_0 = 100$, illuminant C for illumination, test illuminance $E_0 = 1000$ lx (background luminance $L_0 = 320$ cd/m²) corresponding to the experiment by Evans, the normalizing illuminance of the nonlinear color-appearance model $E_{or} = 1000$ lx, and the 22 pseudo object colors with $Y = 0.581$ to 120.0 at each wavelength from 400 nm to 700 nm with every 10 nm interval. The 22 Y values are shown in Table 1. The use of the above pseudo object colors corresponds to the presentation of the spectral stimuli with the luminance L by eq. (2) to the central test aperture in the actual experiment.

Table 1 Twenty two Y values used for deriving $\langle G_0 \rangle$

NO.	Y
1	0.581
2	1.210
3	2.021
4	3.162
5	4.614
6	6.555
7	9.003
8	12.000
9	15.570
10	19.770
11	24.580
12	30.050
13	36.200
14	43.060
15	50.680
16	59.100
17	68.400
18	78.660
19	90.010
20	100.000
21	110.000
22	120.000

$$L = Y E_0 / 100\pi \quad (2)$$

As expected, a caution should be made to illuminate the background not to affect the luminance of the central test stimulus. This can be easily done in the actual experiment.

The computational procedure is given below.

- (1) The values of Q_t and C_t of test stimulus are computed by changing Y value from 0.581 to 120.0 at each wavelength using the nonlinear color-appearance model. The Q_t function is a correlate to the achromatic component of lightness of the test color, and the C_t function corresponds to its

Fig. 2 Determination of the point P with Q_λ , C_λ , corresponding to $\langle Go \rangle$ at 550 nm. The curve 1 corresponds to the relation between Q_t and C_t , and the curve 2 to the relation between $Q_v(C_t)$ and C_t . The point P is the intersection between the two curves 1 and 2.

chroma. The computed results are shown in a Q-C plane. An example is shown in **Fig. 2** to 550 nm. The curve 1 in the figure corresponds to the $Q_t - C_t$ curve at 550 nm. For the specified illumination and background, the curve 1 corresponds to the maximum chroma perception at any specified value of Q_t for each test stimulus. This means that the spectral stimulus at 550 nm can not have chroma perceptions in the right hand side of curve 1.

- (2) At each wavelength, the locus is specified by eq. (3), on which only grayness and chroma perceptions coexist⁹⁾.

$$Q_v(C_t) = q_1(\theta) C_t + q_2(\theta) C_t^2 \times 10^{-4}, \quad (3)$$

where θ is the hue angle in the nonlinear color-appearance model, and the functions $q_1(\theta)$ and $q_2(\theta)$ are shown in Appendix C of our previous article⁹⁾. The values of θ are also given in Table 2 for each wavelength λ . The object colors predicted by eq. (3) correspond to those found on the horizontal line¹³⁾ connecting gray ($s = 50$, $Cr = 100$) and the full color ($s = 0$, $Cr = 100$) at each of the NCS constant hue planes.¹⁴⁾ Though the NCS gray has Munsell Value 5.8/, we specified the gray point by Munsell Value 5/ in the non-linear color-appearance model, and the grayness-chroma curve in eq. (3) is specified by the gray with 5/ as a starting point. This selection was done by con-

Table 2 Relation between λ and θ°

λ (nm)	θ°	λ (nm)	θ°
400	310.3	600	68.09
410	306.9	610	56.61
420	299.2	620	48.87
430	279.3	630	44.13
440	253.7	640	40.92
450	232.9	650	39.02
460	220.2	660	37.89
470	213.9	670	37.33
480	207.2	680	36.94
490	194.1	690	36.66
500	176.1	700	36.57
510	159.1	-565	330.10
520	145.9	-560	337.35
530	137.9	-550	342.30
540	131.6	-540	344.71
550	125.6	-520	347.88
560	118.7	-505	351.87
570	109.6	-500	355.12
580	97.5	-497	358.99
590	82.7	-495	4.03
		-494	8.68
		-493	18.18

sidering the achromatic color with 5/ the gray.

