Paper

Improvement of Indoor Illuminance Distribution with the Thin Phase Hologram

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ABSTRACT

It is possible to improve the distribution of illuminance on a floor by controlling the direction of daylight incident on the floor from a side window which is called a holographic window in this paper. The idea of a thin phase hologram makes possible uniform distribution of illumination throughout the year. The special hologram may be made by annealing technology on a computer simulation algorithm.

1. Introduction

On a fine day, it is lighter near a window and darker in the back of the room. Therefore, energy for lighting and cooling could be saved if light from a window could reach the back of the room. That technique is called the "daylighting system" and is an effective means of reducing both lighting and cooling costs.

There are various daylighting systems that result in energy saving. One of them is the prism system¹⁾. Designing the prism is difficult because of the changing angle of the sun's rays throughout the day and seasons, and effective prisms must be selected experimentally. Another daylighting system employs a hologram, which is called a holographic window²⁾. The hologram is a broadband characteristic that changes the angle of the sun's rays and can passively track the movement of the sun across the sky throughout the day and year.

We attempted to show that the distribution of illuminance on a floor can be improved through the use of holographic windows. According to Mitsuba et.al.³⁾, reflected light from the back wall of a room distributes illuminance uniformly. The intensity of the light was weak because the light hit only the back wall of the room.

In this paper, we considered light which travels from the window to the floor and attempted to use a hologram which uniformly distributes illuminance on the floor for a period of one year. The hologram was installed on the top half of the window, with glass remaining on the bottom half to allow people to see outside.

2. Theory

Fig. 1 is the room model in the simulation. The room is a cuboid stretching lengthwise from north to south. There is a

Fig. 1 Model room used in the computer simulation

window only on the south side. An element of a room divided in minute area is a uniform perfect diffuse surface. Determining the amount of light leaving a element requires a completed specification of the geometric relationships among all elements and a specification of the light leaving every other element. This relationship is based on the luminous flux transfer method and is given by

$$E_i^n = E_{0i} + \sum_{j=1}^m \rho_j F_{ij} E_j \quad (1)$$

where,

- E_i^n : final illuminance distribution of element i
- E_{0i} : initial illuminance distribution of element i
- ρ_j : diffuse reflectance of element j
- F_{ij} : form factor from element i to element j

The form factor F_{ij} is formed from the boundary integral method and is given by

$$F_{ij} = \frac{1}{2\pi S_i} \int_{S_i} \sum_{k=1}^w \beta_{ki} \cos \alpha_{ki} dS_i \quad (2)$$

where, as shown in Fig.2, S_i and S_j are areas of the elements, α is an angle between N_p and $N_{\Delta PAB}$, and β is an angle between the line joining A and B on the uniform diffuse source of area S_i .

Fig. 2 Form factor of the boundary integral method

Eq.1 is expanded into Eq.3 with the term of direct illuminance E_{0i} . n is the reflection order of the surface.

$$E_i^n = E_{0i} + E_{ri}^{(1)} + E_{ri}^{(2)} + \dots + E_{ri}^{(n)} + \dots \quad (3)$$

$E_{ri}^{(n)}$ is the indirect illuminance of element i after n th reflection.

Thus,

$$\begin{aligned} E_{ri}^{(1)} &= \sum_{j_1=1}^m \rho_{j_1} F_{j_1 i} E_{0j_1} \\ E_{ri}^{(2)} &= \sum_{j_2=1}^m \sum_{j_1=1}^m \rho_{j_1} \rho_{j_2} F_{j_2 j_1} F_{j_1 i} E_{0j_1} \\ &\vdots \\ E_{ri}^{(n)} &= \sum_{j_n=1}^m \dots \sum_{j_1=1}^m \rho_{j_1} \dots \rho_{j_n} F_{j_n j_{n-1}} \dots F_{j_1 i} E_{0j_1} \end{aligned}$$

m is the total number of elements of the room.

The diffraction phenomena from the hologram to the room is calculated by the Fresnel-Kirchhoff diffraction formula.

$$\begin{aligned} u(x_i, y_i, z_i) &= -\frac{j}{\lambda} \sum_{k=1}^m g(x_k, y_k, 0) \frac{\exp(jkr)}{r} \\ &\cdot \frac{\cos(\mathbf{n}, \mathbf{r}) - \cos(\mathbf{n}, \mathbf{r}_0)}{2} \cos(\mathbf{n}', \mathbf{r}) \end{aligned} \quad (4)$$

where,

$$r = \sqrt{z_i^2 + (x_i - x_k)^2 + (y_i - y_k)^2}$$

Fig.3 illustrate the geometry of the holographic diffraction phenomena. $\mathbf{r}_0$ is the vector from the source to the

hologram. $\mathbf{r}$ is the vector from the observed point $u(x_i, y_i, z_i)$ to the hologram. $\mathbf{n}'$ is the normal vector in the direction of the observed point $u(x_i, y_i, z_i)$.

