Physical measurements of stray light in steer's excised eyeballs

Yoshikazu NAKANE*

By using steer's eyeballs, which are similar in optical characteristics to human eyeballs, the direct measurements of the amount of stray light in them have been made.

The results are as follows :

- (1) The effect of lapse of time after the steer's eyeballs are taken out is negligible, if it is within 10 hours.
- (2) The quantity of stray light grows greater as the light source moves nearer to a point on the visual line, especially when the light source is within 10° (degree) of the visual point.
- (3) The causes of the stray light are: (1) optical defects of eyes when the light source is within 10° of the visual point; (2) diffusion of light on the surface of the cornea when the angle stated in (1) is more than 10°.
- (4) The smaller the iris grows, the smaller the quantity of stray light.
- (5) When the light source is comparatively large, divide it into 1° units, and the total sum of the quantity of the units is almost equal to that of the quantity of the original light source.

- (6) In order to obtain the ratio of E_θ to E_o , the following equations are used :

$$\theta > 1^\circ \quad E_\theta/E_o = \exp \{(-\ln \theta)^3 - 0.25\} + 0.032 \theta^{-1.8}$$

$$\theta \leq 1^\circ \quad E_\theta/E_o = \exp(-0.17 \theta)$$

E_o : Illuminance of fovea when the light source is on the visual line.

E_θ : Illuminance of fovea when the light source deviates by θ° from the visual line.

1. Interoduction

A series of studies¹⁾⁻⁴⁾⁶⁾ have been made on visual acuity curves, showing the relation between background luminance (threshold condition of the visual consciousness in the visual field of uniform and non-uniform luminance) and size and contrast of visual objects. Moreover other studies⁵⁾ were also made on (1) the relationship between visibility of printed letters (Chinese characters of varied size, number of strokes, and sentences) and the Landolt's Ring visual targets, and (2) the estimation of readability.

It would be necessary to comprehend the effects of light-scattering conditions in human eyeballs in order to elucidate the accomodated luminance in a visual field of uneven luminance and those of stray light caused by a plane of high luminance such as

window, point and area light sources, etc.

Concerning the decrease of visual acuity caused by a highly luminant light source, several studies were made by Holladay⁸⁾, Stiles⁹⁾, Vos¹⁰⁾, Boynton¹¹⁾ and Fry¹²⁾, who made use of a method in which the light source luminance was replaced by the so-called an equivalent veiling luminance, i.e. a luminance which causes a decrease of visual acuity equal to the decrease caused by a highly luminant light source when a light veil having a uniform luminance is hung between a visual object and its observer's eyes. Their experiments were physiological and psychological measurements, and the results of such experiments does not give the very amount of stray light that might actually exist in the fovea centralis.

On the other hand, Boynton and DeMott¹³⁾⁻¹⁶⁾ measured the amount of stray light in the eyeball using excised steer's eyeballs. According to the measuring method employed by Boynton and DeMott, however, substantially a large part of the retina part including the fovea centralis was cut off, as shown in Fig. 1, and the measurements were taken of the light led out from there. Moreover,

* Department of Housing and Design, Faculty of Science of Living, Osaka City University. Sugimoto-cho, Sumiyoshi-ku, Osaka, Japan.

A part of this paper has been published in the Reference (7) in Japanese with English summaries.

Fig. 1 Schematic optical diagram of Boynton's measuring apparatus.

it is not certain whether or not all the light coming to the fovea centralis was captured. There is a possibility that the measurements were taken with the images not focused on the retina, and not in the state of focus.

When a person sees a visual object, the diameter of his pupil is adjusted to its luminance, but in the Boynton and DeMott's measurements of excised eyeballs, the effect of the iris might not be well taken into account. In examining the amount of stray light which is induced when the person actually looks at an object, it should have been necessary to have a grasp of how the amount of stray light incident to the fovea centralis part changes when the amount of the light coming in is adjusted, throttled by the iris, so that the light passing on the perimeter of the cornea is shut off.

