Shape Recovery of Glossy Surface from Shading Image and its Directional Derivatives

Hidekazu TANAKA*, Yuji IWAHORI** and Naohiro ISHII**

* Toyota College of Technology,
2-1, Eisei-cho, Toyota, 471 JAPAN

** Nagoya Institute of Technology,
Gokiso-cho, Showa-ku, Nagoya, 466 JAPAN
Received February 18, 1994.

ABSTRACT

A new estimation approach is proposed to reconstruct the glossy surface shape from shading images.

The reflectance property of an object is uniform glossy surface described by Phong reflectance model, and the light source produces a uniform parallel beam and the image projection is orthographic. This approach is a kind of wide range recovery technique and the illuminating direction is varied slightly, and the directional derivatives of image density are obtained. The local surface orientation parameters of the curved and polyhedral objects are estimated from the image density and the directional derivatives by a nonlinear optimization method. The estimated gradient values for Lambertian surface are used as the initial values in the optimization, and they are determined only by the directional derivatives of the image density.

The effectiveness of this approach is demonstrated in the shape reconstruction simulations for the polyhedral and curved objects.

keywords shape from shading, photometric flow fields, glossy surface, directional derivatives

1. Introduction

A new approach is proposed to reconstruct the surface shape from shading images. The imaging conditions are supposed that the reflectance property of the object surface material is glossy non-Lambertian, and the light source produces a uniform parallel beam and image projection is orthographic.

Horn formulated *Shape from Shading*, one of the useful techniques that reconstruct the shape of the three-dimensional object from its two dimensional shading image¹⁾²⁾. Woodham introduced *Photometric Stereo* to determine the local Lambertian surface shape of the object from three measured images under the independent and distant light sources of illumination³⁾. Iwahori *et al.* extended the photometric method to the object illuminated by a point light source to determine the local Lambertian surface shape of the object and the distance from the lens⁴⁾. The shape recovery techniques from the image density and its directional derivatives⁵⁾⁸⁾¹⁰⁾ are proposed to reconstruct more surface elements in comparison with *Photometric Stereo*. These techniques require the condition that the reflectance property of the object surface is uniform Lambertian.

The extended *Photometric Stereo* developed the shape recovery algorithm for the glossy surface object⁷⁾. The property of the surface reflectance which depends on Phong reflectance function⁹⁾ is supposed in this method. The estimation of these reflectance parameters and the shape recovery by the estimated parameters are performed from three image density values illuminated by independent and distant light sources. Tagare *et al.* proposed a theory of *Photometric Stereo* for a large class of non-Lambertian reflectance property⁹⁾.

In this paper, the wide range recovery techniques from the image density and its directional derivatives are proposed to reconstruct the glossy or specular surface shape. Phong reflectance function is also supposed as the reflectance property of the object surface. The surface shape *i.e.*, the gradient parameters of the local surface element of the object are determined by a nonlinear optimization approach¹²⁾. The object function for optimization consists of the sum of square errors with the image density and its directional derivatives. The initial values of the gradient parameters for optimization are selected by the Lambertian estimates. The Lambertian estimates stand for the estimated gradient values for Lambertian surface object, and are derived only by the directional derivatives of the image density. The illuminating direction, the coefficient of

reflectance are not required for these estimates¹⁾. Four images are required to reconstruct the surface shape, by the changing illuminating directions.

Computer simulations show the evaluation of errors in the approximation of the directional derivatives and the quantization process of the image density. The effectiveness of this approach is demonstrated for the polyhedron and the curved objects with glossy surfaces.

2. Illuminating Equation

The imaging conditions are supposed that the reflectance property of an object surface material is glossy non-Lambertian. The light source is a distant source of illumination and produces a uniform parallel beam and the image projection is orthographic as shown in Figure 1.

Suppose the reflectance property of the object surface material obeys Phong reflectance function, then the image density is described as,

$$d(x, y) = c\{s \cos^n \varphi + (1 - s) \cos \theta\} \quad (1)$$

where $d(x, y)$ is an image density (*i.e.* image irradiance) obtained at an image pixel $d(x, y)$ on the image plane reflected at the surface element (X, Y, Z) from the incident light source. The factor c represents the proportional constant including the strength of the light source, the surface albedo, camera parameter *etc.*, $\cos^n \varphi$ defines the specular component and $\cos \theta$ defines the diffuse component. The factor s represents the ratio of the strength of the specular component to that of the diffuse component, and n represents the width (*i.e.* sharpness) of the specular peak.

