All Sky Model as a Standard Sky Luminance Distribution
—Part 2 A Numerical Expression of Sky Luminance Distributions for All Sky Conditions —

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ABSTRACT

The aim of this research work is to establish a standard sky model for designing excellent daylighting schemes that cover all sky conditions from clear sky to overcast sky.

In the previous paper (Part 1) 1), the "normalized global illuminance" was defined as a function of the measured global illuminance and the solar altitude for estimating the sky luminance distribution.

In this paper (Part 2), the "Relative All Sky Model" is introduced. It offers formulas to show the relative sky luminance distribution as a function of the normalized global illuminance. The equation of the zenith luminance concerning the Relative All Sky Model is also a function of the normalized global illuminance. It is called the "All Sky Zenith Luminance". An absolute standard sky luminance distribution model called the "All Sky Model" is introduced that is a multiplication of the Relative All Sky Model and the All Sky Zenith Luminance. The All Sky Model can be calculated from the normalized global illuminance, i.e., from the measured global illuminance and the solar altitude or from the measured global illuminance, the horizontal diffuse illuminance, and the solar altitude. The global illuminance and horizontal diffuse illuminance are easily obtained from even the simplest daylight measurements. The All Sky Model can produce an excellent standard sky for daylighting design for any place where an advanced daylight environment is needed.

KEYWORDS: sky luminance distribution, standard sky, daylighting design, All Sky Model

1. Introduction

The sky luminance distribution is the most basic information for daylighting design. The aim of this research work is to propose a sky luminance distribution model as a standard sky for daylighting design.

In the previous papers (Part 1) 1), we described sky luminance distribution measurement and a method of arranging the measured data. Furthermore, we discussed indices for estimating the sky luminance distribution for the whole range, from clear sky to overcast sky, based on measured values and calculated values regarding the daylight and solar radiation other than the sky luminance distribution. After comparison of various indices, the normalized global illuminance defined as the quotient that divided global illuminance with the global illuminance of clear sky was selected as an index for estimating the sky luminance distribution.

In this paper (Part 2), we will propose, firstly, a numerical model to show the sky luminance distribution for all sky conditions, from clear sky to overcast sky, by a relative value to the zenith luminance as a function of the normalized global illuminance based upon measured data of the sky luminance distribution. Secondly, we will propose a zenith luminance model that shows the zenith luminance at an arbitrary solar altitude and an arbitrary sky condition as a function of the corresponding normalized global illuminance. Finally, we will propose a formula to show the sky luminance distribution for all sky conditions in absolute values. This all-sky luminance distribution in absolute values was named "All Sky Model" and had been presented at various domestic and overseas conferences, and so on 2) - 8). Since then, the study results have been rearranged and integrated, and the theoretical structure has been improved slightly. The formulas have also been modified. This paper shows the formularization of the relative sky luminance distribution and the formularization of the zenith luminance will be described anew, and then the new All Sky Model that is created from these two formulas.

2. Formularization of Relative Sky Luminance Distribution

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It is considered that the formula for the relative sky luminance distribution should express all sky conditions continuously from clear sky to overcast sky by the same sky index. At the same time, it must be consistent with the results obtained by actual measurement. Hence, the existing various sky luminance distributions were compared to choose a form of equation that is considered to be able to show all sky conditions continuously. This is called the basic equation. Then, a regression analysis of the sky luminance distribution data obtained by actual measurement was applied to the basic equation, and a numerical equation showing the relative sky luminance distribution for all sky conditions was proposed. This is called the Relative All Sky Model.

2.1 Basic Data for Formulation

Two-hundred eighty sets of the average sky luminance distribution, which represented for every 5° of solar altitude by every 5% of ordering index, were obtained from about 20,000 samplings, as described in the previous paper, and were called the sky luminance distribution of the individual sky. They were applied as the basic data for formalization of the sky luminance distribution. The sky luminance distribution was assumed to be symmetry for the solar meridian.

2.2 Basic Equation for Formulation

The sky luminance distribution of the CIE Standard Clear Sky is expressed as a product of an equation \( f_0 \) to show diffusion indicatrix of sky elements with the angular distance between the sun and the sky elements and an equation \( \Phi(y) \) to show a gradation regarding the altitude of the sky elements \(^{10} \)

\[
L_{\text{CR}}(y_0, y, \zeta) = \frac{L_{\text{CR}}(y_0, y, \zeta)}{L_{\text{CR}}(y_0)} = \frac{\Phi(y) \cdot f_0(\zeta)}{f_0(\pi/2)} \cdot f(x/2 - y) \]  \hspace{1cm} (1)