The curve 2 in Fig. 2 shows the relation between $Q_v(C_t)$ and C_t . Moving along the $Q_v(C_t) - C_t$ curve to the right direction, the grayness decreases and to the contrary the chroma increases in the color appearance of the object colors on it.

- (3) The values of Q_λ , C_λ of the point P in Fig. 2 is determined corresponding to the intersection of curves 1 and 2 in Fig. 2. The point P corresponds to the color perception with the maximum chroma but with the minimum grayness in the test color at each wavelength. The values Q_λ and C_λ are considered the Go point at the wavelength λ under consideration. They were estimated from 400 nm to 700 nm, and are shown in Table 3.
- (4) The values of Q_λ and C_λ in Table 3 are further transformed to colorimetric values $\langle x_\lambda \rangle$, $\langle y_\lambda \rangle$, Y_λ by using inverse transformation of the nonlinear color-appearance model. In the transformation, again illuminant C, $Y_0 = 100$, and $E_0 = E_{or} = 1000$ lx are used. By confirming the sufficiently well agreement between the derived $\langle x_\lambda \rangle$, $\langle y_\lambda \rangle$ values and the input x_λ , y_λ values in step 1, the check of the whole computations done were made. The fourth column of Table 3, the values of Y_λ are shown.

Table 3 Values of Q_λ , C_λ , Y_λ , and $\log (100/Y_\lambda)$ corresponding to the $\langle Go \rangle$ values for spectral colors

λ	Q_λ	C_λ	Y_λ	$\log (100/Y_\lambda)$
400	-14.0	292.0	7.67	1.12
410	-15.5	282.0	7.14	1.15
420	-20.0	262.0	5.66	1.25
430	-26.0	229.0	4.39	1.36
440	-29.0	219.0	4.24	1.37
450	-25.0	248.0	6.43	1.19
460	-16.0	280.0	12.76	0.89
470	-13.0	255.0	14.81	0.83
480	-12.0	199.0	14.14	0.85
490	-8.0	158.0	16.49	0.78
500	-6.0	140.0	17.49	0.76
510	-5.0	142.5	17.92	0.75
520	1.0	153.5	23.19	0.63
530	6.0	150.5	27.63	0.56
540	11.0	164.0	31.87	0.50
550	16.5	160.5	36.81	0.43
560	22.0	163.5	42.37	0.37
570	27.5	156.5	48.13	0.32
580	33.0	148.0	54.24	0.27
590	36.0	139.0	56.90	0.24
600	31.0	140.5	47.00	0.33
610	20.0	144.5	32.43	0.49
620	7.0	148.5	19.80	0.70
630	-0.5	156.0	14.58	0.84
640	-4.5	163.5	12.31	0.91
650	-8.5	168.0	9.94	1.00
660	-8.5	172.0	9.92	1.00
670	-9.0	173.5	9.62	1.02
680	-10.0	175.0	9.04	1.04
690	-10.5	175.5	8.77	1.06
700	-10.5	176.0	8.76	1.06

- (5) At each wavelength, the $\langle Go \rangle$ values, the estimates of Go , are given by eq. (4).

$$\langle Go \rangle = \log [100/Y_\lambda] \quad (4)$$

The value 100 corresponds to the tristimulus value Y of the background. The value $100/Y_\lambda$ in eq. (4) corresponds to Lo/Lt in eq. (1). The values of $\langle Go \rangle$ are shown in the fifth column of Table 3. The real line in Fig. 1 corresponds to the $\langle Go \rangle$ values. Its height was adjusted by using the method of least squares for 400 nm to 700 nm to have a good conformity between Go and $\langle Go \rangle$.

By inspecting Fig. 1, the agreement between Go and $\langle Go \rangle$ is pretty good except for the wavelength region shorter than 440 nm and the steep change of Go near 580 nm. The observed Go curve

in Fig. 1 is based upon experiment only done by Evans himself. In addition, the accurate visual observation is generally difficult in the spectral range less than 440 nm. Considering a pretty good agreement⁹⁾ between the experimental results and the predictions in the Helmholtz-Kohlrausch effect for the spectral range, the cause of the deviation between Go and $\langle Go \rangle$ below 440 nm can not be reduced to only the predicting method of $\langle Go \rangle$ used. In fact, the originally measured value of Go by Evans¹²⁾ gave a saturated characteristic instead of a uniform increase to high values as shown in the dotted line in Fig. 1 below 440 nm.