Fig. 3 Geometry of the holographic diffraction phenomena

The hologram changes the distribution of an aperture (the window) to that of a complex amplitude $u(x_i, y_i, z_i)$. The distribution of the light which passes through the hologram is the complex amplitude distribution. Therefore, such systems that use illuminance distribution have been seen to obey the intensity :

$$E_{0i}^H = \frac{|u(x_i, y_i, z_i)|^2 \times E_0^H}{\sum_{p=1}^m |u(x_p, y_p, z_p)|^2} \quad (5)$$

E_0^H is input illuminance distribution in the hologram.

Thus,

$$E_{0i} = E_{0i}^G + E_{0i}^H \quad (6)$$

where, E_{0i}^G is the illuminance distribution of the light which passes through a glass window.

The hologram used in this study is a phase hologram. A phase hologram records the phase with the amplitude of 1. Therefore, the uneven hologram surface represents a complex amplitude distribution. As the transmittance of a phase hologram is about 1, the light efficiency of a phase hologram is higher than that of an amplitude hologram which records transmittance of a light. Thickness of the thin hologram was one wavelength of the recording wavelength. In the case of the thin phase hologram, the value of diffraction efficiency is 100% in recording the interference fringe⁴⁾.

One method of devising a phase hologram is that the photosensitive material records the gray level as shown in Fig.4 and the material is bleached to remove the silver particles. In another technique, the electron beam shaves the surface of the material.

Fig. 4 Computer generated hologram and relative value of gray level

We made the hologram in this study to increase the average illuminance of the floor and uniformly distribute illuminance. The optimum phase of this hologram is solved by SA (Simulated Annealing)⁹⁾. SA is a computational optimizing method that is analogous to the annealing of metals. To be optimized, the system consists of a variable and a cost function representing the system configuration.

The initial illuminance is calculated from the distribution of the hologram by the Fresnel-Kirchhoff diffraction formula, and the final illuminance E_i^n is calculated from the initial illuminance by the interreflection equation. Eq.7 represents the cost function used in the evaluation of the distribution of the final illuminance E_i^n .

$$E_{cost} = \sqrt{\sum_{i=1}^m (E_i^n - 1000)^2 / N} \quad (7)$$

N is the number of elements of the floor. Complex amplitude distributions of a few elements in the phase hologram are changed by the quantized phase. The cost value is calculated by the cost function (Eq.7) whereby $\Delta E (= E_{cost} - E_{0cost})$, the cost value after phase change E_{cost} minus the cost value before phase change E_{0cost} , is computed. As $\Delta E < 0$, the value of the phase change is accepted. As $\Delta E \geq 0$, the value is accepted on probability P .

$$P = \exp(-\Delta E / T) \quad (8)$$

Next, phases of other elements on the hologram are changed. Similarly, the cost value is calculated, and the value is accepted or rejected. This process is repeated on lowering the temperature. Eq.9 is the temperature gradient.

$$T' = aT (0 < a < 1) \quad (9)$$

Initial temperature and final temperature in Eq.9 are experimentally decided parameters.

3. Simulation

Fig.1 is the room model in the simulation. The room is a cuboid stretching lengthwise from north to south. There is a window only on the south side. The hologram is installed on the top half of the window, as shown in Fig.5.

Fig. 5 Configuration of the holographic window

The reflectivity of the ceiling is 80%, that of the wall is 86% and that of the floor is 50%. The reflection order of the surface is 20 times. The room is divided as shown in Fig. 1 into $16 \times 8 \times 8$ elements by the finite element procedure. The size of the room is $2.0 \times 2.6 \times 2.6$ [m]. The size of the hologram is 0.65×1.95 [m], and the hologram whose elements are 30×90 is calculated on the computer. The resolution of the hologram is 300 [line/mm].

Fig.6 is a flow chart of the program for the computer simulation. The hologram is made using one program.

Fig. 6 Flowchart of program in the computer simulation

The initial temperature is $T_0=1000$ on the SA. The temperature schedule is Eq.10.

$$T_{New} = 0.99 \times T_{Old} \quad (10)$$

The distribution of illuminance in the room was computed once every hour from 0 p.m. to 4 p.m., every hour, on the vernal equinox day, autumnal equinox day, summer solstice, and winter solstice.

An optimization hologram for uniformly distributing illuminance on the floor was computed according to average values for a period of a year. Therefore, the hologram could uniformly distribute illuminance for a year.