\( L_{\text{CR}}(y_0, y, \zeta) \): Relative luminance of a sky element in the CIE Standard Clear Sky,
\( L_{\text{CR}}(y_0, y, \zeta) \): Corresponding luminance of the sky element in the CIE Standard Clear Sky,
\( L_{\text{CR}}(y_0) \): Corresponding zenith luminance of the CIE Standard Clear Sky,
\( \Phi(y) \): Gradation function,
\( f_0(\zeta) = 1 - \exp(-0.32/sin(y)) \): Gradation function,
\( f_0(\zeta) = 0.91 \pm 10 \pm 3 \cdot \zeta + 0.45 \cdot \cos^2 \zeta \): Diffusion indicatrix,
\( y_0 \): Solar altitude [rad],
\( y \): Altitude of the sky element [rad],
\( \zeta \): Angular distance between the sun and the sky element [rad].

There are several other equations that express the sky luminance distribution from clear sky to overcast sky that have forms similar to equation (1).

In Homogeneous Sky \(^{10} \) by Kittler, R., the diffusion indicatrix is expressed as follows.

\[
f_0(\zeta) = 1 + a \cdot \exp(-3 \cdot \zeta) - 0.009 + b \cdot \cos^2 \zeta \]  \hspace{1cm} (2)

\( a \) and \( b \): Coefficients.

In All-Weather Model \(^{10} \) by Perez, R., the relative luminance of an arbitrary sky element is expressed as follows.

\[
L_{\text{CR_{AW}}} = \{1 - a \cdot \exp(\cos(y)) \} \cdot \{1 + c \cdot \exp(d \cdot \zeta + e \cdot \cos^2 \zeta)\} \]  \hspace{1cm} (3)

\( L_{\text{CR_{AW}}} \): Relative luminance of a sky element in the All Weather Model,
\( y \): Zenith angle of the sky element [rad],
\( a, b, c, d, e \): Coefficients.

In Sky Luminance Model \(^{10} \) by Perraudin, M., the relative luminance of an arbitrary sky element is expressed as follows.

\[
L_{\text{CR_{Sky}}} = \{a - b \cdot \sin^2(y)\} \cdot \{c + d \cdot \exp(-3 \cdot \zeta) + e \cdot \cos^2 \zeta\} \]  \hspace{1cm} (4)

\( L_{\text{CR_{Sky}}} \): Relative sky luminance of a sky element in the model of Perraudin, M.,
\( a, b, c, d, e \): Coefficients.

Although these equations use independent sky indices in order to determine each coefficient of \( a \) and \( b \), \( a, b, c, d, e \), their forms are basically about the same. Following them, the diffusion indicatrix to show all sky luminance distribution from clear sky to overcast sky is expressed in this study as follows so that it becomes 1 when \( \zeta \) is 90°.

\[
f_0(\zeta) = 1 + b \cdot \exp(c \cdot \zeta - \exp(c \cdot \pi/2)) + d \cdot \cos^2 \zeta \]  \hspace{1cm} (5)

\( f_0(\zeta) \): Diffusion indicatrix
\( b, c, d \): Coefficients

Regarding \( \Phi(y) \), the CIE Standard Clear Sky and the All-Weather Model by Perez, R. use equations of the same form. The divisor of both \( \Phi(y) \) becomes zero when the altitude of the sky element is zero. \( \Phi(y) \) in this study is expressed as follows so that the divisor does not become zero and also so that continuity from the clear sky to the overcast sky is taken into account.

\[
\Phi(y) = 1 / \{1 + a/\exp(a2 \cdot \sin(y))\} \]  \hspace{1cm} (6)

\( \Phi(y) \): Gradation function,
\( a1 \) and \( a2 \): Coefficients

The corresponding sky luminance distribution is expressed by the following equation, which is based upon equations (5) and (6).

\[
L(y, \zeta) = \Phi(y) \cdot f_0(\zeta) = 1/(1 + a/\exp(a2 \cdot \sin(y))) \cdot \{1 + b \cdot \exp(c \cdot \zeta - \exp(c \cdot \pi/2)) + d \cdot \cos^2 \zeta\} \]  \hspace{1cm} (7)

\( L(y, \zeta) \): Corresponding luminance of the sky element,
\( a1, a2, b, c, d \): Coefficients.

It is necessary for coefficients \( a1 \) and \( a2 \) to correspond to the property of \( \Phi(y) \) that expresses the gradation of
luminance in regard to the altitude of the sky element, that is, the tendency that the value of $\Phi(y)$ decreases in the overcast sky and increases in the clear sky when the altitude of the sky element is low. Coefficient $b$ is a property proportionate to the luminance in the vicinity of the sun and it is necessary that it shows a tendency to increase for the clear sky and to decrease for the overcast sky. Coefficient $c$ shows the changes in inclination of luminance in the vicinity of the sun. It is necessary to show a tendency to be small for the clear sky when the change in inclination of the luminance in the vicinity of the sun is large and to be large for the overcast sky. Coefficient $d$ shows the back scattering on the surface of the earth and it is necessary to show a tendency to be large for the clear sky and to be small for the overcast sky.