Next we consider the reason of the deviation between Go and $\langle Go \rangle$ near 580 nm. The Q_v (C_t) function in eq. (3) used for estimating the $\langle Go \rangle$ function is derived from the analysis of object colors between gray and full color at each hue plane of the NCS color scheme. However, the steep change of Q_v (C_t) function was not found near 580 nm in the NCS color scheme. This is the reason why the $\langle Go \rangle$ function did not show the steep change near 580 nm.

Now, we consider the meaning of Go or Lo/Lt . The quantity Lt is the minimum luminance needed to cancel the blackness found in the test color stimulus at $Lt = 0$ completely. The color perception of the test stimulus at Go consists of only chroma component. The Lt value is high near 580 nm (yellow hue) and low near 450 nm and 650 nm. The Go value changes to the contrary to Lt . The characteristic of Go function suggests that yellow spectral color stimuli are weaker in the power for canceling the blackness perceptions existing in them than those existing in other hues.

This canceling power corresponds to the chromatic strength of test color stimulus reported by Evans¹²⁾.

3. Y Values of Full Colors in Munsell Color Scheme

The Munsell Values of full colors are different for different Munsell Hues. Approximately, they are estimated 5/ to 5R, 9/ to 5Y, 5/ to 6/ to 5G, and 3/ to 4/ to 5PB¹⁵⁾. Evans^{12, 16)} already stated that the differences of Munsell Values of full colors are caused by the change of chromatic strength for different Hues as found from the Go function in Fig. 1. As the chromaticity coordinates of full colors in the above 4 Munsell-Hue planes are very close to those of spectral colors, the following spectral colors were substituted for the above Munsell full colors; 700 nm for 5R, 580 nm for 5Y, 510 nm for 5G, and 400 nm for 5PB. The values Y_λ for the 4 wavelengths in Table 3 were specified as the Y values of full colors in the four Munsell Hues considered. The value Y_λ corresponds to the $\langle Go \rangle$ value directly. The results of Y (V/) estimated are 8.8 (3.5/) for 5R, 54.2 (7.7/) for 5Y, 17.9 (4.8/) for 5G, 7.6

(3.2/) for 5PB. The numbers in the parenthesis show the Munsell Values V/λ of the estimated Y value.

The Munsell Values of full colors for the four Munsell Hues predicted from $\langle Go \rangle$ change similarly to those already shown in the Munsell scheme. A detailed inspection shows that the predicted Munsell Values are uniformly about 1 Value lower than those found in the Munsell scheme. The cause of the above deviations between the predicted and the specified Munsell Values may be caused by the use of Munsell Value 5/ for the complete gray in the nonlinear color-appearance model. The gray of the NCS scheme ($s = 50$, $Cr = 0$) estimated by using a white background has the Munsell Value 5.8/. If the NCS gray point is used for the gray point in the nonlinear color appearance model, the Value estimates for full colors from $\langle Go \rangle$ increase by about 0.8 Value. This may eliminate the above 1 Munsell Value deviations.

4. Purity Discrimination

We consider that a spectral color stimulus is added to an achromatic one. The total luminance of the mixed color stimulus is always kept constant by adjusting the luminance of achromatic stimulus irrespective of the added quantity of spectral color stimulus. When the luminance of added spectral color stimulus is very little, the mixed color stimulus is still perceived as achromatic. By adding an appreciable amount of the spectral color stimulus, the chromatic component in the mixed color stimulus is clearly perceived corresponding to the hue of the added spectral color. In an intermediate amount of mixing spectral color stimulus, the purity threshold exists.