Accordingly, the experiments were planned as follows; (1) the direct measurements of the amount of stray light in the fovea centralis are taken by inserting a light receiving element into the fovea centralis of the excised steer's eyeballs, to determine the relationship between the position of the light source and the amount of stray light, and then, (2) the effect of the iris obtained by placing an artificial iris just in front of the eyeballs, is measured, and moreover, (3) by examining the effect of the size of the light source which may take place on the amount of stray light, treating method is studied, when the sizes and configurations of the light sources are different.

The steer's eyeballs were selected for the study as their optical characteristics are very close to those of the human eyes¹⁷⁾.

2. Method of experiments

Excised steer's eyeballs were stored in physiological salt solution kept at 0°C during the period of their transportation and until they were used in the experiment. The measurements were taken of 147 excised eyeballs. The room temperature at the time of measurements was about 20°C. Just before making measurements of stray lights, a hole 4 mm. in diameter was made in the fovea centralis retina of the steer's eyeball in order to expose its vitreous body. A silicon photocell (SBC), which had been calibrated for spectral luminous efficiency, was set on the focal plane in the hole. The light receiving angle of the SBC was set at 1° to accommodate it to the range of high visual acuity of human eyes. A light shielding plate having a circular hole for adjusting the size of the light source was placed in front of a slide projector having a diffuser plate,

Fig. 2 Schematic optical diagram of the measuring apparatus.

and the size of the light source was such that a 1° visual angle was maintained. An optical diagram of the measuring apparatus is shown in Fig. 2. The angle of incidence of the light source (deflexion angle θ° : the light source deviates θ° from the visual line) to the eyeball was changed by rotating the eyeball on a rotating table with the light source kept fixed.

Moreover, in order to examine the effect of the size and configuration of the light source on the amount of stray light, measurements were made by changing the size of the light source as mentioned hereunder; (1) the angular size of the light source is altered by changing the distance from the eyeball to the light source, and then, (2) as the light source is on the visual line (deflexion angle $\theta=0^\circ$), another measurement is made by placing a black or white circular reflecting plate (visual angle $\alpha=2\sim40^\circ$) in front of the light adjusting room, the inner surface of which was finished in white (nearly perfectly diffusing) or in black (nearly perfectly absorbing).

3. Results of measurement

3.1 Effect of lapse of time after excising the eyeballs on the illuminance in fovea centralis

Fig. 3 shows the relation between the deflexion angle and the illuminance in fovea centralis due to lapse of time after excising of the eyeballs. The illuminance in fovea centralis measured one hour

Fig. 3 Relationships between deviation angle of light source (deflexion angle) and ratio of illuminance in fovea centralis due to fluctuation as a function of time after excising eyeballs.

after the excising of eyeballs (E_{01}) with the light source at the visual point ($\theta=0^\circ$) was taken as standard in this experiment.

Taking E_{01} as 100%, the variation of illuminance ratio during five hours was approximately equal to zero, but decreased by 4% after ten hours and by 20% after twenty-seven hours. The effect of stray light in the eyeball on the fovea centralis ($\theta=10^\circ$) was investigated for each lapse of time.

The foregoing results indicate that up to the first 10 hours after the excise of the eyeballs, the effect of the lapse of time is not very large and negligible up to 5 hours time.

3.2 Relation between deflexion angle and illuminance in the fovea centralis retina

Based on the results obtained in section 3.1, the relation between deflexion angle and illuminance in fovea centralis measured within eight hours after the excise of eyeballs (the average lapse of time was 4.0 hours) is shown in Fig. 4. In the 1° de-

Fig. 4 Relationship between deflexion angle (θ°) and ratio of illuminance in fovea centralis.

flexion angle range, direct light from the light source was also included in the light receiving element, while above 1° range only stray light in the eyeball was measured. The intensity of stray light in the eyeball below 10° deflexion angle is different from that of above 10° . When the light source ap-

proaches the visual line, the intensities of stray light increased. Since the deviations of the measured values are relatively small, differences among individual eyeballs used are considered to be small.