The relative sky luminance distribution for all sky conditions called the Relative All Sky Model is expressed as follows referring to the CIE Standard Clear Sky.

$$L_r(\gamma, \zeta) = \frac{L(\pi/2, \pi/2 - \gamma)}{L_r(\pi/2, \pi/2)}$$  \hspace{1cm} (8)

$L_r(\gamma, \zeta)$: Relative luminance of a sky element in the Relative All Sky Model,  
$L(\gamma, \zeta)$: Corresponding luminance of the sky element and  
$L_r(\pi/2, \pi/2)$: Corresponding zenith luminance ,  
$\Phi(y)$: Gradiation function,  
$\mathcal{K}(\zeta)$: Diffusion indicatrix .

2.3 Determination of Coefficients in the Basic Equations

A regression analysis was applied using the basic data based upon equation (8) in order to determine the coefficients $a_1$, $a_2$, $b$, $c$, and $d$. When $\Phi(y)$ is put as $A$, $L(\gamma, \zeta)$ is shown by the following equation.

$$L(\gamma, \zeta) = A \cdot [1 + b \cdot \{\exp(c \cdot \zeta) - \exp(c \cdot \pi/2)\} + d \cdot \cos^2\zeta]$$  \hspace{1cm} (9)

In order to determine the coefficients $A$, $b$, $c$, and $d$, regression analyses independent of each other were made from the sky luminance distribution data of 280 individual skies. After that, the results of the 280 regression analyses were applied to each coefficient, and, furthermore, the regression analyses were implemented to determine the coefficients.

2.3.1 Regression Analysis of the Relative Sky Luminance Distribution of the Individual Sky

The altitudes of the sky elements in the sky luminance distribution data obtained by actual measurement were 6°, 18°, 30°, 42°, 54°, 66°, 78°, and 90°. Data for the sky elements at the 6° altitude were excluded from the regression analysis since there was a possibility that they were influenced by the surrounding buildings. Also, for each of the seven altitudes of the sky elements, the average luminance of the two sky elements at the same angular distances from the sun was used for the regression analysis. The number of data finally used for the regression analysis is shown in Table 1.

<table>
<thead>
<tr>
<th>Altitude of sky elements [deg]</th>
<th>Number of measuring points</th>
<th>Number of data used for regression analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>30</td>
<td>0</td>
</tr>
<tr>
<td>18</td>
<td>30</td>
<td>16</td>
</tr>
<tr>
<td>30</td>
<td>24</td>
<td>13</td>
</tr>
<tr>
<td>42</td>
<td>24</td>
<td>13</td>
</tr>
<tr>
<td>54</td>
<td>18</td>
<td>9</td>
</tr>
<tr>
<td>66</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>78</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>90</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

It is desirable to have larger number of data for the regression analysis. Therefore, the coefficients were sorted and judged, giving priority to the results of the regression analysis for the data of the sky elements at the altitudes of 18°, 30°, 42°, and 54°, and these results were examined with the data for 66°, 78°, and 90°, for which the number of data were small.

The regression analysis was applied with equation (8) on each altitude of the same sky element to obtain values for the coefficients $(A', b', c', d')$ and coefficients of correlation $r$ for each of the seven altitudes from 18° to 90°. Initially, attention was paid to $b$ to select, among seven $b$s in total, those with the top two largest coefficients of correlation out of four kinds of $b$ corresponding to the sky element altitudes of 18°, 30°, 42°, and 54°. The average of them was put as $b$ provisionally.

This $b$ was substituted as a constant in the equation and the second regression analysis was made using the basic data to obtain seven values of the coefficients $A'$, $c'$, and $d'$ per sky element altitude. Similar to the case of $b$, a provisional $c'$ was obtained based upon $c'$.

Furthermore, a third regression analysis was applied with $b'$ and $c'$ substituted as constants, and $d'$ was obtained from $d'$ obtained in a similar method.

A provisional diffusion indicatrix based upon the basic equation for the individual sky was obtained using these provisional $b'$, $c'$, and $d'$.

Then, the fourth regression analysis was applied in order to determine the coefficient $A'$ based upon the basic data with the provisional $b'$, $c'$, and $d'$ substituted as constants to obtain seven values of the coefficient $A'$ for each altitude of the sky elements with corresponding coefficients of correlation. By these obtained seven values of $A'$, the gradation function can be expressed as a function of the altitude of the sky element by an equation with coefficients $a_1$ and $a_2$. 