The purity threshold $\Delta P_{c\lambda}$ is given by eq. (5).

$$\Delta P_{c\lambda} = \Delta L_{\lambda} / (L_w + \Delta L_{\lambda}), \quad (5)$$

where ΔL_{λ} is the luminance of the added spectral color stimulus at the purity threshold, and L_w is the luminance of achromatic color stimulus. As already stated the total luminance ($L_w + \Delta L_{\lambda}$) is kept constant throughout in the experiment.

Many studies were reported so far in the purity threshold^{3, 4)}. Pridmore⁴⁾ summarized the results of various experimental and theoretical studies on the purity discrimination. Figure 3 shows the experimental results corresponding to the two kinds of different experimental conditions.

The curve 1 in **Fig. 3** corresponds to the purity-threshold experiment reported by Priest et al.¹⁷⁾. The curve 2 in the figure corresponds to the purity difference on the equi-chroma colors, which are experimentally determined by Uchikawa et al.¹⁸⁾ in the color gamut beyond the purity threshold. In other words, the curve 1 in **Fig. 3** may correspond to the equi-chroma loci at the purity

Fig. 3 Two kinds of discrimination curves on colorimetric purity. The curve 1 corresponds to the discrimination experiment at threshold, and the curve 2 to the experiment estimating a constant perceived chroma above the threshold.

threshold. It is also believed that the difference exists between the color-differences at and beyond the color-difference threshold. On the purity discrimination, the differences at and beyond the purity threshold are found as shown in curves 1 and 2 in **Fig. 3**.

The two curves in **Fig. 3** generally agree in shape. However, its detailed inspection clarifies that the purity discrimination at threshold (curve 1) changes steeper than that beyond threshold (curve 2) in the spectral range 500 nm to 570 nm.

The nonlinear color-appearance model does not expect to predict the color discrimination at threshold. By this reason, the colorimetric purity curve was estimated to the object colors corresponding to an equi-chroma perception beyond the threshold using the model. The predicted results are expected to be similar to the experimental results by Uchikawa et al.¹⁸⁾ (curve 2 of **Fig. 3**).

The conditions of computation are as follows; Y value of achromatic background $Y_0 = 20$, illuminant C for illumination, $E_0 = E_{or} = 1000$ lx. The test samples are the object colors with $Q = 0$ and $C = 4$ in the nonlinear color-appearance space with different dominant wavelengths.

The following procedure was used for computation.

- (1) To each dominant wavelength λ (its hue angle θ_{λ}), the following quantities are specified.

$$\begin{aligned} Q &= 0, \\ T &= C \cos \theta_\lambda, \\ P &= C \sin \theta_\lambda, \end{aligned} \quad (6)$$

where $C = 4$. The relation between λ and θ_λ is already given in Table 2.

- (2) To each wavelength, the values Q , T , P in eq. (6) are further transformed to the corresponding

Table 4 Values of x'_λ , y'_λ , and $\log [1/\Delta P_{c\lambda}]$ corresponding to the equal chroma locus with $Q = 0$, $C = 4$ (Munsell colors with 5/0.7) in the nonlinear color-appearance model.

λ	x'_λ	y'_λ	$\log [1/\Delta P_{c\lambda}]$
400	0.3078	0.3052	3.26
410	0.3072	0.3050	3.25
420	0.3060	0.3048	3.21
430	0.3034	0.3051	3.09
440	0.3015	0.3074	2.99
450	0.3013	0.3102	2.94
460	0.3013	0.3120	2.86
470	0.3014	0.3130	2.64
480	0.3017	0.3141	1.79
490	0.3025	0.3166	1.57
500	0.3044	0.3203	1.50
510	0.3069	0.3237	1.40
520	0.3093	0.3262	1.30
530	0.3109	0.3277	1.24
540	0.3123	0.3287	1.18
550	0.3137	0.3296	1.13
560	0.3153	0.3305	1.07
570	0.3175	0.3314	0.97
580	0.3204	0.3321	1.13
590	0.3231	0.3312	1.20
600	0.3242	0.3286	1.30
610	0.3244	0.3261	1.39
620	0.3242	0.3243	1.45
630	0.3239	0.3231	1.50
640	0.3237	0.3223	1.54
650	0.3235	0.3219	1.56
660	0.3234	0.3216	1.57
670	0.3234	0.3214	1.58
680	0.3233	0.3213	1.59
690	0.3233	0.3213	1.59
700	0.3233	0.3212	1.59
-565	0.3115	0.3071	2.74
-560	0.3130	0.3081	2.39
-550	0.3140	0.3089	2.17
-540	0.3145	0.3093	2.07
-520	0.3151	0.3099	1.95
-505	0.3159	0.3106	1.84
-500	0.3166	0.3113	1.78
-497	0.3174	0.3121	1.77
-495	0.3184	0.3132	1.74
-494	0.3193	0.3142	1.71
-493	0.3210	0.3166	1.66