3.3 Relation between pupil diameter and illuminance in fovea centralis

Depending on the strength of luminance persons may adjust the amount of incident light on their eyes by dilating or contracting their pupils. However, the pupil of the excised eyeball remained dilated. The measurement was therefore made by placing a diaphragm having an aperture as artificial iris just before the eyeball to simulate the performance of the human eye. The results of the intensities of stray lights in the eyeball measured by adjusting the diaphragm are shown in Fig. 5.

Fig. 5 Relationship between deflexion angle (θ°) and ratio of illuminance in fovea centralis due to iris aperture (fixed conditon) fluctuation.

Moreover, the results of the actual measurements of the illuminance at the fovea centralis at the deflecting angle of the light source of $\theta=0^\circ$ when an artificial iris is placed just in front of the eyeball are listed in Table 1.

Table 1 Changes of the Illuminance at the Fovea Centralis due to the Diameter of the Iris (the deflecting angle of the light source, $\theta=0^\circ$)

Diameter of the iris (mm ϕ)	Measured values of the illuminance at the fovea centralis (lx)										
	1	2	3	4	5	6	7	8	9	10	Average
2	240	260	240	260	250	245	270	240	250	240	250
4	910	950	910	970	940	900	1,000	880	920	920	930
8	2,660	2,800	2,690	2,880	2,790	2,650	2,950	2,620	2,800	2,690	2,750
Without iris*	4,000	4,200	4,000	4,300	4,150	3,950	4,400	3,900	4,100	4,000	4,100

(*: Equivalent to 16 mm ϕ)

When the aperture was decreased, the stray light in the eyeball was found to be gradually confined to the periphery of the light source.

3.4 Relation between size of light source and illuminance in the fovea centralis

The effects of the size of a light source on the intensity of the stray light in the eyeball was examined by measuring the illuminance in the fovea centralis at each deflexion angle (θ) with a light source having visual angles (α) 0.5° , 1° , 2° , 4° and 8° . At $\theta=0^\circ$, measurement was made by using a reflex type circular light source having the light source visual angles (α) 2° , 4° , 6° , 8° , 10° , 20° and 40° . The results are shown in Fig. 6 and Table 2.

The effect of the size of a light source on the ratio of the illuminance in fovea centralis was not as large as that produced by varying the pupil's diameter. However, the effect of the forms of the light source on the amount of the stray light is large as shown in Table 2.

Fig. 6 Relationship between deflexion angle (θ°) and ratio of illuminance in fovea centralis due to size of light source.

Table 2 Result of measurement using reflex type circular light source target placed normal to the visual line.

		A	B	C	D	E
		(lx)	A/1,650	1.00—B	C×110,000	(lx)
I. Light adjusting room (white)	no target condition	1,650	1.00			
	1° black target	1,590	0.964	0.036	3,960	4,100
	2° black target	1,430	0.867	0.133	14,630	15,000
	4° black target	910	0.551	0.449	49,390	50,000
	6° black target	460	0.279	0.721	79,310	80,000
	8° black target	310	0.188	0.812	89,320	90,000
	10° black target	260	0.158	0.842	92,620	(93,000)
	20° black target	220	0.133	0.867	95,370	(96,000)
	40° black target	190	0.115	0.885	97,350	(98,000)
II. Light adjusting room (black)	no target condition	0	0.	—	0	—
	2° white target	230	0.139	—	15,290	15,000
	4° white target	760	0.461	—	50,710	50,000
	10° white target	1,400	0.848	—	93,280	(93,000)
	20° white target	1,440	0.873	—	96,030	(96,000)
	40° white target	1,470	0.891	—	98,010	(98,000)
	hemisphere in white target	1,650	1,000	—	110,000	(110,000)

A: Average of the measured illuminance in the fovea centralis in the light adjusting room

B: The ratio between the illuminance in the fovea centralis for various visual angles and illuminance due to the entire hemisphere in white light.