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2.3.2 Regression Analysis for the Relative All Sky Model Corresponding to All Sky conditions

Two hundred eighty independent regression analyses regarding equation (8) were repeated based upon the basic data. From the obtained regression coefficient, regression analysis of the relationship between \( b \) and normalized global illuminance was applied to determine \( b \) as follows.

\[
b = 24.3 \times (1.6 \times \text{Nevg})^{59} \times \exp(-0.20 \times \text{Nevg} \times (1.1 - \text{Nevg})^{1.5})
\]  \hspace{1cm} (10)

Then, \( b \) was substituted in equation (8) and the regression analysis was repeated to determine \( c \) as shown in the following equation.

\[
c = -3.05/\{1 + 24 \times \exp(-7.7 \times \text{Nevg})\}
\]  \hspace{1cm} (11)

Furthermore, \( b \) and \( c \) per equations (10) and (11) were substituted in equation (8) and the regression analyses were repeated based upon the basic data on all individual skies to determine \( d \) as shown in the following equation.

\[
d = 0.46/\{1 + 630 \times \exp(-9.9 \times \text{Nevg})\}
\]  \hspace{1cm} (12)

As described above, all of \( b \), \( c \), and \( d \) were determined.

In order to determine \( A \), equations (10), (11) and (12) were substituted in equation (8), and the regression analyses were repeated on all skies.

The value of \( A \) corresponding to the sky element altitude of 90° as obtained through regression analyses is indicated by \( A_{90} \). With the value of the obtained \( A \), for each altitude of the sky elements divided by \( A_{90} \), as shown in the following equation, regression analysis was applied on each individual sky.

\[
\frac{A_{\gamma}}{A_{90}} = 1/\{1 + a1/\exp(a2 \times \text{sin} \gamma)\}
\]  \hspace{1cm} (13)

\( A_{\gamma} \): Coefficient \( A \) in case of altitude \( \gamma \) of a sky element [\( \cdot \)],

\( A_{90} \): Coefficient \( A \) in case of altitude 90° of the sky element [\( \cdot \)].

After repeated regression analysis, \( a1 \) and \( a2 \) were determined as follows.

\[
a1 = 5.5
\]  \hspace{1cm} (14)

\[
a2 = 1.82 \times \text{Nevg}^2 - 5.82 \times \text{Nevg} + 2.26
\]  \hspace{1cm} (15)

2.4 Comparison of the Relative All Sky Model with Previous Models and Measured Values

Based upon the regression analyses described above, the relative sky luminance distribution for the skies from clear sky to overcast sky can be expressed continuously. The Relative All Sky Model can be expressed in the following equation, which is a function of the normalized global illuminance and the solar altitude.

\[
L_{\gamma, \gamma, \zeta, \text{Nevg}} = \frac{\Phi(\gamma) \times f(\zeta)}{\Phi(\pi/2) \times f(\pi/2 - \gamma,.)}
\]  \hspace{1cm} (16)

\( L_{\gamma, \gamma, \zeta, \text{Nevg}} \): Relative luminance of a sky element in the Relative All Sky Model,

\[
\Phi(\gamma) = 1/\{1 + a1/\exp(a2 \times \text{sin} \gamma)\}
\]

\[
f(\zeta) = 1 + b \times \{\exp(c \times \zeta) - \exp(c \times m/2)\} + d \times \cos^2 \zeta,
\]

\[
a1 = 5.5,
\]

\[
a2 = 1.82 \times \text{Nevg}^2 - 5.82 \times \text{Nevg} + 2.26,
\]

\[
b = 24.3 \times (1.6 \times \text{Nevg})^{59} \times \exp(-0.20 \times \text{Nevg} \times (1.1 - \text{Nevg})^{1.5}),
\]

\[
c = -3.05/\{1 + 24 \times \exp(-7.7 \times \text{Nevg})\},
\]

\[
d = 0.46/\{1 + 630 \times \exp(-9.9 \times \text{Nevg})\},
\]

\( \text{Nevg} \): Normalized global illuminance (= \( \text{vg} \times \text{Sevg}(\gamma) \)),

\( \text{Evg} \): Global illuminance [klx],

\( \text{Sevg}(\gamma) \): Standard global illuminance [klx],

\[
\text{Sevg}(\gamma) = -36.78 \times \gamma^5 + 188.79 \times \gamma^4 - 375.95 \times \gamma^3 + 306.20 \times \gamma^2 + 15.47 \times \gamma + 0.83 \text{ [klx]},
\]

\( \zeta \): Angular distance between the sun and the sky element [rad],

\( \gamma \): Altitude of the sky element [rad],

\( \gamma_0 \): Solar altitude [rad].

