Paper

Linear Uniform Colour Space Composed of Simple Transformations of Tristimulus Values X , Y and Z

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ABSTRACT

A new linear uniform colour space has been developed by systematic combinations of tristimulus values X , Y and Z . For colours with constant value of 6 in Munsell space, numerical computer analyses have been made to minimize the deformations of hue circles from uniform circles in the new space by optimizing the coefficients in the formulae of linear transformations. By appropriate transformations, the shapes of hue circles which are distorted in the $L^*a^*b^*$, $L^*u^*v^*$ and Cube-Root spaces come close to uniform circles, and the warps of hue circles with moderate chroma are reduced to $1/4 \sim 1/5$ as compared with those in the traditional colour spaces.

In this new space, hue and chroma can be approximated by metric hue angle and metric chroma, respectively, with considerable accuracy for colours of moderate chroma.

1 Introduction

Chromaticity diagram $Y-(x, y)$, recommended by the CIE in 1931, has been widely used as the colour space to specify colours, and these chromaticity coordinates are also used for psychophysical representation of colours defined by the rule of Munsell system.^{1) 2) 3)}

It is, however, evident that uniformity of the space is not sufficient, because the aim of this colour space is simply to indicate the chromaticity of colours. Furthermore, the coordinates of a colour on the chromaticity diagram do not correspond with hue, lightness and chroma, which represent the attributes of the perceived colour, and the geometrical distance in this colour space does not express the perceived colour difference.

The CIE, therefore, recommended the following two colour spaces in 1976 in order to improve the uniformity, namely, $L^*a^*b^*$ uniform colour space designed to attain uniform distribution of coordinates of colours located uniformly in the space defined by Munsell system, and $L^*u^*v^*$ uniform colour space designed to make MacAdam discrimination ellipses shaped more like uniform circles.^{1) 4) 5) 7)} Cube-Root space, with consideration given to RGB response in the visual system, was also proposed.⁶⁾

However, each of these colour spaces has its merits and demerits. Therefore, the development of a new uniform colour space by further improvement on uniformity, that is able to represent appropriately the perceived aspects of colours, has been expected for the quantitative specification of colours.^{8) 9) 10) 15)}

There are Munsell system, Natural Colour System and Ostwald colour system to specify the psychological attributes of colours. Among these, Munsell system is currently prevalent and its correspondence with the CIE XYZ or

$Y-(x, y)$ has been precisely made. This correspondence is also defined under JIS (Japanese Industrial Standard).

The aim of this study is to create a new colour space with excellent uniformity, while maintaining linearity, using linear transformations of tristimulus values X , Y and Z . This colour space is designed to optimize uniformity of the distribution of colour coordinates for the colours located uniformly on a plane with a constant value of $V=6$ in the three-dimensional space defined by the rule of Munsell system, in another words, Munsell space.

It has been pointed out in past studies that the uniformity of Munsell system is not perfect, however, any quantitative remedy for the improvement has not yet been indicated.^{11) 12) 13) 14)}

Therefore, in this study, the analyses are carried out by using the data set forth by JIS.

The reasons for choosing the plane rendered by the value condition of $V=6$ are as follows; this lightness is average in terms of perceived brightness; the number of colours indicated is large; and it is possible to review the uniformity in a wide range because chromaticity coordinates are defined up to high chroma for all hues.

2 Evaluation measure of uniformity

An evaluation standard could be a proper geometrical measure which can indicate the degree of correspondence between attributes representing the perceived aspects of colours and colorimetric quantities, and may show the accuracy of expressing the perceived colour differences by the metric distances of colorimetric quantities in appropriate correspondence with their attributes.

If hue, value, and chroma are in correspondence with hue

Figure 1 Coordinates of Munsell colours with value of 6 in the (x, y) chromaticity diagram, $L^*a^*b^*$ uniform space, $L^*u^*v^*$ uniform space and Cube-Root space.

angle, lightness and the distance from the origin or the metric chroma, respectively, namely, a hue circle with constant value and equal chroma is shown as a uniform circle on a plane of constant value, then the variation of hue and the variation of chroma will independently be in correspondence with the change of hue angle alone and the change of the distance from the origin or radius of hue circle, respectively.

Therefore, it can be evaluated that the closer the hue circle is to a uniform circle, the more superior it is in uniformity of colour space, and that the farther the hue circle is from a circle with constant radius, the less superior it is in uniformity of colour space.