(A/1,650), where 1,650 is the measured illuminance in the fovea centralis due to the hemispherical white lighted light-adjusting room.

C: (1.00—B)

Estimated ratio of the illuminance in the fovea centralis due to the value measured under showing the black circular targets at hemisphere white light.

D: Calculated illuminance in the fovea centralis.

(C×110,000 lx) for condition I, (B×110,000 lx) for condition II. Where 110,000 is the estimated illuminance in the fovea centralis for a hemispherical slide projector light source, due to the value measured with 1° angular size slide projector light source.

E: The measured illuminance in fovea centralis due to the slide projector light source.

4. Discussion

4.1 Effect of lapse of time after the excising of eyeballs

Boynton et al. investigated the effect of lapse of time after excising of eyeballs on stray light in the eyeball by using a cat's eye, and stated¹³⁾ that the illuminance in fovea centralis rapidly increased with time lapse. In particular, a remarkable increase was observed at the deflexion angle of 7~8°.

The results obtained in the present study differed from those reported by Boynton et al. The difference may be attributed to the fact that the eyeballs used in the present study were well preserved in a physiological salt solution at 0°C. Thus no remarkable effects were observed on the measurement within 10 hours after the excising of the eyeballs. The eyeball used by Boynton however, might have effects of injury since he used a single eyeball for

the measurement of the time lapse. Thus, the results obtained here would be more reliable than those reported in the literature.

4.2 Effect of deviation of light source from the visual line on illuminance in fovea centralis retina

Fig. 4 shows a considerable difference in the manner of emergence of the stray light in the eyeball, when the deflexion angle was greater than the borderline of 10°.

The distortion of the peripheral part of the optical system of the eyeball, may be considered to be the cause of occurrence of the stray light which is prominent when the light source is near the visual line and decreases rapidly when the light source is removed from the visual line. This may be the result of cutting lights passing the peripheral part of the eyeball and thus bringing about marked decrease of the stray light in the eyeball. The results obtained when a diaphragm is placed just before the eyeball as shown in Fig. 5, where the amounts of the stray light decreased at a definite rate with increase of deflexion angles, the light source having been removed from the visual line.

The cause of the emergence of such stray light is considered to be the results of scattering of light both over the surface and inside of the eyeball (crystalline lens, vitreous body, surface of retina, etc.). However, the effect of the scattering of light over the surface of the retina is considered to be negligible, because no change in the illuminance in the fovea centralis was observed even when the retina on the lateral side of the eyeball was perforated and the light falling on that part was allowed to escape. Moreover, in view of the fact that kerato-leukoma was the most conspicuous change noted macroscopically when the measurement was made on the effects of time lapse, the main cause of the stray light which becomes conspicuous when the deflexion angle is large, is considered to be the result of scattering of light over the surface of cornea.

4.3 Effect of pupil's diameter on the illuminance in fovea centralis

Fig. 5 shows a marked decrease of its stray lights when the iris aperture is decreased and also a somewhat parallel movement toward the direction of small deflexion angles. This is considered to be caused by the fact that the light passing through the periphery of the cornea which is much distorted is shut out when the iris contracts, as explained in section 4.2.

In the actual visual environment, the pupil is dilated when an object is not very bright but contracts when the object is bright or a light source approaches the visual line to increase the illuminance in fovea centralis.

Seeing that the diameter of the pupil of the eyeball is determined by the illuminance at the fovea centralis¹⁸⁾, the relationship between the deflexion

Table 4 Some of the results of calculation at 0° deflexion angle.