Note: \( f(\zeta) \) was put as \( \text{Nevg}=1.0 \) in cases of \( \text{Nevg}>1.0 \).

Then, the Relative All Sky Model was compared with the previous sky luminance distribution models for validation.

![Graph showing comparison of relative diffusion indicatrixes.](image)

**Fig.1** Comparison of relative diffusion indicatrixes.

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For the previous models, the CIE Standard Clear Sky, Perraud et al.'s Blue Sky, and Perez's Clear Sky were selected and compared with the case in which the normalized global illuminance was put as 1 in the Relative All Sky Model. The results are shown in Fig. 1.

The diffusion indicatrix of the Relative All Sky Model in the case of $N_{\text{avg}}=1$ (clear sky) was perfectly in accordance with the diffusion indicatrix of the CIE Standard Clear Sky.

In the model of Perez, $R_s$, the sky luminance is higher compared to the other models when the angular distance between the sun and the sky elements is larger than $90^\circ$. Also, it shows the tendency that the luminance is lowest in the range from $90^\circ$ to $25^\circ$ among the sky elements with the angular distance between the sun and the sky elements below $90^\circ$ and that the luminance increases drastically in the vicinity of the sun.

The coefficients of correlation between the Relative All Sky Model and the sky luminance distribution measured are shown in Fig. 2. The coefficients of correlation are more than 0.95 for almost all sky conditions. The coefficients of correlation are slightly smaller in cases of the solar altitude below $12.5^\circ$ and a relative ordering index in the vicinity of 60%. This is considered to be almost no problem for its practical application.

Also, examples of comparisons of relative sky luminance distributions between measured skies and calculated values in cases of the solar altitude in the range from $30^\circ$ to $35^\circ$ are shown in Fig. 3. It is considered that they match fairly well as a whole from the clear sky to the overcast sky.

In view of the above, it is considered that the Relative All Sky Model is an appropriate relative sky luminance distribution model which expresses the sky luminance distribution continuously from the clear sky to the overcast sky.

**Fig. 2** Coefficients of correlation between the Relative All Sky Model and measured skies.

**Fig. 3** Comparison of relative sky luminance distributions between measured skies and estimated skies (solar altitude: $30^\circ$–$35^\circ$).
3. All Sky Zenith Luminance

So far, the Relative All Sky Model that expresses the relative sky luminance distribution with an index of the normalized global illuminance has been proposed. It is necessary to know its absolute value for its practical application. For this purpose, it is proposed to show the zenith luminance at an arbitrary solar altitude and an arbitrary sky condition as a function of the normalized global illuminance. This is called the All Sky Zenith Luminance hereinafter.

3.1 Basic Data for All Sky Zenith Luminance and Their Arrangement

Data regarding the zenith luminance were extracted from the data used for formulating the Relative All Sky Model. The total number of data was approximately 20,000.

Firstly, the data of the zenith luminance and the normalized global illuminance were sorted into fourteen solar altitude zones, that is, by intervals of 5° in solar altitude from 5° to 75°. Then, each solar altitude zone was divided into eleven groups of the normalized global illuminance, these being one interval centered on 0.05 and ten more intervals 0.1 apart in the range from 0.1 to 1.0, and averages of the zenith illuminance for each group were obtained. These were made the basic data for formulating the zenith luminance shown in Fig. 4.

3.2 Basic Equation for Formularization of Zenith Luminance

There has been no equation proposed to express the zenith luminance continuously for all sky conditions. Nakamura et al. proposed the following form for the zenith luminance of the clear sky:\(^{10,8}\)

\[ L_{\infty}(\gamma_c) = A \cdot \sin^2(\gamma_c) + B \]  
\[ L_{\infty}(\gamma_c): \text{Zenith luminance of clear sky [kcd/m}^2\].

Furthermore, Nakamura et al. proposed the equation (19) that compounded the equations to express the clear sky and the overcast sky for the zenith luminance of the intermediate sky:\(^{12,17}\)

\[ L_{\infty}(\gamma_c) = A \cdot \sin^2(\gamma_c) + B \cdot \tan^2(G \cdot \gamma_c) + C \]  
\[ L_{\infty}(\gamma_c): \text{Zenith luminance of overcast sky [kcd/m}^2\].

Referring to the basic data and each equation in the above, the following equation which is a function of the solar altitude and the normalized global illuminance was decided to express the All Sky Zenith Luminance.

\[ L_{\infty}(\gamma_c, \text{ Nevg}) = A \cdot \sin^2(F \cdot \gamma_c) + B \cdot \tan^2(G \cdot \gamma_c) + C \]  
\[ L_{\infty}(\gamma_c, \text{ Nevg}): \text{All Sky Zenith Luminance [kcd/m}^2\].