Hence, the level of closeness of the hue circle to a circle with

constant radius shall be regarded as a significant measure to indicate the uniformity of the colour space.

Based on such concept, the locus of coordinates of colours with equal chroma which are located on the plane of constant value in Munsell space has been tracked in a space being evaluated. In other words, the degree of closeness of the hue circle to a circle with constant radius has been studied, and the evaluation of uniformity has been carried out by using a geometrical measure which shows the relative degree of this closeness.

On the plane of constant value in Munsell space, if a colour on a hue circle corresponding to a particular chroma is expressed by coordinates (x, y) in the colour space which is being evaluated, the mean radius $\langle R \rangle$ of the hue circle in

this colour space will be shown as follows:

$$\langle R \rangle = \sum_{i=1}^N \sqrt{(X_i^2 + Y_i^2)} / N \quad (1)$$

N is the number of colours aligned on the hue circle.

The relative gap between this hue circle and a circle with constant radius will also be given by σ , that is called the relative deviation.

$$\begin{aligned} \langle \Delta R \rangle^2 &= \sum_{i=1}^N [\sqrt{(X_i^2 + Y_i^2)} - \langle R \rangle]^2 / N \\ \sigma &= \langle \Delta R \rangle / \langle R \rangle \end{aligned} \quad (2)$$

The closer the σ is to zero, the smaller is the gap between the hue circle and a circle with constant radius, in other words, colours of equal chroma shall be aligned in proximity to an annular ring of constant radius in the space. Hence, the radius or the distance from the point of origin corresponds to the chroma appropriately.

For reference purposes, coordinates of colours aligned uniformly on the plane of $V=6$, $\Delta H=2.5$, and $\Delta C=2.0$ (V as value, ΔH as hue spacing and ΔC as chroma separation) in Munsell space are shown in (x, y) chromaticity diagram, $L^*a^*b^*$ uniform space, $L^*u^*v^*$ uniform space and Cube-Root space as in Figure 1.

Mean radius of the hue circle $\langle R \rangle$ with value $V=6$ and chroma $C=8$ and the relative deviation σ or the distance between coordinates of the colours and the circumference of a circle with mean radius $\langle R \rangle$ are obtained as shown in Table 1.

The gap between the hue circle and a circle with constant radius is rather wide in any of these spaces.

Especially, the relative deviation shown in $L^*a^*b^*$ uniform space is up to 22.8%, and the warp is much larger than what is expected from its shape of the hue circle with chroma $C=6$.

Table 1 Mean radius $\langle R \rangle$ and relative deviation σ which indicates the degree of warp of the hue circle with $V=6$ and $C=8$ from a uniform circle in the $L^*a^*b^*$, $L^*u^*v^*$ and Cube-Root uniform colour spaces.

Colour Space	$L^*u^*v^*$	$L^*a^*b^*$	Cube-Root
Radius $\langle R \rangle$	57.983	41.385	42.458
Deviation σ	0.1631	0.2280	0.2017

3 Construction of uniform space by linear transformations

3.1 The study of mathematical analysis method

Analyses are carried out by combining tristimulus values X , Y and Z linearly, and also by selecting the proper coefficients of linear transformations so as to optimize the improvement of uniformity in terms of hue and chroma on the colour space.

This study limits its consideration to uniformity of only hue and chroma on the plane of constant value, because we understand that value V in Munsell space and lightness function L^* in the CIE uniform colour space have been studied for many years to attain good correspondence with psychological brightness perception.

Let a be the horizontal coordinate, b the vertical, on the plane of constant value in a linear colour space. Then these can be expressed as linear functions of tristimulus values X , Y and Z such as:

$$\begin{aligned} a &= k_{11} X + k_{12} Y + k_{13} Z \\ b &= k_{21} X + k_{22} Y + k_{23} Z \end{aligned} \quad (3)$$

The improvement of uniformity may be brought about by changing coefficients from k_{11} to k_{23} in sequence in order to make the hue circle shaped closer to a uniform circle. However, in reality, an enormous amount of calculation is required and this operation is extremely difficult to execute.