A	B	C × D	E	F	G	H				
0 ~ 0.5°	0	1 × 1.00	1.00	4,100 lx	4,100 lx	1°				
0.5 ~ 1.0°	0.79°	3 × 0.88	3.64	14,924	15,000	2°				
1.0 ~ 2°	1.58°	12 × 0.70	12.04	49,364	50,000		4°			
2 ~ 3°	2.55°	20 × 0.36	19.24	78,884	80,000			6°		
3 ~ 4°	3.5°	28 × 0.088	21.704	88,986	90,000				8°	
4 ~ 5°	4.5°	36 × 0.022	22.496	92,234	(93,000)					10°
5 ~ 6°	5.5°	44 × 0.0075								
6 ~ 7°	6.5°	52 × 0.0029								
7 ~ 8°	7.5°	60 × 0.0013								
8 ~ 9°	8.5°	68 × 0.0008								
9 ~ 10°	9.5°	76 × 0.0006	23.155	94,936	(96,000)					20°
10 ~ 11°	10.5°	84 × 0.00046								
11 ~ 12°	11.5°	92 × 0.00038								
12 ~ 13°	12.5°	100 × 0.00033								
13 ~ 14°	13.5°	108 × 0.00029								
14 ~ 15°	14.5°	116 × 0.00026								
15 ~ 16°	15.5°	124 × 0.00023								
16 ~ 17°	16.5°	132 × 0.00020								
17 ~ 18°	17.5°	140 × 0.00018								
18 ~ 19°	18.5°	148 × 0.000165								
19 ~ 20°	19.5°	156 × 0.000150	23.450	96,145	(98,000)					40°

- A: Field of ring type light source from visual line.
B: Deviation angle of light source from visual line (θ°).
C: Size of the large source area in reference to the unit area of 1°.
D: Ratio of stray light incident on the fovea with reference to the case when $\alpha=1^\circ$, $\theta=0^\circ$.
E: Accumulated illuminance ratio up to the field of this ring type light source, that is $\Sigma(C \times D)$.
F: Calculated fovea illuminance due to each light source.
[(E × 4,100) lx, where 4,100 is the measured illuminance incident on the fovea centralis for $\alpha=1^\circ$, $\theta=0^\circ$.]
G: Total feasured illuminance incident on the fovea centralis.
(G): Estimated illuminance in fovea centralis.
H: Angular size of circular light source (α°).

of the eyeball and deflexion angles may be expressed as follows:

$$\text{if } \theta > 1^\circ, \text{ then } E_\theta/E_0 = \exp\{(-1n\theta)^3 - 0.25\} + 0.032\theta^{-1.8} \dots\dots\dots(1)$$

$$\text{if } \theta \leq 1^\circ, \text{ then } E_\theta/E_0 = \exp(-0.17\theta) \dots\dots\dots(2)$$

where E_0 : illuminance in fovea centralis (1x)
when the light source is on the visual line ($\theta=0^\circ$).

E_θ : illuminance in fovea centralis (1x)
when the light source deviates θ° from the visual line.

As shown in Fig. 8, the results agree quite well with the measured values.

The ratio of illuminance in fovea centralis ($\theta > 1^\circ$) is equal to that of the quantity of the stray light in the eyeball at each deflexion angle. However, the ratio of illuminance in the fovea centralis ($\theta < 1^\circ$) also includes a component of the direct light from the light source. This component of the direct light is expressed by equation (3) as will be given the next section, and by subtracting the value obtained by calculation according to equation (3) from that obtained by equation (2), the quantity of stray light alone may be determined. The very ratio of the stray light in the eyeball may be obtained also for the range of ($\theta \leq 1^\circ$).

The first term on the right side of equation (1) is considered to express the stray light caused by a distortion of the optical system of eyeballs, and the second term expresses the intensity of the stray light caused by scattered lights over the surface of the eyeballs.

4.6 Studies on the direct light from the light source and the stray light in the eyeballs

When both the visual angle of a light source and the light receiving angle are 1°, the direct light from the light source as well as the stray light in the eyeball are mixed into the illuminance in the fovea centralis at a deflexion angle below 1°. If the luminance of a light source is uniform, and the sensitivity of the light receiving element is uniform all over the light receiving planes, the quantity of light produced by an image in the light source is directly proportional to the ratio of the overlapped areas of the image and the light receiving element. Equation (3) expresses the ratio of the overlapped areas of the source image and the light receiving element by the relation between the ratio and the deflexion angle of the light source.