A, B, C, D, E, F, and G. Coefficients.

Equation (20) is to be called the basic equation for the All Sky Zenith Luminance.

3.3 Determination of Coefficients in the Basic Equation of the All Sky Zenith Luminance

Based upon the relationship between the solar altitude and the zenith luminance by the normalized global illuminance as shown in Fig. 4, regression analyses using equation (20) were applied for each solar altitude. First, the regression analysis was applied for each solar altitude and obtained the values of provisional coefficients (put as \(A', B', C', D', E', F'\) and \(G\)).

Then, based upon the obtained values of the provisional coefficients \(F\) and \(G\), a regression analysis was applied for each solar altitude to obtain \(F=0.7\) and \(G=0.7\).

Next, \(F=0.7\) and \(G=0.7\) were substituted in the equation (20) and regression analyses with the basic data were repeated, as was done with \(F\) and \(G\), to obtain \(D=1.3\) and \(B=1.3\).

Then, \(D, E, F,\) and \(G\) were substituted in equation (20) and the regression analyses were continued. Regarding the provisional coefficient \(A'\) obtained as a result, a further regression analysis was applied as a function of the normalized global illuminance to obtain the following equation.

\[ A = -1231.9 \cdot \text{Nevg}^2 + 2713.8 \cdot \text{Nevg} - 1302.3 \cdot \text{Nevg}^4 - 440.19 \cdot \text{Nevg}^2 + 188.91 \cdot \text{Nevg}^2 + 63.044 \cdot \text{Nevg} \]  
\[ A = -114.78 \cdot \text{Nevg}^2 + 4281.1 \cdot \text{Nevg} \]  

Then, \(A, D, E, F,\) and \(G\) were substituted in equation (20) and the regression analysis was continued to obtain the following equation for \(B\):

\[ B = 962.58 \cdot \text{Nevg}^2 - 2123.2 \cdot \text{Nevg} + 1121.5 \cdot \text{Nevg} + 1005.5 \cdot \text{Nevg}^2 - 114.78 \cdot \text{Nevg}^2 + 4281.1 \cdot \text{Nevg} \]  

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Furthermore, $A, B, D, E, F,$ and $G$ were substituted in equation (20) and the regression analysis was continued to obtain the following equation for $C$.

$$C = -1.86 \cdot \text{Nevg}^2 + 2.43 \cdot \text{Nevg} + 0.04 \cdot \text{Nevg}$$  \hspace{0.5cm} (23)

Reflecting the above, with the determined $D, E, F,$ and $G$ put as constants, the All Sky Zenith Luminance is expressed by the following equation, in which $A, B,$ and $C$ are the functions of the solar altitude and the normalized global illuminance.

$$L_{\text{z}}(\gamma_s, \text{Nevg}) = A \cdot \sin^{13} (0.7 \cdot \gamma_s) + B \cdot \tan^{13} (0.7 \cdot \gamma_s) + C \quad [\text{klx/m}^2]$$  \hspace{0.5cm} (24)

$L_{\text{z}}(\gamma_s, \text{Nevg})$: All Sky Zenith Luminance [klx/m$^2$],

$$A = -1231.9 \cdot \text{Nevg}^6 + 2713.8 \cdot \text{Nevg}^5 - 1302.3 \cdot \text{Nevg}^4$$
$$- 440.19 \cdot \text{Nevg}^3 + 188.91 \cdot \text{Nevg}^2 + 63.044 \cdot \text{Nevg},$$

$$B = 962.58 \cdot \text{Nevg}^6 - 2123.2 \cdot \text{Nevg}^5 + 1121.5 \cdot \text{Nevg}^4$$
$$+ 160.55 \cdot \text{Nevg}^3 - 114.78 \cdot \text{Nevg}^2 + 4.281 \cdot \text{Nevg},$$

$$C = -1.86 \cdot \text{Nevg}^2 + 2.43 \cdot \text{Nevg} + 0.04 \cdot \text{Nevg},$$

$\text{Nevg}$: Normalized global illuminance ($= \text{Evgr} / \text{Sevg}(\gamma_s)$),

$\text{Evgr}(\gamma_s)$: Global illuminance [klx],

$\text{Sevg}(\gamma_s)$: Standard global illuminance [klx],

$$\text{Sevg}(\gamma_s) = -36.78 \cdot \gamma_s^5 + 188.79 \cdot \gamma_s^4 - 375.95 \cdot \gamma_s^3 + 306.20 \cdot \gamma_s^2$$
$$+ 15.47 \cdot \gamma_s + 0.83 \quad [\text{klx}],$$

$\gamma_s$: Solar altitude [rad].