Rotating the coordinates around the origin point does not create a change in uniformity. Hence, one of these coefficients can be set to zero by rotating the coordinates with the selection of a proper angle according the equations below.

$$\begin{aligned} a' &= a \cos \theta + b \sin \theta \\ b' &= -a \sin \theta + b \cos \theta \end{aligned} \quad (4)$$

For example, if an angle is chosen to set k_{21} to zero, the number of coefficients necessary will be five as shown in the following equations.

$$\begin{aligned} a' &= k'_{11} X + k'_{12} Y + k'_{13} Z \\ b' &= \quad \quad \quad + k'_{22} Y + k'_{23} Z \end{aligned} \quad (5)$$

Moreover, both scaling up and down the coordinates does not change uniformity, thus the matter of concern will only be the relative relation among the coefficients. In other words, the right side of the equations can be enclosed by one of the coefficients, for example k'_{22} , and the remaining four variables can be adjusted relatively.

$$\begin{aligned} a' &= K (k''_{11} X + k''_{12} Y + k''_{13} Z) \\ b' &= K (\quad \quad \quad + Y + k''_{23} Z) \\ K &= k'_{22}, \quad k''_{ij} = k'_{ij} / k'_{22} \end{aligned} \quad (6)$$

As shown above, the improvement of uniformity can be made by adjusting the four independent variables, which indicates that execution of the four independent operations is sufficient in terms of mathematical analyses.

In this sequential operation of changing four coefficients in order to find an optimum solution, the relationship of each operation with geometrical structure of the space is difficult to grasp, thus, the process of the improvement of uniformity can not be clearly observed.

As it is proven above, four independent operations are sufficient in terms of mathematical procedure. Therefore, analyses should be carried out by using operations that have clear geometrical meanings.

3.2 Selection of colour space model and procedure of geometrical operations

A number of colour space models have been devised to this point of time. Here, Cube-Root space with Reilley's coefficient is chosen as the model for mathematical analysis, considering the fact that this space has been constructed in contemplation of opponent colour responses and has been reviewed by CIE colorimetric committee.⁶⁾

Analysis to obtain the optimum solution is attempted by executing geometrical operations on this model in sequence.

As provided in the following equations, Cube-Root space is a space which indicate opponent colour responses a and b by proper transformations of tristimulus values X , Y and Z .

$$\begin{aligned} a &= 106.0 (R^{1/3} - G^{1/3}) \\ b &= 42.34 (G^{1/3} - B^{1/3}) \\ R &= 1.1084 X + 0.0852 Y - 0.1454 Z \\ G &= -0.0010 X + 1.0005 Y + 0.0004 Z \\ B &= -0.0062 X + 0.0394 Y + 0.8192 Z \end{aligned} \quad (7)$$

In order to carry out analyses in a linear space, we introduce the following functions by omitting the coefficients of $1/3$ power, and by letting two coefficients in front of the brackets be K and KA_1 , respectively.

$$\begin{aligned} a &= K (R - G) \\ &= K (1.1094 X - 0.9153 Y - 0.1458 Z) \\ b &= KA_1 (G - B) \\ &= KA_1 (0.0052 X + 0.9611 Y - 0.8188 Z) \end{aligned} \quad (8)$$

Moreover, tristimulus values X , Y and Z are replaced by X_c , Y_c and Z_c as in the following formulae to set the coordinates of achromatic colour on the origin, because the coordinates of achromatic colour have not been located on the origin in the original Cube-Root space.

$$\begin{aligned} X_c &= S (x - x_0) = X - S x_0 \\ Y_c &= S (y - y_0) = Y - S y_0 \\ Z_c &= S (z - z_0) = Z - S z_0 \\ S &= X + Y + Z \end{aligned} \quad (9)$$

x , y and z , and x_0 , y_0 and z_0 are chromaticity coordinates of a colour and the illuminant, respectively.

Since the hue circle in this colour space is somewhat distorted and shaped as an oval, in order to bring its shape closer to a uniform circle, the following four geometrical operations are executed.

- (1) Adjust the coefficient of vertical axis and change its ratio relative to the horizontal axis. : $[A_1]$

- (2) Set the angle of the major/longer axis of the hue circle that forms with the horizontal axis. : $[\theta]$
- (3) Adjust the multiplication coefficient of the minor axis, which is perpendicular to the major axis inclined an angle of θ , so that the length of both minor and major axis will be equal. : $[k]$
- (4) Multiply the vertical axis with a coefficient to attain proper ratio of vertical axis to horizontal axis. : $[A_2]$

Operation (4) is independent from (1) since operations (2) and (3) are executed in between. In other words, it is not possible to substitute operation (4) by operation (1).