Fig. 8 Relationship between deflexion angle (θ°) and ratio of illuminance in fovea centralis. (Comparison between measured values and those obtained from experimental equations).

$$S_{L\theta}/S_k = \left\{ r^2 \cdot \sin^{-1} \frac{h}{r} + R^2 \cdot \sin^{-1} \frac{h}{R} - r \cdot h \cdot \sin \left(\cos^{-1} \frac{h}{r} \right) - R \cdot h \cdot \sin \left(\cos^{-1} \frac{h}{R} \right) \right\} \div S_K \dots\dots\dots (3)$$

where, S_K : The area of light receiving element of 1° light receiving angle is taken as standard, the light receiving angle of 1° , will have an area of $\pi/4$.

$S_{L\theta}$: The area of light source image overlapping on that of light receiving element (light source deflexion angle $= \theta^\circ$).

r : The radius of the light receiving element.

R : The radius of the light source, when the diameter of light receiving element of 1° light receiving angle is taken as standard.

$$h: \frac{2}{\theta} \sqrt{S(S-r)(S-R)(S-\theta)} \dots\dots\dots \left(\begin{array}{l} \text{co-ordinate } Y \text{ of} \\ \text{the intersection} \end{array} \right)$$

$$S = \frac{1}{2}(r+R+\theta)$$

The result obtained by subtracting the ratio of area in **Fig. 9** from the ratio of illuminance in the

Fig. 9 Ratio between overlapping area of light source image and area of the light receiving element due to fluctuation of deflexion angle (θ°).

eyeball shown in Fig. 4 and Fig. 6 may therefore be regarded as the ratio of the stray light in the eyeball.

4.7 Comparison between present study results and those in the literature concerning stray light in the eyeball

The results of the experiments made by Boynton, Enoch & Bush¹³⁾, and Fry & Alpern¹²⁾ are shown in Fig. 10. The ratio of illuminance in the fovea centralis (at any glare angle) reported by Boynton is greater than obtained in the present study. This is attributed to the fact that Boynton used a large light source of 4.76° and a light receiving angle of about 4° . Therefore, his experimental apparatus is not suitable for obtaining basic data for the design of illumination since the conditions used are far from those encountered with human eyes, often about 1° in diameter in size of the foveola.

Fry et al.'s experiment is a psychological experiment based on the comparison of both eyes of a man, where the ratios of luminance of the veiling glare approach the results obtained in the present study, i.e. the aperture of F5.6. However, the ratios they obtained are smaller than those of the present study when compared with the results shown in Fig. 7 and when taking into consideration variations of pupil diameter depending on lightness. The differences may be the result of deviation in the

Fig. 10 Relationship between deflexion angle (θ°) and ratio of stray light illuminance incident on the fovea centralis. (Results of other researchers in the literature.)

accommodation of both eyes. It is rather questionable whether the left and right eyes accommodate themselves quite independently from each other.

Regarding the change in the efficiency of penetration of light into the fovea centralis according to the position on the human cornea, the existence of Stiles-Crawford's effect is recognized.

Comparison between the present experimental re-

Fig. 11 Effect of the light passing positions.

sults (See Table 3) and Stiles et al.'s results in terms of the effect of the angles between the visual axis and the lines connecting the fovea centralis to the respective positions on the surface of the cornea and the effect of the light passing such positions on the illuminance on the fovea centralis is as shown in Fig. 11, and approximately equal results were obtained.

Although Stiles et al. explained the cause of Stiles-Crawford's effect to be the probable existence of an angular characteristic in the cones, the author considers that the effect is caused by the scattering of penetrating light due to the strain in the periphery of the eyeball optic system, based on the result of measurement using steer's eyeballs.