In the condition that the coefficient c is constant for all the surface elements of the object, this model represents the property of reflectance from diffuse surface to glossy surface with the two reflectance parameters s and n . If the parameter s is set to 0.0, the reflectance property is Lambertian, and if s is close to 1.0, its reflectance property is close to the specular.

Let three vectors at a surface element be $\mathbf{n}_s$ as a unit vector to the illuminating direction, $\mathbf{n}_o$ as a unit object surface normal vector and $\mathbf{v}$ as a unit vector to the viewing direction.

The unit illuminating direction vector $\mathbf{n}_s$ is defined as $(u, u, w)^T$, and its components are restricted by the equation, $u^2 + v^2 + w^2 = 1$.

The unit surface normal vector $\mathbf{n}_o$ is defined as $\frac{1}{\gamma}(-p, -q, 1)^T$, where, p and q are the first order partial derivatives of z with respect to x or y , they are defined as $p = \partial z / \partial x$ and $q = \partial z / \partial y$. And the variable γ means $\sqrt{p^2 + q^2 + 1}$.

The unit viewing vector $\mathbf{v}$ is defined as $(0, 0, 1)^T$.

The variable θ is the angle between the illuminating direction vector $\mathbf{n}_s$ and the surface normal vector $\mathbf{n}_o$, and it is defined as follows,

$$\cos \theta = (\mathbf{n}_o \cdot \mathbf{n}_s) \quad (2)$$

$(\mathbf{n}_o \cdot \mathbf{n}_s)$ denotes the scalar product to two vectors $\mathbf{n}_o$ and $\mathbf{n}_s$.

The variable φ is the angle between the light reflection direction vector $\mathbf{n}'_s$ and the viewing direction vector $\mathbf{v}$,

$$\cos \varphi = (\mathbf{n}'_s \cdot \mathbf{v}) \quad (3)$$

and this equation is described as follows,

$$\cos \varphi = 2(\mathbf{n}_o \cdot \mathbf{n}_s)(\mathbf{n}_o \cdot \mathbf{v}) - (\mathbf{n}_s \cdot \mathbf{v}) \quad (4)$$

Then, next equations are derived

$$\cos \varphi = 2 \frac{-pu - qv + w}{\gamma^2} - w \quad (5)$$

$$\cos \theta = \frac{-pu - qv + w}{\gamma} \quad (6)$$

Substituting these equations into Equation (1), the relation between the image density $d(x, y)$ and the gradient parameters p and q may be obtained. This relation is non-linear. It is impossible to solve it with respect to p and q analytically, even if the additional equations or restrictions are given. Therefore, we search the optimal parameters p and q to minimize the objective function by the numerical analysis method such as the Newton method.

Figure 1: Coordinate System

3. Shape Recovery Algorithm by Non-linear Optimization Approach

A shape recovery problem is defined as the searching problem to obtain the optimal solutions which are closed to the true gradient parameters p and q of the surface element to minimize the appropriate object function by a nonlinear optimization approach,

3.1 Objective Function for Optimization

Let the objective function for optimization be the sum of square errors with the image density and its directional derivatives. The image density error is defined as the difference between the image density model by Equation (1) and the observed one. The directional derivative errors of the image density are defined as the difference between the differentiations of Equation (1) with respect to the component of illuminating direction vector n_s and the corresponding approximated derivative values from observed image density values. The optimization is to minimize the following sum of square errors for the surface element of the object.

$$f(p, q) = (d - d^*)^2 + \lambda \left\{ (d_u - d_u^*)^2 + (d_v - d_v^*)^2 + (d_w - d_w^*)^2 \right\} \quad (7)$$

where an image density at a pixel (x, y) illuminated from the direction $n_s = (u, v, w)^T$ is represented as $d = d(u, v, w)^T$, while, $d^* = d^*(u, v, w)^T$ is an observed one, and λ is an appropriate coefficient. By varying illuminating direction $(u, v, w)^T$ with small offsets δu , δv and δw , we obtain three observed image density values at the same pixel, $d^*(u, v, w)$, $d^*(u + \delta u, v, w)$, $d^*(u, v + \delta v, w)$ and $d^*(u, v, w + \delta w)$.