Procedures for these operations are shown in Figure 2.

3.3 Improvement of uniformity by geometrical operations

Actual operations are attempted on the hue circle of chroma $C=8$ on the plane of value $V=6$. The reasons for selecting the hue circle of $C=8$ are as follows ; this level of chroma is slightly higher than moderate chroma and is often used as proper colour ; it can be concluded that the hue circle of $C=8$ represents the average level of chroma for all hues, as the same results are obtained in several attempts of the same operation to attain average uniformity among hue circles of $C=2$ to 14 ; and the geometrical meaning is clear and simple when analyses on the hue circle of $C=8$ alone is carried out.

Adjusting the vertical and horizontal axis' correction factor A_1 , the gradient of the major axis θ , minor and major axis' correction factor k , and another vertical and horizontal axis' correction factor A_2 sequentially, we carried out numerical analyses using computer to attain the least distortion of the shape of the hue circle with value $V=6$ and chroma $C=8$.

After long calculation, relations between these coefficients and relative deviations are obtained as shown in Figure 3.

Fortunately, the curves which indicate the change of relative deviations are simple in any coefficient change. Thus, there is only one local minimal point in each case.

Therefore, carrying out this analysis of finding the local minimum among the group of local minimums by changing these coefficients in sequence, we get the final and optimum solution, which has a minimum distortion of hue circle, of $A_1 = 0.38$, $\theta = 47.0$ (degree), $k = 1.40$, and $A_2 = 1.24$.

Executing geometrical operations by using these values, the following expression of linear transformations for the specification of colour is obtained.

$$\begin{aligned} a &= K (1.3464 X_c - 1.1840 Y_c - 0.1149 Z_c) \\ b &= K (-0.2716 X_c + 0.7636 Y_c - 0.4215 Z_c) \end{aligned} \quad (10)$$

Hue circle in this case has a mean radius of $\langle R \rangle = 13.4367K$ and the relative deviation, which indicates the warp of this hue circle from a circle with constant radius, of $\sigma = 0.042601$, by which it is clear that the distortion of the

Figure 2 Geographical procedures for the improvement of uniformity in colour space.

- (a) Original Hue circle before transformation.
- (b) Scale of ordinate is multiplied by A1.
- (c) Proper angle of long axis is determined.
- (d) Short axis is multiplied by k.
- (e) Scale of ordinate is multiplied by A2.

hue circle of $C=8$ is eliminated considerably.

If rotation of the coordinates around the origin is executed by equations (4) so that the coordinates of colours with hue of 5Y are aligned on the vertical axis, or if the coefficient of X_c in b axis has to be zero, the expression (10) will be transformed into the following.

$$\begin{aligned} a &= K (1.3735 X_c - 1.3116 Y_c - 0.0293 Z_c) \\ b &= K (0.5144 Y_c - 0.4359 Z_c) \end{aligned} \quad (11)$$

3.4 Corrections required to make coordinates of achromatic colour located at the origin

In expression (11), Sx_0 , Sy_0 or Sz_0 is subtracted from each tristimulus value in order to locate the coordinates of the achromatic colour at the origin. Assigning the following coordinate values of the achromatic colour (N6) or the coordinates of the gray colour of Munsell value $V=6$ illuminated by standard illuminant C, the value of correction is indicated as in the following equation.

Figure 3 Aspect on the variation of the relative deviation σ , i.e. distortion factor of the hue circle according to the change of coefficient A_1 , angle of long axis θ , coefficients k and A_2

$$\begin{aligned}
 Sx_0 &= X_0 \quad (S/S_0) = 98.072 \quad (S/S_0) \\
 Sy_0 &= Y_0 \quad (S/S_0) = 100.000 \quad (S/S_0) \\
 Sz_0 &= Z_0 \quad (S/S_0) = 118.225 \quad (S/S_0) \\
 S/S_0 &= 0.3005
 \end{aligned} \quad (12)$$

Correction values are expressed by the following formulae.

$$\begin{aligned}
 \Delta a &= K (1.3735 Sx_0 - 1.3116 Sy_0 - 0.0293 Sz_0) \\
 &= 0.02341 K \\
 \Delta b &= K (+0.5144 Sy_0 - 0.4359 Sz_0) \\
 &= -0.02833 K
 \end{aligned} \quad (13)$$

These values are very small which are equivalent to only 2.3 to 2.8% of radius for the hue circle of $C=8$. This shows that correction values for locating the achromatic colour at the origin become closer to zero as the coefficients of equations expressing the colour space are adjusted in order to make the hue circle shaped closer to a circle with constant radius.