chromaticity coordinates x'_λ , y'_λ by using the inverse transformation of the nonlinear color-appearance model. The results are shown in Table 4.

The locus of the derived chromaticity coordinates is shown by a real line in Fig. 4, and that of the estimated chromaticity coordinates of color samples with 5/0.7 is shown by a dashed line. As expected, the both curves are similar to each other.

Fig. 4 Comparison between the chromaticity locus of the object colors with $Q = 0$, $C = 4$ (real line) and that of the Munsell colors with 5/0.7 (dashed line)

- (3) The value of $\Delta P_{c\lambda}$ is derived by eq. (7). It is already well known that eq. (5) is completely equivalent to eq. (7).

$$\Delta P_{c\lambda} = y_\lambda^d \times Pe / y'_\lambda, \quad (7)$$

where y_λ^d is the y chromaticity coordinate of dominant wavelength λ , and Pe is the excitation purity of x'_λ , y'_λ .

- (4) Derivation of $\log (1/\Delta P_{c\lambda})$. The computed results are also shown in Table 4. Figure 5 shows the values of $\log (1/\Delta P_{c\lambda})$ from 400 nm to 700 nm, and the results are quite similar to curve 2 in Fig. 3 as expected.

It is not a good agreement between Fig. 5 and curve 1 of Fig. 3, because the nonlinear color-appearance model does not expect to predict color discrimination at threshold. This insufficient agreement may suggest the necessity of adding some mechanisms to the nonlinear color-appearance model to make it possible to predict the discrimination phenomena at threshold.

Fig. 5 Relation between colorimetric purities for object colors with $Q = 0$ and $C = 4$ and their dominant wavelengths.

The Go function in Fig. 1 and the purity discrimination in Fig. 3 are completely different phenomena, and they were estimated by using completely different experimental methods. Even so, spectral characteristics of the two phenomena are quite similar. Evans¹²⁾ already pointed out that the functions Go and $\log(1/\Delta P_{C\lambda})$ could be illustrated by the wavelength characteristic of the chromatic strength of spectral color stimuli and were closely related each other.

As clearly found in Figs. 3 and 5, the low value of $\log(1/\Delta P_{C\lambda})$ in the range 570 nm to 580 nm naturally suggests that ΔL_λ is high as compared with other wavelengths for constant value of $L_w + \Delta L_\lambda$. To the contrary, ΔL_λ is low for the spectral range near 400 nm. In other words, the chromatic strength of yellow hue spectral stimuli are weaker than those with other hues, for example blue, purple and red hues. In the present paper, this Evans' insight was theoretically confirmed by predicting the two phenomena by using the single nonlinear color-appearance model.

5. Helmholtz-Kohlrausch Effect

The luminance of each spectral color stimulus is determined by the flicker photometry and the CIE 1924 spectral luminosity function $V(\lambda)$ ²⁰⁾ is effective to compute the luminance value of any color stimulus. However, two spectral color stimuli kept at the same lumi-

nance but with different wavelengths are generally different in brightness. For example, the brightness of 450 nm color stimulus is higher than that of 570 nm one in their direct comparison, even though they are kept at the same luminance. By this reason, CIE reported the luminous efficiency function $V_{b,2}(\lambda)$ for heterochromatic brightness matching in 1988²⁾.

The deviations between $V_{b,2}(\lambda)$ and $V(\lambda)$ correspond to the Helmholtz-Kohlrausch effect or brightness/luminance (B/L) ratio effect for luminous colors. Several experimental studies are given in the reference of the technical report²⁾.