The theoretical directional derivatives of the image density are defined as $d_u = \partial d / \partial u$, $p = \partial d / \partial v$ and $d_w = \partial d / \partial w$ respectively. The variables d_u^* , d_v^* and d_w^* are approximated values of d_u , d_v and d_w respectively, and obtained from the observed image density values by the following.

$$\begin{aligned} d_u^* &= \frac{d^*(u + \delta u, v, w) - d^*}{\delta u} \\ d_v^* &= \frac{d^*(u, v + \delta v, w) - d^*}{\delta v} \\ d_w^* &= \frac{d^*(u, v, w + \delta w) - d^*}{\delta w} \end{aligned} \quad (8)$$

Four image density values, $d^*(u + \delta u, v, w)$, $d^*(u, v + \delta v, w)$ and $d^*(u, v, w + \delta w)$ at each image pixel (x, y) are necessary for this approximation.

3.2 Nonlinear Optimization Approach

The Newton method is an available approach because of the rapid convergence and solutions can be determined if sufficient good initial estimates are given. However, it often fails to converge to solutions from poor initial estimates. And the Hessian matrix consists of the second partial derivatives of the object function, we have to perform extremely laborious and time-consuming operation to obtain the elements of Hessian.

In this paper, we consider DFP method (Davidon, Fletcher and Powell) and BFGS method (Broydon, Fletcher, Goldfarb and Shanno) as the representatives of the quasi-Newton methods¹²⁾, in the reason that they have sufficiently rapid and stable convergence.

The termination condition of convergence are given in the following,

$$|f(p, q)| < 10^{-6} \quad (9)$$

It is defined as an allowable error between the estimated values and true ones.

3.3 Selection of Initial Values

The selection of initial values for optimization is important for good solution and rapid convergence. In this shape recovery problem, the solutions are near the Lambertian estimates for the almost surface elements. Therefore, Lambertian estimates are good initial values of optimization.

If the reflectance property is Lambertian, the image density value $d_L(x, y)$ is determined as the following equation, and it is a case of $s = 0$ in Equation (1),

$$d_L(x, y) = c \frac{-pu - qv + w}{\gamma} \quad (10)$$

In this condition, the gradient parameters p , q are derived analytically as follows¹⁰⁾,

$$\begin{aligned} p_L &= -\frac{d_{Lu}}{d_{Lw}} \\ q_L &= -\frac{d_{Lv}}{d_{Lw}} \end{aligned} \quad (11)$$

where, d_{Lv} and d_{Lw} are the directional derivatives obtained by varying the illuminating direction for the object of Lambertian reflectance property, i.e., $d_{Lu} = \partial d_L / \partial u$, $d_{Lv} = \partial d_L / \partial v$ and $d_{Lw} = \partial d_L / \partial w$ respectively.

3.4 Algorithm for Shape Recovery

The algorithm for shape recovery by the nonlinear optimization approach is presented in this section. Suppose that, the reflectance parameters (n , s , c) of the object surface and the offset values δu , δv and δw of the unit illuminating direction vector $(u, v, w)^T$ are known *a priori*.

The offset values are defined as follows, In the Cartesian coordinates, suppose that the light source position is $(U, V, W)^T$, and three directional displacement values ΔU , ΔV and ΔW of light source position are known values. The offset values δu , δv and δw of the unit vector are defined as $\delta u = \Delta U/R$, $\delta v = \Delta V/R$ and $\delta w = \Delta W/R$. The distance R between the light source and the origin of the coordinates is defined as $R = \sqrt{U^2 + V^2 + W^2}$.

Repeat the following procedure steps for all pixels corresponding to the surface elements of an object.

1. Pick up the image density values
Picking up an image density value $d^*(u, v, w)$ at an image pixel (x, y) , and three image density values $d^*(u + \delta u, v, w)$, $d^*(u, v + \delta v, w)$ and $d^*(u, v, w + \delta w)$ by varying the illuminating direction with offsets.
2. Calculation of directional derivatives
Calculating the directional derivatives d_u^* , d_v^* and d_w^* from these four image density values.
3. Optimization
Search the optimal gradient parameters p and q to minimize the objective function defined by the sum of square errors of the image density and its directional derivatives. Quasi-Newton method in the nonlinear optimization approach is adopted for this purpose. The initial values of the gradient parameters (p, q) for optimization are the Lambertian estimates (p_L, q_L) given by Equation (11).

4. Simulation

In the first, errors in approximating the directional derivatives by the numerical difference method and errors in quantization of input images are evaluated by the computer simulations. Next, the selection of initial values of the gradient parameters (p, q) for optimization are evaluated. The candidates of the initial values are null set of parameters $(0, 0)$ and the Lambertian estimates (p_L, q_L) by the Equation (11). And, the shape recovery simulations of the polyhedron and the curved object with the glossy reflectance property are demonstrated based on these evaluation results.

