Some error sources and their corrections in the measurement of relative spectral sensitivity by means of two—tandem—monochromator method—on effective pass band and slit width correction—

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The measurement of relative spectral sensitivity of detectors by two-tandem-monochromator method consists of two measuring steps; The measurement of spectral transmittance of a monochromator, and the determination of spectral sensitivity by the combination of the monochromator with the known spectral transmittance and the calibrated standard radiance lamp. This report discusses the two main factors which cause systematic errors in the second step. The one is the effective pass band which is an important quantity in order to estimate the flux which passes through the monochromator. An improved method to give the accurate dispersion curve and the effective pass band is described in detail. This method can exclude the errors which may be the amount of about $\pm 5\%$ in measuring the relative spectral sensitivity. The second one is the corrections to the errors caused by the finite slit width. An equation for the correction and the calculated results based on the measurement of the spectral sensitivity of some detectors are given. The errors are estimated more than 1% in the wavelength region longer than 600nm in case of S-11 type and less than 0.3% in almost whole wavelength region involved in case of S-20 type.

1. Introduction

The relative spectral sensitivity distribution of photodetectors is measured mostly on the basis of the black coated detectors such as a thermopile and thermocouple¹⁾. Although the principles of the measurement is simple and clear, the actual procedure in the measurement has a difficulty in examining either spectral or spatial uniformity of the sensitivity of a reference detector, in handling the low sensitivity, and in obtaining good shield condition against the surrounding thermal radiation. In contrast, the two-tandem-monochromator method²⁾ has advantages in the situation that much higher sensitive detector is applicable and it takes shorter time for the measurement. However, this method requires basic and more detailed studies on the spectral measurement. In this report, we deal with

the following subjects for the purpose of the accurate spectral sensitivity distribution measurement. First, the error analysis is made in the estimation of the effective pass band which is one of the necessary quantities to determine the incident monochromatic flux on the tested detector surface which is placed behind the monochromator. Our analysis revealed that the systematic error is estimated by the amount of about 5% in case of adopting the pass band simply derived from dispersion curve. Secondly, the error due to finite band width is analyzed. The correction to the error, the slit width correction, carried out on the measured spectral sensitivity distribution by means of the two-tandem-monochromator method. The amount of the correction changes with the wavelength and as the example, the estimated errors are shown for the spectral sensitivity distribution of S-20 type and S-11 type photodetectors.

2. Concept of the pass band

In the spectral sensitivity distribution measurement of a photodetector, the output signal of the photodetector I_{ST} is expressed by

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$$I_{ST}(\lambda_0) = k \int_0^\infty J_{ST}(\lambda) S(\lambda) \tau(\lambda_0, \lambda) d\lambda, \dots\dots\dots(1)$$

where λ_0 is the wavelength for which the monochromator is set, λ is the wavelength of the radiation passing through the monochromator, $J_{ST}(\lambda)$ is the known spectral radiance distribution of the light source, $\tau(\lambda_0, \lambda)$ is the transmission characteristics of the monochromator, and $S(\lambda)$ is the spectral sensitivity of the photodetector under measurement. k shows a geometrical factor, which is considered to be constant over the wavelength region concerned and will be excluded hereafter. The transmission characteristics $\tau(\lambda_0, \lambda)$ means the fraction of the transmitted radiation of the wavelength λ , when the wavelength scale is set at λ_0 . Since $\tau(\lambda_0, \lambda)$ is almost zero except the adjacent wavelength region to λ_0 , it is expressed approximately by

$$\tau(\lambda_0, \lambda) = T(\lambda_0) t(\lambda_0, \lambda). \dots\dots\dots(2)$$

This approximation implies that the transmission characteristics of the monochromator is determined by the overall spectral transmittance $T(\lambda_0)$ due to the used optical elements, and by the geometrical interruption $t(\lambda_0, \lambda)$ due to the slits in the monochromator. Considering Eq. (2), Eq. (1) is approximated to

$$I_{ST}(\lambda_0) \doteq J_{ST}(\lambda_0) T(\lambda_0) S(\lambda_0) \int_0^\infty t(\lambda_0, \lambda) d\lambda \dots\dots\dots(3)$$

Since the effective pass band shown as the integration in the right hand side of Eq. (3) changes with setting wavelength λ_0 of the monochromator, it is necessary to clarify its dependence on λ_0 . When the monochromator is scanned with constant mechanical slit width, the effective pass band is usually determined by taking the following procedures. First, the mechanical scale used for the wavelength selection of the monochromator is calibrated to the wavelength scale for many line spectra whose wave-

lengths are precisely known. Secondly, the effective pass band is obtained by use of the derivative of the dispersion curve given by the above procedure or by use of the full width at half maximum based on the measurement of $t(\lambda_0, \lambda)$ (to be strict, $t(\lambda, \lambda_0)$ for several line spectra. In the following section the pass band is estimated for the double prism monochromator adopted by the authors to measure the spectral sensitivity distribution of photodetectors.