3.4 Comparison of Measured Value and Calculated Value of Zenith Luminance

The measurements of zenith luminance were compared with the All Sky Zenith Luminance obtained by equation (24) based upon the normalized global illuminance calculated with the corresponding solar altitude and global illuminance. Fig. 5, Fig. 6, and Fig. 7 show examples of the zenith luminance measured and the All Sky Zenith Luminance calculated from the normalized global illuminance and the solar altitude for the cases of solar altitude zones from 10° to 15°, from 30° to 35°, and from 50° to 55°. It is considered that the measured values and calculated values matched well in any solar altitude zone.

3.5 Comparison with Previous Zenith Luminance Models

The All Sky Zenith Luminance in cases of the normalized global illuminance being 0.9, 1.0, and 1.05 were compared with the clear sky zenith luminance proposed by Kittler, R. 18, Dogniaux, R. 18, Krochmann,
All Sky Zenith Luminance across the range of 1.0 through 1.05 of the normalized global illuminance (definitely clear sky).

In a similar method but regarding the overcast sky, the All Sky Zenith Luminance in the cases of the normalized global illuminance of 0.15, 0.2, and 0.3 were compared with the zenith luminance proposed by Kittler, R.21, Krochmann, J.22, and Nakamura, H. et al.18. The results of the comparisons are shown in Fig. 9. According to the figure, the previous zenith luminance for the overcast sky is included in the range of the All Sky Zenith Luminance. Also the value in the case of 0.3 of the normalized global illuminance (slightly bright overcast sky) is close to the value obtained through the proposed equation of Nakamura, H. et al.

The number of proposed equations for the intermediate sky is few. Nakamura, H. et al. proposed the zenith luminance of the intermediate sky together with luminance distribution of the intermediate sky1417.

The intermediate sky of Nakamura, H. et al. is the average of skies, excluding clear sky, overcast sky, and skies that are thought to be close to these, and his definition is different from the sky condition equivalent to the intermediate sky in this study. Therefore, they cannot be compared easily. However, the All Sky Zenith Luminance in the cases of three kinds of the normalized global illuminance, 0.3, 0.5, and 0.7, was compared with the zenith luminance of the intermediate sky by Nakamura, H. et al. The results are shown in Fig. 10. In general, the zenith luminance has a tendency to be low in the overcast sky, to get higher gradually in accordance with a shift from the overcast sky to the intermediate sky, and to get lower gradually again in accordance with a shift from the intermediate sky to the clear sky. The All Sky Zenith Luminance shown in Fig. 10 includes the zenith luminance at the highest values. It is reasonable that each of them is different to some extent from the zenith luminance of the intermediate sky by Nakamura, H. et al. that is an average of wide range.

Although there are small numbers of other proposed equations for the zenith luminance for skies similar to the intermediate sky, it is considered inappropriate to make comparison with them since each definition of sky similar to the intermediate sky is different.

As a result of the comparisons shown above with the measured values and various previous proposed equations, the All Sky Zenith Luminance proposed in this study is considered to be appropriate.

As a result of this study, it has become feasible to estimate the relative sky luminance distribution and zenith luminance corresponding to all sky conditions by means of the normalized global illuminance that can be obtained from the global illuminance. As a result, the all sky luminance distribution can be expressed in absolute values. In other words, it is now possible to construct the All Sky Model easily with measurements of the global illuminance alone.

Actual measurement of the global illuminance is comparatively easy. Also, when the measured data of the global illuminance cannot be obtained, the global illuminance can be obtained by multiplying the luminous efficacy by the global irradiance. The global irradiance is measured at the Meteorological Observatories. Recently, the authors proposed an equation to estimate the daylight illuminance from the irradiance55. It is possible to utilize these other sources of information.
4. All Sky Model

4.1 Composition of All Sky Model

The All Sky Model in absolute values can be obtained as a product of the Relative All Sky Model and the All Sky Zenith Luminance. This is shown in equation (25).

\[ L_{VA}(\theta_v, \gamma, \zeta, N_{eyg}) = L_{V}(\theta_v, \gamma, \zeta, N_{eyg}) \times L_{VZ}(\gamma_a, N_{eyg}) \text{ [kcd/m}^2\text{]} \]

(25)

\[ L_{VA}(\theta_v, \gamma, \zeta, N_{eyg}) \]: All Sky Model \text{ [kcd/m}^2\text{]}. \]

\[ L_{V}(\theta_v, \gamma, \zeta, N_{eyg}) \]: Relative All Sky Model [-]. \]

\[ L_{VZ}(\gamma_a, N_{eyg}) \]: All Sky Zenith Luminance [kcd/m}^2\text{]. \]