If a slight correction is made to the formulae of (13), or to its original formulae (11), so as to make both Δa and Δb equal to zero, the correction for locating the achromatic colour at the origin is no longer necessary.

In other words, it is sufficient to make both a and b equal to zero in expression (11) without subtracting Sx_0, Sy_0 , and Sz_0 , when the tristimulus values of illuminant X_0, Y_0 and Z_0 are

assigned to tristimulus values X, Y and Z .

In case that this limiting condition is applied, the relations among each coefficient are expressed as follows.

$$\begin{aligned}
 K(k_{11} X_0 - k_{12} Y_0 - k_{13} Z_0) &= 0 \\
 K(+k_{22} Y_0 - k_{23} Z_0) &= 0
 \end{aligned} \quad (14)$$

After correcting the coefficients under this limitation, and by keeping one of these coefficients constant, for example k_{22} , it is required to re-adjust the remaining four coefficients in order to minimize the slight increase of distortion occurring in the hue circle by this correction. However, practically, two of the coefficients will be determined by themselves by adjusting the other two, due to the limiting condition of expression in preceding formulae (14).

Carrying out computerized numerical analyses, by changing k_{11} and k_{12} in sequence, the optimum solution will be expressed by the following formulae under the limited condition expressed in the equation (14) applied to equation (11).

$$\begin{aligned}
 a &= K (1.37109 X - 1.30994 Y - 0.02937 Z) \\
 b &= K (+0.51440 Y - 0.43510 Z)
 \end{aligned} \quad (15)$$

This solution does not require the correction to locate the

Figure 4 Variation of the relative deviation σ according to the change of coefficients Γ and γ in the process of mathematical analysis to minimize the distortion of the hue circle.

coordinates of achromatic colour at the origin since it is already included.

Comparison between the space expressed by equation (11), which requires this correction, and that of (15), which does not, are shown in Table 2 in terms of the mean radius $\langle R \rangle$ and the relative deviation σ of the hue circle with $V=6$ and $C=8$, respectively.

Table 2 Mean radius $\langle R \rangle$ and relative deviation σ which indicates the degree of warp of the hue circle with $V=6$ and $C=8$ from uniform circle in the new uniform colour space NC— I

Colour Space	Corrected Space	Uncorrected Space
Radius $\langle R \rangle$	40.2265	40.1640
Deviation σ	0.042601	0.042845

Although the distortion of the hue circle is only slightly smaller in the space that requires the correction among the origin, the difference is only up to 0.024%.

In equations (15), the coefficients are determined so that they can be applied to the specification of colours under the illumination of standard illuminant C.

In order to transform these equations so that they can be applied to illumination of any illuminant, by dividing X , Y and Z by tristimulus values of standard illuminant shown in formulae (12), and also by multiplying the coefficients with tristimulus values of standard illuminant, a general expression is given by the following.

$$\begin{aligned}
 a &= K' \Gamma [X/X_0 - \gamma Y/Y_0 - (1 - \gamma) Z/Z_0] \\
 b &= K' [Y/Y_0 - Z/Z_0] \\
 K' &= k_{22} Y_0 K = 0.51440 Y_0 K \\
 \Gamma &= 1.37109 X_0 / 0.51440 Y_0 \\
 \gamma &= 1.30994 Y_0 / 1.37109 X_0
 \end{aligned} \quad (16)$$

While equations (15) and (16) are the same, if we change the coefficients of Γ and γ to further investigate the change

of the relative deviation σ , which indicates the distortion of hue circles, we see results as shown in Figure 4.

The value of Γ is 2.61404 and γ is 0.97418, when the relative deviation σ obtained from numerical analyses is minimized. However, if we use these values and the tristimulus values of standard illuminant C in equation (16), we can confirm that it becomes the same formulae as equation (15).

Moreover, if the mean radius of the hue circle of colours with $V=6$ and $C=8$ are approximately 40, considering its correspondence with the CIE uniform colour space and if K' of b coordinates are adjusted to be a round number, then $K' = 154$.

Also, in order to define the 3-dimensional distance in this space, we must define the lightness coefficient. Again, considering the correspondence with the CIE formulae, we shall use the lightness function L^* of the CIE uniform colour space.