Table 1 Angle of sun on three seasons at five different time periods

Time in p.m.	Vernal equinox day and autumnal equinox day [degree]		The summer solstice [degree]		The winter solstice [degree]	
	Azimuth ϕ	Angle of elevation θ	Azimuth ϕ	Angle of elevation θ	Azimuth ϕ	Angle of elevation θ
0	0.00	55.00	0.00	78.45	0.00	31.56
1	25.04	52.30	52.48	72.58	15.89	29.85
2	45.19	45.19	74.21	61.53	30.41	25.00
3	60.16	35.40	85.93	49.43	42.91	17.65
4	71.68	24.18			53.45	8.49

Table 1 shows five different angles of the sun for each of three seasons. A unit is a degree. An azimuth was 0 degree for the south, which is 90 degrees for the west. 0 p.m. is meridian passage. The azimuth is ϕ and the angle of elevation is θ as shown in Fig.7.

Fig. 7 Angle of elevation and azimuth

In calculation of illuminance it is considered that horizontal illuminance in the summer solstice at meridian passage is 100000[lx].

Table 2 Mean illuminance of daylight on the window

Time in p.m.	Vernal equinox day and autumnal equinox day [lx]	The summer solstice [lx]	The winter solstice [lx]
0	63163.3	20437.7	85791.5
1	60081.1	18373.7	81644.3
2	51137.7	12545.9	69034.9
3	37292.5	4105.7	47151.8
4	20433.1		14629.7

Table 2 shows illuminance of daylight on the window. A unit is lx. According to JIS, the illuminance standard of an office is 750-300[lx] and that of a school is 750-200[lx]. The object was to make a hologram that provided uniform illuminance distribution of about 1000[lx].

Although daylight is white light, in this computer simulation, we do not use the wavelength of daylight, but that of laser light ($\lambda=632.8\text{nm}$) because of the characteristics of the hologram. The room is situated at latitude 35°N .

3. Results

Fig.8-a,b is the distribution of illuminance of the floor at 2 p.m. on the vernal equinox day and autumnal equinox day.

Fig. 8-a Room with the all glass window

Fig. 8-b Room with the hologram

Fig. 8 Illuminance distribution of the floor at 2 p.m. on the vernal equinox day and autumnal equinox day.

Fig.8-a shows a room with the full glass window. These words "full glass window" will hereinafter be abbreviated "glass window". Fig.8-b shows a room with the hologram inserted on the top half of the window. The longitudinal axis shows illuminance and the horizontal axis shows an element of the floor. Because extremely bright illuminance is not desired, the maximum value of the longitudinal axis is 1200lx. The direction of Fig.8-a,b corresponds to Fig.1.

Fig. 9 Minimum illuminance of the floor on the vernal equinox day and the autumnal equinox day

Fig. 10 Minimum illuminance of the floor on the summer solstice

Fig. 11 Minimum illuminance of the floor on the winter solstice

Fig.9-11 compared the minimum illuminance of a room with the glass window with the minimum illuminance of a room with a hologram for the purpose of determining improvement of illuminance of the back of the room. The illuminance of the room with the hologram was better than that of the room with the glass window at all times.

Table 3 shows the cost value of Eq.7 in the case of the room with the glass window and that with the hologram.

Table 3 Cost Value

Time in p.m.		Glass E_{cost}	Hologram E_{cost}
The vernal equinox day and the autumnal equinox day	0	2484	1609
	1	2158	1434
	2	1453	1009
	3	853	702
The summer solstice	4	752	714
	0	1312	1013
	1	1158	970
	2	1102	893
The winter solstice	3	959	941
	0	1762	1286
	1	1057	825
	2	700	487
	3	555	422
	4	830	798

The cost value of the hologram was less than that of the glass at all times. Therefore, optimized illuminance distribution can be decided by the function of cost.

5. Conclusion

The optimization of a phase hologram provided almost uniform distribution of illumination on the floor for a year. Because the lower half of the window is glass, the influence of the changing angle of the sun's rays is great and the illuminance distribution of the floor was not perfectly uniform. However, the back of the room with the hologram on the top half of the window was light and the distribution of illumination of the floor was more uniform in contrast to that with the glass window only. These results indicate that the distribution and level of illumination in a room may be improved by making use of the phase hologram.

The optimized phase distribution of the hologram was obtained by using Eq.7 in the cost function. The higher illuminance was reduced and the lower illuminance was increased. However, we thought that changing the constant of Eq.7, i.e., 1000 into a variable in proportion to the illuminance would be better because the change of illuminance varied greatly throughout the day. The temperature scheduling of the SA was decided experimentally, but a hologram which distributed illuminance on the floor in a uniform way was achieved. We think that making the hologram by SA is one of the best ways.

As reported in this paper, our purpose was to make a phase hologram for maximally illuminating a floor and to distribute such illuminance uniformly. However, light from this hologram glared for an occupant. Therefore, with a view to decreasing glare, the light from the hologram must be directed above the height of a man.

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