4.2 All Sky Model Composed from Global Luminance and Diffuse Illuminance

The All Sky Zenith Luminance can be estimated by dividing the diffuse illuminance measured by the diffuse illuminance (relative value) that can be obtained by integrating the Relative All Sky Model, as shown in the following equation.

\[ L_{VZ}(\gamma_a, N_{eyg}) = \frac{E_{VD}}{\int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} L_{V}(\theta_v, \gamma_a, \zeta, N_{eyg}) \sin \gamma \cos \zeta \, dy \, d\zeta} \text{ [kcd/m}^2\text{]} \]

(26)

The All Sky Model in case of the 35° solar altitude is shown in Fig. 11.
Evd: Diffuse illumiance [klx], \( \alpha \): Azimuth angle of a sky element [rad].

Accordingly, the All Sky Model can be composed from the diffuse illumiance and the Relative All Sky Model as follows.

\[
L_n(\gamma, \phi, \phi, \zeta, \psi, \phi) = \frac{L_n(\gamma, \phi, \phi, \zeta, \psi, \phi)}{\sin \gamma \cos \phi} \left[ \frac{\text{klx}}{\text{m}^4} \right]
\]

\[
= \int_{\gamma=0}^{\gamma=90} \int_{\phi=0}^{\phi=360} \frac{L_n(\gamma, \phi, \phi, \zeta, \psi, \phi)}{\sin \gamma \cos \phi} \sin \gamma \cos \phi \, d\gamma \, d\phi
\]

\[
\text{(27)}
\]

For estimating the All Sky Zenith Luminance from the diffuse illumiance as shown in equation (26), the diffuse illumiance and the global illumiance that is simultaneously measured are required. The Relative All Sky Model is estimated after obtaining the normalized global illumiance from the global illumiance. That is to say, the All Sky Model can be constructed from the simultaneously measured global illumiance and diffuse illumiance.

When daylight is measured, the global illumiance and the diffuse illumiance are measured simultaneously in most cases. Also, IDMP (International Daylight Measurement Programme) by CIE requires all measurement stations to measure these items as part of the basic set. Therefore, there is a comparatively large amount of simultaneously measured global illumiance and diffuse illumiance, and it is comparatively easy to obtain both together. Also, the numerical calculation of equation (26) is not difficult.

It is discretionary to use the global illumiance alone or to use the global illumiance and the diffuse illumiance for composing the All Sky Model. The meteorological observatory is implementing fixed-time measurement of the global illumiance at a fairly large number of locations. We have already mentioned that the global illumiance can be estimated by the global illumiance and the luminous efficacy. It is also possible to obtain the diffuse illumiance by multiplying the horizontal diffuse illumiance, which can be obtained through dividing the global illumiance, by the luminous efficacy.

5. Conclusions

In this research work, the Relative All Sky Model which allows handling the relative sky illumiance distribution of all sky conditions on a computer and the corresponding All Sky Zenith Luminance were proposed. Owing to these, it has become possible to express the sky illumiance distribution in absolute values. This was proposed as the All Sky Model. As a result, it has become possible to estimate the absolute values of the sky illumiance and its distribution at any location for which the global illumiance is available.

(1) The All Sky Model can be expressed by a comparatively simple numerical equation as a function of the normalized global illumiance and the solar altitude.

(2) The All Sky Model matches fairly well the previously proposed equations that express particular sky conditions.

(3) The error between the basic data applied for formularization the All Sky Model and the All Sky Model is small.

(4) The All Sky Model enables easy calculation using a computer.

(5) As shown in the above, the All Sky Model is believed to be eminently suitable for practical use.

Regarding the sky illumiance distribution, emphasis was put mainly on the study of the relative sky illumiance distribution in the past. It seems that the studies of the absolute values of the sky illumiance and its distribution, which are indispensable for practical use, were inadequate. Even the two CIE Standard Skies recommend only the relative values. The form of the All Sky Model is simple. It enables the estimation of the sky illumiance distribution for all sky conditions in absolute values using the global illumiance, which is the most basic measurement data of daylight.

In addition, the method proposed in this research work enables the construction of the All Sky Model with the global illumiance, which is regularly measured at the meteorological observatories, and appropriate luminous efficacies. In other words, it enables estimation of the sky illumiance distribution in absolute values for locations where daylight measurement is unavailable.

In recent years, obtaining various meteorological data, such as the standard meteorological data for calculating heat loads for air conditioning, and the Expanded AMeDaS weather data, has been facilitated. The results of this study can be applied easily to these meteorological data. It enables the assimilation of the daylight environment and the indoor thermal environment and the implementation of coordinated and integrated environmental planning and design.

References


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