Based on these studies, the B/L ratio effect is estimated by eq. (8) below. However, in eq. (8), the Judd modified spectral luminosity function $V_M(\lambda)$ is used instead of $V(\lambda)$. The $V_M(\lambda)$ function initially reported by Judd²¹⁾ and recommended its final version by CIE in 1990, because of the shortage in sensitivity of the $V(\lambda)$ function for the spectral range less than 460 nm.

$$\delta(\lambda) = \log [V_{b,2}(\lambda) / V_M(\lambda)] - \log [V_{b,2}(570) / V_M(570)] \quad (8)$$

The value of $\delta(\lambda)$ is normalized to zero at 570 nm. The computed results by eq. (8) is shown by the real line in Fig. 6. The dotted and the dott-dash line in Fig. 1 are the predicted lightness / luminance factor (L/Y) ratios⁹⁾ by using the nonlinear color-appearance model^{10,11)} and the hypothesis⁸⁾ on the perceived lightness of chromatic object colors. They give the estimates of the B/L ratios of the real line on the same spectral range. The dotted line in Fig. 6 is derived on the basis of the Wyszecki experi-

Fig. 6 Three kinds of estimations on the Helmholtz-Kohlrausch effect. Real line corresponds to $\delta(\lambda)$ in eq. (7). Dot-dash line is estimated on the basis of the Wyszecki experiments⁶⁾, and dashed line on the basis of the Sanders-Wyszecki experiment⁷⁾.

ment⁶⁾ using real object colors, and the dott-dash line is derived on the basis of the Sanders-Wyszecki experiment⁷⁾ using pseudo-object colors. The agreement between the three curves in Fig. 6 is remarkably good. Again, the Helmholtz-Kohlrausch effect is high near 400 nm and 700 nm as compared with near 570 nm. This again shows that the chromatic strength is low at near 570 nm as compared with other wavelengths.

The chromatic strength is the ability of spectral color stimulus to cancel the blackness in the Go function, and to the contrary in the Helmholtz-Kohlrausch effect the chromatic strength of spectral color stimulus is the ability to induce the brightness or the lightness perception. This two concepts on the chromatic strength are considered to be equivalent.

It should be noted that the effect in Fig. 6 is relatively smaller (about one third) than those found in Fig. 1 on the same logarithmic scale. This is easily found by comparing the effect difference between 400 nm and 570 nm in the figures. At present, the authors suspect that the small magnitude of the Helmholtz-Kohlrausch effect found in Fig. 6 is mainly caused by the method used for representing the effect. On this problem, we will discuss in our future study.

6. Conclusion

The conclusions in the present studies are summarized as follows.

- (1) It was made clear that all the different four phenomena, the Go function, the change of Munsell Values of full colors for different hues, purity discrimination, and the Helmholtz-Kohlrausch effect, are caused by the same common factor, the chromatic strength of spectral color stimulus already reported by Evans^{3,12)} in 1967.
- (2) The chromatic strength of spectral colors near 570 nm is lower than other spectral colors. This means that the ability for canceling the achromatic component (blackness) found in color stimulus is low in the spectral colors near 570 nm. This situation corresponds to the Go function and the Y values of most saturated colors in the Munsell scheme. For the constant chroma contour, the chromatic strength of spectral colors is understood as follows. The luminance value ΔL_k in eq. (5) at about 570 nm for inducing a constant chroma perception is higher (lower in chromatic strength) than other wavelengths. In the Helmholtz-Kohlrausch effect, the chromatic strength of spectral colors are found by the low ability for increasing the brightness or lightness in the color stimulus at about 570 nm as compared with other wavelengths.
- (3) The above four phenomena in item (1) were well predicted by the nonlinear color-appearance model^{10,11)} and the method for deriving the perceived lightness of chromatic colors⁸⁾. The nonlinear

color-appearance model has a wide range of application including the prediction of chromatic adaptation.

- (4) The authors now believe that the Go function is a fundamental quantity on the color appearance of object and luminous colors. The importance of the studies by Evans should be recognized in the colors community.

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