While our analyses up to this point have been aimed at improving the uniformity concerning colours with $C=8$ existing on the plane of $V=6$, it is desirable for the result to be applied to colours on other value planes. Hence, it is necessary to adjust K' function according to each value, so that the mean radius of hue circles of other colours with the same chroma are almost equal to that of colours with $V=6$.

As a result of making such adjustments to coefficients, the linear uniform colour space can be formulated in the following equations:

$$\begin{aligned}
 a &= K' [2.614040 (X/X_0) \\
 &\quad - 2.546545 (Y/Y_0) \\
 &\quad - 0.067495 (Z/Z_0)] \\
 b &= K' [1.000000 (Y/Y_0) \\
 &\quad - 1.000000 (Z/Z_0)] \\
 L^* &= 116 (Y/Y_0)^{1/3} - 16 \\
 K' &= 154 [8 / (6L^*/L_6^* + 2)]^2 \\
 \text{or } K' &\doteq 154 [8 / (V + 2)]^2
 \end{aligned} \quad (17)$$

Here, L^* is the lightness function of the CIE uniform colour space, L_6^* corresponds to L^* of value 6 and V is the value function of Munsell colour. The value of K' will be approximately the same when calculated with any of these value functions.

In the final result, the level of accuracy of numerical analyses is raised and coefficients are calculated up to the 6th digit to the right of the decimal point.

Figure 5 indicates the mean radius of hue circles with chroma $C=8$ and value $V=1-9$, in cases where K' has a constant value of 154 and K' is a function of value V as in equation (17). As the value of K' does not differ when it is calculated with L^* or V , it is difficult to differentiate the two on the graph.

Because of that, the calculated result of only V will be expressed in Figure 5. When K' is a function of V , we can see that the mean radii of hue circles with $V=2-8$ are all roughly 40.

The mean radius is slightly larger than 40 when value $V=1$ and $V=9$. This is due to the fact that colours with chroma $C=8$ with these two values are scarce and calculations could only be made for colours corresponding to limited and specific hues. These values do not represent the essential values of the mean radii of hue circles.

In the meantime, the linear uniform colour space for colour specification that has been acquired here shall be named New Colour Space—I, or abbreviated as NC—I.

Figure 5 Mean radii of hue circles with chroma of 8 and values of 1-9 in the cases that K' has constant value of 154 and that K' is a function of value V .

4 Comparison between linear uniform colour space and conventional colour spaces

Coordinates of Munsell colours aligned on planes of value $V=1-9$ in linear uniform colour space NC—I are shown in Figure 6.

Mean radii $\langle R \rangle$, values of relative deviations σ and the mean relative deviations $\langle \sigma \rangle$ of hue circles of Munsell colours that are aligned on the plane of value $V=6$ and chroma $C=2-14$, by each chroma, in $L^*a^*b^*$ uniform colour space, $L^*u^*v^*$ uniform colour space, Cube-Root space and linear uniform colour space NC—I are shown in Table 3 (a).

Moreover, mean radii $\langle R \rangle$ and values of relative deviations σ of hue circles of colours with value $V=1-9$ and chroma $C=8$ in these colour spaces are shown in Table 3 (b).

Meanwhile, colours with complete colour coordinates on hue circles with chroma of $C=8$ are only those of $V=4-7$, thus hue circles of other values are incomplete as coordinate values are not partially assigned. For this reason, values corresponding to those other than $V=4-7$ are indicated in parentheses () in Table 3.

Table 4 shows the hue angles θ corresponding to 5 principal hues with value $V=6$, chroma $C=2-14$ and mean deviation σ_θ for each of these angles by chroma in linear uniform colour space NC—I. Furthermore, Table 5 indicates the mean values of hue angles of these 5 colours $\langle \theta \rangle$, the separation between principal hue angles and adjacent hue angles $\Delta \theta$ and mean deviations of these angles σ_θ in $L^*a^*b^*$ uniform colour space, $L^*u^*v^*$ uniform colour space, Cube-Root space and linear uniform colour space NC—I.

Generally, the gaps between the hue circles and uniform circles are smaller in the new linear uniform colour space compared to those in other colour spaces.

Particularly, the relative deviation against the hue circle is extremely small when chroma is in the medium range, and the value is reduced to 1/4 - 1/5 of conventional colour spaces.