We suppose the components of an unit illuminating vector $\mathbf{n}_s$ are set to $(0.052926, 0.084681, 0.995002)^T$ in this simulation. And the offset values δu , δv and δw are known *a priori*.

4.1 Simulation for Error Analysis

The numerical difference in approximating the directional derivatives is defined by the ratio between the difference of two image density values and the offset value. The ideal offset value is infinitesimal, but the actual offset is finite in numerical difference method, and the range of its value is evaluated by the computer simulation. In the simulation, the offset value is the same for all components of the illuminating vector $\mathbf{n}_s$ i.e., $\delta u = \delta v = \delta w$.

Figure 2 shows the iterations for convergence by varying the offset values in the shape recovery simulation. The data marked by (○) are by the BFGS method and the data marked by (●) are by the DFP method in Figure 2. The convergence ratios in the range of offset value shown in Figure 2 are 100%.

From this evaluation, the appropriate offset value is 0.01, and the nonlinear optimization approach of BFGS method is better than that of DFP method in the iteration counts.

Figure 2: Iterations for Convergence and Offset Values

The errors in the quantization of input images are shown in Figure 3. Figure 3 shows the convergence ratio and iterations for convergence by varying the resolution bits of quantization from eight bits to thirty-two bits in the shape recovery simulation. The data marked by (◇) are the convergence ratios and the data marked by (○) are the iterations for convergence, and both data are obtained by BFGS method in Figure 3.

It is apparent that the lower resolutions than twelve bits can not hold 100% ratio of convergence and a lot of iterations for convergence are required.

From these evaluations, the offset value is suitable at 0.01, and for the resolution of quantization twelve bits are required at least.

Resolution: variable, Offset: 0.01

Initial values: Lambertian estimates

Figure3: Iterations for Convergence and Density Resolutions by BFGS Method

4.2 Simulation for Selection of Initial Values

How much the selection of initial values of the gradient parameters (p, q) affects the shape recovery performance is evaluated. The candidates of the initial values are null set of parameters (0,0) and the Lambertian estimates (p_L, q_L) by the Equation (11) are evaluated.

As a result, the gradient parameters (p, q) are recovered in a sufficient order of convergence either from the parameters (p_L, q_L) or the Lambertian estimates (0,0). The convergence ratio holds 100% in both cases, The distinction of two cases is the number of iterations for convergence.

Table 1 shows the iterations for convergence. The adopted optimization approach is BFGS method in this simulation. From the result shown in Table 1, the fewer iterations in the case of the Lambertian estimates than that of the null set of (0,0). Table shows the tendency to increase the iterations with increasing the factor s (the percentage of the strength of the specular component),

It is expedient to select the Lambertian estimates (p_L, q_L) for the initial values of optimization,

Table 1: Iterations for Convergence and Initial Value Selections

Parameters		Iterations	
Ratio s	Sharpness n	Values (p_L, q_L)	Values (0,0)
0.0	1	0.0	11.1
0.1	20	2.8	13.9
0.3	16	4.2	14.9
0.6	10	8.1	16.8

4.3 Simulation for Shape Recovery

The recovery of the surface shape of the polyhedron and the curved object is demonstrated by the computer simulations in this section,

In the simulation of the polyhedron, the values of the unit illuminating direction vector are set to $(0.052926, 0.084681, 0.996002)^T$, and the case of curved objects, the values of the vector are set to $(0.189896, 0.249863, 0.949478)^T$. Three offset values are set to the same values 0.01, and the resolution of quantized image density is twelve bits in both cases,

Figure 4 shows a sample graph of four errors of the objective function. This sample is obtained at a surface element corresponding the image pixel (24,36) in the shading image. The four curves are the members of the objective function defined by the Equation (7). Each curve is a set of the plotted points that the value of the corresponding error is less than 0.1. The intersection of the four curves may be an optimal solution. The points marked by (○), (□) and (●) mean the initial estimates, the searched optimal estimates and the true values. Optimization program searches the variables that the objective function consisted by the previous four errors has the least values by the BFGS method. The points marked by (○) are the initial sets of the variables for the searching. Successful convergence makes the mark by (□) and (●) overlapped. Note that the points marked by (□) and (●) are overlapped in this sample graph.