5. Conclusion

The results are summarized as follows:

(1) The effect of time lapse after the excising of steer's eyeballs is negligible at least up to 10 hours.

(2) The quantity of stray lights increases as the light source approaches a point on the visual line, especially when the light source is within 10° from the visual point.

(3) The causes of the stray light are: (1) optical defects of eyes when the light source is within 10° of the visual point; (2) diffusion of light on the surface of the cornea at angles greater than 10° .

(4) The smaller the iris aperture, the smaller the quantity of stray lights.

(5) When a large light source is divided into 1° units, the total sum of the stray light from each unit is almost equal to the quantity of the original light source.

(6) In order to obtain the ratio of E_θ/E_0 , the following equations are used:

$$\theta > 1^\circ \quad E_\theta/E_0 = \exp\{(-1n\theta)^3 - 0.25\} + 0.032\theta^{-1.8}$$

$$\theta \leq 1^\circ \quad E_\theta/E_0 = \exp(-0.17\theta)$$

where, E_0 : Illuminance of fovea when the light source is on the visual line ($\theta=0^\circ$).

E_θ : Illuminance of fovea when the light source deviates θ° from the visual line.

Acknowledgements

The author is very grateful to Dr. Katsuzo Ito for suggesting this investigation as well as for his constant guidance throughout the course of the work. I also wish to express my thanks to Dr. Akira Ohshino and Dr. Michio Matsuyama for their advices in the course of the work. Further, I wish to express my thanks to Mr. Tadashi Doi for his assistance in the course of the experiments.

References

- (1) K. Ito, Y. Nakane and K. Iwanaga: Reports of the Annual Meeting 1967 Architectural Institute of Japan, 606~607. (in Japanese)

- (2) K. Ito and Y. Nakane: Reports of the Annual Meeting 1969 Architectural Institute of Japan, 103~104. (in Japanese)
- (3) K. Ito and Y. Nakane: Reports of the Annual Meeting 1971 Architectural Institute of Japan, 57~58. (in Japanese)
- (4) Y. Nakane and K. Ito: Transactions of the Architectural Institute of Japan, **229** (1975), 101~110. (in Japanese)
- (5) Y. Nakane: Transactions of the Architectural Institute of Japan, **229** (1975), 111~120. (in Japanese)
- (6) Y. Nakane and K. Ito: J. of Light & Visual Environment, 2-1, Illum. Engng. Inst. Japan, (1978), 38~44.
- (7) Y. Nakane and T. Doi: Transactions of the Architectural Institute of Japan, **272**, (1978), 87~96. (in Japanese)
- (8) L. L. Holladay: J. Opt. Soc. Am., **12-4**, (1926), 271~319.
- (9) W. S. Stiles: Proc. Roy. Soc., **104B**, (1929), 322~351.
- (10) J. J. Vos and J. Boogaard: J. Opt. Soc. Am., **53-7**, (1963), 869~873.
- (11) R. M. Boynton and F. J. J. Clarke: J. Opt. Soc. Am., **54-1**, (1964), 110~119.
- (12) G. A. Fry and M. Alpern: J. Opt. Soc. Am., **43-3**, (1953), 189~195.
- (13) R. M. Boynton J. M. Enoch and W. R. Bush: J. Opt. Soc. Am., **44**, (1954), 879~886.
- (14) D. W. DeMott and R. M. Boynton: J. Opt. Soc. Am., **48-1**, (1958), 13~22.
- (15) D. W. DeMott and R. M. Boynton: J. Opt. Soc. Am., **48-2**, (1958), 120~128.
- (16) D. W. DeMott: J. Opt. Soc. Am., **49-6**, (1959), 571~579.
- (17) Opt. Tech. Hand Book: Asakura Shoten, (1968), 747~769. (in Japanese)
- (18) Opt. Tech. Hand Book: Asakura Shoten, (1968), 733~734. (in Japanese)

Received 2 Apr. 1979; Revision Received 21 Jun. 1979