The distortions of hue circles with low chroma ($C=2-4$) and high chroma ($C=12-14$) are still rather large; particularly the high chroma portion of 5Y direction is compressed, while on the other hand, the high chroma portion of 5PB direction is expanded.

This is due to the non-linearity of opponent response in visual system. We have limited our analyses to the linear space in this study, while this can be improved by introducing non-linearity in colour space.

When reviewing hue angles corresponding to 5 principal colours in the linear colour space, the mean deviation σ_θ indicating the change of hue angles created by the difference of chroma is large at 5B, which can be attributable to non-linearity of Y-B response.

However, when reviewing the separation between hue angles $\Delta \theta$ corresponding to the 5 principal colours, it is clear that uniformity of intervals of angles is superior in linear uniform colour space than in conventional colour spaces.

While the introduction of a linear uniform colour space has been aimed at improving uniformity in relation to chroma, we can see that it is also superior in attaining uniformity in relation to hue, compared to conventional colour spaces.

The objective of this study has been to establish a new colour space with better uniformity, while maintaining

Figure 6 Coordinates of Munsell colours with values of 1-9 in the uniform linear colour space NC-I

linearity on the constant value plane, yet in order to further improve uniformity, we should introduce an expression with consideration given to nonlinear response of human visual system.

However, the linear colour space is advantageous in terms of mathematical operation, because linearity between coordinate values of the colour space could be maintained.

Therefore, it can be said that this colour space is exceptionally superior in uniformity compared to the CIE $Y-(x, y)$ space, and will prove to be highly valuable.

5 Conclusion

The linear uniform colour space introduced in this study is composed of linear combinations of tristimulus values X, Y

Table 3 Mean radii $\langle R \rangle$ and relative deviations σ , which indicate the degrees of warp of the hue circles from uniform circles in the $L^*a^*b^*$, $L^*u^*v^*$, Cube-Root uniform colour spaces and new linear uniform colour space NC-I

(a) In the case that $V=6$ and $C=2-14$

Colour	Space	$L^*a^*b^*$	$L^*u^*v^*$	Cube-Root	NC-I
Mean Radius $\langle R \rangle$ $V=6$	C=2	10.339	15.226	10.599	10.624
	4	20.592	29.967	21.160	20.729
	6	31.142	44.604	31.992	30.802
	8	41.385	57.983	42.458	40.164
	10	51.659	70.724	52.910	49.428
	12	62.133	82.982	63.559	58.867
	14	73.901	94.459	74.406	68.223
Deviation σ $V=6$	C=2	0.2781	0.2679	0.2443	0.2149
	4	0.2430	0.2177	0.2200	0.1347
	6	0.2250	0.1822	0.2030	0.0648
	8	0.2280	0.1631	0.2017	0.0428
	10	0.2222	0.1458	0.1936	0.0727
	12	0.2126	0.1365	0.1853	0.1258
	14	0.2385	0.1395	0.1840	0.1778
$\langle \sigma \rangle$	Mean	0.2353	0.1790	0.2046	0.1191

(b) In the case that $V=1-9$ and $C=8$.

As for $V=1,2,3,8,9$, numerical values are putted in parentheses, because hue circles with chroma of 8 are imperfect due to partial lack of chromaticity coordinates of the colours on these planes.

Colour	Space	$L^*a^*b^*$	$L^*u^*v^*$	Cube-Root	NC-I
Mean Radius $\langle R \rangle$ $C=8$	$V=1$	(42.939)	(28.802)	(46.715)	(46.297)
	2	(38.168)	(33.899)	(41.740)	(41.040)
	3	(41.088)	(48.319)	(43.241)	(41.306)
	4	42.511	52.851	43.702	40.034
	5	42.025	56.138	43.128	40.386
	6	41.385	57.983	42.458	40.164
	7	41.108	59.917	42.173	40.371
	8	(43.104)	(63.794)	(44.123)	(41.928)
	9	(53.863)	(73.776)	(51.234)	(46.138)
Deviation σ $C=8$	$V=1$	(0.2113)	(0.1837)	(0.3481)	(0.1983)
	2	(0.0926)	(0.2391)	(0.2055)	(0.2113)
	3	(0.1454)	(0.1636)	(0.1648)	(0.1734)
	4	0.1856	0.1345	0.1712	0.1204
	5	0.2051	0.1454	0.1834	0.0668
	6	0.2280	0.1631	0.2017	0.0428
	7	0.2236	0.1703	0.1974	0.0496
	8	(0.2069)	(0.1734)	(0.1863)	(0.0562)
	9	(0.1153)	(0.1213)	(0.0833)	(0.0515)
$\langle \sigma \rangle$	Mean	0.1793	0.1660	0.1935	0.1078

and Z , so that the hue circle for Munsell colours with $V=6$ and $C=8$ has become nearest to a uniform circle with constant radius.