Resolution: 12 bits, Offset: 0.01

Initial values: Lambertian estimates

Figure 4: Example Graphs of Objective Functions

Figure 5: Recovered Shape of Polyhedron and Curved Object

Figure 5 shows the results of the recovered shape of the polyhedron and the curved object (ellipsoid). The needle diagrams (true) show the true gradient parameters distribution. The needle diagrams (optm) show the optimized distribution, and the needle diagrams (init) show the selected initial distribution for optimization. These figures show the tendency to increase the errors of the initial values for optimization with increasing the factor s (the percentage of the strength of the specular component). The recovered shapes are almost the same. These figures show the effectiveness of this approach for the polyhedral and curved objects.

5. Conclusion

A new approach is proposed to extend the shape recovery problem from the shading images.

Under the condition of the uniform parallel light source and the orthographic projection, even if the property of the object surface reflectance is uniform glossy, the gradient parameters of the local surface element of the object are determined by the nonlinear optimization approach.

The sum of square errors with the image density and its directional derivatives is adopted as the objective function for optimization. The initial values of the gradient parameters for optimization are the Lambertian estimates, and the optimal solutions are obtained with fewer iterations.

The Lambertian estimates are derived from only the directional derivatives of the image density. The other information, for example, an illuminating direction, coefficients of the reflectance are not necessary for the initial values selection.

Feasibility studies by computer simulations show that the suitable offset is 0.01 and the resolution bits of quantization is at least twelve. The shape recovery of the polyhedron and the curved object is demonstrated.

In this paper, Phong reflectance function is supposed as the reflectance property of the object surface. It is a typical example of reflectance function, and other functions are also applicable and effective for our optimization method, if the function models are described obviously.

This paper describes the theoretical aspect of our shape recovery approach, it remains to develop further extensions for the actual objects and to demonstrate the experiments based on our method.

References

- [1] Horn, B.K.P., "Obtaining Shape from Shading Information", in *The Psychology of Computer Vision* P.H., Winston, (ed.) McGraw-Hill, pp. 115-155 (1975).
- [2] Horn, B.K.P., "Understanding Image Intensities", *Artificial Intelligence*, **8**, 2, pp.201-231 (1977).
- [3] Woodham, R.J., "Photometric Method for Determining Surface Orientation from Multiple Images", *Optical Engineering*, **19**, 1, pp. 139-144 (1980).
- [4] Iwahori, Y., Kamei, H., Yamaguchi, S., "Direct Shape Reconstruction of 3-D Object from Multiple Shading Images", *J. Light & Vis. Env.*; **vol. 11**, 1, pp. 13-21 (1987).
- [5] Wolff, L.B., "Shape Understanding from Lambertian Photometric Flow Fields", *IEEE Proc, CVPR*, pp. 46-52 (1989).
- [6] Phong, B.T., "Illumination for Computer Generated Pictures", *Comm. of ACM*, **vol. 18**, 6, pp. 311-317 (1975).
- [7] Iwahori, Y., Hiratsuka, N., Kamei, H., Yamaguchi, S., "Extended Photometric Stereo for an Object with Unknown Reflectance Property", *Trans. IEICE (in Japanese)*, **vol. J71-D**, 1, pp. 110-117 (1988).
- [8] Tanaka, H., Iwahori, Y. and Ishii, N., "Determining Surface Orientation by Uni-Directional Photometric Flow Fields", *IAPR Workshop on MVA'90*, pp. 63-66 (1990).
- [9] Tanaka, H.D. and deFigueiredo, R.J.P., "A Theory of Photometric Stereo for a Class of Diffuse Non-Lambertian Surfaces", *IEEE Trans. Pattern Anal. Mach. Intell.*, **PAMI-13**, 2, pp. 133-152 (1991).
- [10] Tanaka, H., Iwahori, Y. and Ishii, N., "Surface Orientation from Shading Image and Its Directional Derivatives under Unknown Illuminating Direction", *ICARCV '92*, pp. CV6.3.1-CV6.3.5 (1992).
- [11] Tanaka H., Iwahori Y. and Ishii, N., "Determining Surface Orientation from Directional Derivatives of Shading Image by Slightly Varied Illuminating Directions", *Trans. IEICE (in Japanese)*, **vol. J76-D-II**, 5, pp. 1076-1079 (1993).
- [12] Dennis Jr. J.E. and Schnabel R.B., "A View of Unconstrained Optimization", in *Handbooks in Operations Research and Management Science*, vol 1, Optimization, (Eds. Nemhauser *et al.*), North-Holland, pp. 1-72 (1989).