In conventional colour spaces, the hue circle is remarkably

Table 4 Hue angles θ of principal 5 hues with $V=6$ and $C=2-14$, and deviations σ_θ of these hue angles in the new uniform colour space NC-I

Hue		5 R	5 Y	5 G	5 B	5 P
Hue Angle θ [deg]	C=2	23.606	93.804	159.985	221.141	310.765
	4	23.341	92.442	160.593	224.679	310.790
	6	22.642	90.321	161.053	228.378	310.707
	8	21.619	89.125	161.794	231.663	310.547
	10	20.616	88.213	162.501	234.286	310.518
	12	19.515	87.894	163.228	236.372	310.719
	14	18.602	87.233	163.856	238.104	310.815
$\langle \theta \rangle$	Mean	21.420	89.862	161.858	230.660	310.695
σ_θ	Deviation	1.7816	2.2821	1.3118	5.7789	0.1083

Table 5 Mean hue angles $\langle \theta \rangle$ of principal 5 hues with $V=6$ and $C=2-14$, interval $\Delta \theta$ of adjacent hue angles and mean deviations σ_θ of these hue angles in the $L^*a^*b^*$, $L^*u^*v^*$, Cube-Root uniform colour spaces and new uniform colour space NC-I

Colour Space	Hue	5 R	5 Y	5 G	5 B	5 P
$L^*a^*b^*$ [deg]	Interval	R-P	Y-R	G-Y	B-G	P-B
	$\langle \theta \rangle$	27.736	92.640	161.951	227.375	314.437
	$\Delta \theta$	73.299	64.904	69.311	66.424	87.062
$L^*u^*v^*$ [deg]	σ_θ	0.6240	2.1147	1.6626	2.8467	1.3072
	$\langle \theta \rangle$	15.822	69.381	149.785	220.080	288.753
	$\Delta \theta$	87.069	53.559	80.404	70.295	68.673
Cube-Root [deg]	σ_θ	1.6840	1.8188	1.7773	4.3797	0.1286
	$\langle \theta \rangle$	22.057	76.526	161.714	210.364	302.022
	$\Delta \theta$	80.035	54.469	85.188	48.650	91.658
NC-I [deg]	σ_θ	1.0095	1.3433	1.8053	0.9377	1.4726
	$\langle \theta \rangle$	21.420	89.862	161.858	230.660	310.695
	$\Delta \theta$	70.725	68.448	71.996	68.802	80.035
	σ_θ	1.7816	2.2821	1.3118	5.7789	0.1083

far from a uniform circle. For this reason, while the distance from the origin point is called the metric chroma, it is difficult to correspond this distance with the perceived chroma.

Moreover, colours with the same chroma but different hues are indicated in the space as having not only different hue angles, but also different metric chromas.

Therefore, it has been impossible to make the metric hue and chroma distances correspond with perceived hue and chroma differences, respectively, and to indicate them uniformly in traditional spaces.

Although hue circles for colours of high and low chromas are still somewhat distorted due to lack of consideration on non-linearity in receptor level of perception and in opponent colour response process, this new linear uniform colour space allows colours of moderate chromas to be expressed on the curves of nearly uniform circles.

In this linear colour space, we are able to separately treat variations in hue and chroma as changes in hue angle and radius of hue circle, respectively, thereby allowing colours, as well as colour differences, to be specified in a manner that is closer to the characteristics of colour perception of human beings.

Moreover, the equations derived in this study include the compensation for chromatic adaptation of von Kries type with dividing tristimulus values of colours by tristimulus values of background illuminant.

Therefore, they may also be applied to the specification of colours under the illumination of any illuminant.

For further improvement on the uniformity of this colour space to specify colours and colour differences appropriately with proper correspondence to the functions of human visual system, we must introduce theoretical equations according to the stage theory which take into account the non-linear characteristics in the sequential response processes of human visual system.

We plan to develop such a non-linear uniform colour space in the future, with this linear colour space as its basis.

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