3. Estimation of the monochromator pass band

3.1 Calibration of the monochromator wavelength scale

The calibration of the wavelength scale of the monochromator plays an important role to estimate the pass band accurately. Fig. 1 shows the schematics of the double prism monochromator which was used in the experiment. The optical configuration of the monochromator is a Czerny-Turner mount in which the wavelength is selected by rotating the prism tables. The wavelength scale of the monochromator was calibrated with more than 100 line spectra from several kinds of low pressure discharge lamp. As shown in Fig. 1, mechanical scale which relates to the wavelength is read as the scale on the drum W, the rotating angle θ_d of the worm which drives the worm wheel attached directly to the prism table. The relation between the wavelength λ_0 and the rotation angle θ_d was approximated by the following polynomial terms as shown in

$$\theta_d = a_0 + \frac{a_1}{\lambda_0} + \frac{a_2}{\lambda_0^2} + \dots, \dots\dots\dots(4)$$

where $a_0, a_1, a_2 \dots$ are the coefficients by which the relation between θ_d and λ_0 is approximated in the sense of the least square errors. Since the

Fig. 1 Schematic of the double prism monochromator used in the experiment; S_1 ; entrance slit, S_2 ; middle slit, S_3 ; exit slit P_1, P_2 ; first and second prism, H_1, H_3 ; collimator mirror, H_2, H_4 ; focussing mirror, A_1, A_2 ; flat mirror, W ; wavelength driving drum.

Fig. 2 Difference between each point of the wavelength scale calibrated with the line spectra and that of the wavelength scale obtained from the approximation equation.

calibrated wavelength range was extended over 250 nm to 800 nm in the monochromator, a set of the coefficients was not enough to give a good approximation over the whole wavelength range. This was overcome by dividing the wavelength range into three regions which are from 250 nm to 400 nm, from 350 nm to 650 nm, and from 600 nm to 850 nm, and by applying the three sets of the coefficients corresponding to these regions. **Fig. 2** shows the deviations in nm scale between the wavelength scale of the monochromator obtained experimentally by use of the line spectra and that calculated from Eq. (4) on the basis of these data. In **Fig. 3** the calibration curve of the wavelength scale, $\theta_d(\lambda_0)$, and its derivative $d\theta_d/d\lambda$ are shown.

Fig. 3 Calibration curve of the drum scale to the wavelength and its derivative with respect to the wavelength.

3.2 Pass band and dispersion of the monochromator

In the prism monochromator whose wavelength is selected by moving the position of the slit, the first derivative of the calibrated mechanical scale with respect to setting wavelength λ_0 determines the pass band of the monochromator. On the contrary, it is not so straightforward to obtain the pass band in the prism monochromator the wavelength of which is selected by turning the dispersion prism. Suppose that corresponding to the selected wavelength λ_0 , readings of the scale of the

drum be θ_d and the rotating angle of the prism be θ_p (which is regarded as the direction of the normal to the incident surface of the prism), and that corresponding to $\lambda_0 + d\lambda_0$, those be $\theta_d + d\theta_d$ and $\theta_p + d\theta_p$, respectively. At the selected wavelength of λ_0 , the incident radiation with the wavelength $\lambda = \lambda_0$ and $\lambda = \lambda_0 + d\lambda$ come out from the second prism with angles of i and $i + di$ respectively. Accordingly, the pass band $\Delta\lambda$ follows

$$\Delta\lambda = \frac{C}{\frac{di(\lambda)}{d\lambda}}, \quad \dots\dots\dots(5)$$

where C is the constant which is independent of $\lambda_0(=\lambda)$, and determined by the focal length of the employed focussing elements and the slit widths. In the rotatable mount type of the prism monochromator, the dispersed angles for the radiation of the wavelengths $\lambda = \lambda_0$ and $\lambda = \lambda_0 + d\lambda$ at the selected wavelength scale λ_0 of the monochromator is the same amount to that at the selected wavelength $\lambda_0 + d\lambda_0$ of the monochromator in which the prism is rotated by the angle of $d\theta_p$. In contrast, di differs from $d\theta_p$. The relation between them is given by

$$di = 2 \left(1 + \frac{\tan i \tan(2\beta - \gamma)}{\tan \gamma \tan \omega} \right) \cdot d\theta_p = \kappa(\lambda) d\theta_p, \quad \dots\dots\dots(6)^*$$

where the angles are labeled in Fig. A-1 in the appendix, and the coefficient $\kappa(\lambda)$ is used to denote the dependence on λ . From Eqs. (5) and (6), the effective pass band $\Delta\lambda_{\text{eff}}$ is expressed by

$$\Delta\lambda_{\text{eff}} = \frac{C}{\kappa(\lambda)} \frac{1}{d\theta_p(\lambda)/d\lambda}. \quad \dots\dots\dots(7)$$

When the widths of the entrance, the middle and the exit slit denoted by S_1, S_2 , and S_3 are kept equal, and the entrance slit of the monochromator is illuminated by the radiation of line spectrum, Eq. (8) is given of the necessary rotating angle $\Delta\theta_p$ for the prism in which the image of the entrance slit is shifted completely from the position of the exit slit.

$$\Delta\theta_p = \frac{S_1}{f} \frac{1}{\kappa(\lambda)}, \quad \dots\dots\dots(8)$$

where f is the focal length of the collimator in the monochromator. **Fig. 4** shows the experimental con-

Fig. 4 Block diagram of the experimental configuration for the measurement of $2\Delta\theta_d$.

* see appendix

Fig. 5 A measured example of the change of the photocurrent with respect to θ_d .

figuration employed to measure $\Delta\theta_p$. A grating monochromator is used to select a monochromatic line in well isolated state to the entrance slit of the prism monochromator whose dispersion is small especially in the long wavelength region. Fig. 5 shows a measured example of the change in the photocurrent obtained by rotating the prism table. The horizontal axis is scaled as the number of pulses required to drive a step motor for the rotation of the prism, and the lines marked with u_1 and u_2 are obtained by the estimation of the least square method. If the linear relation holds between θ_d and θ_p , and the image of the entrance slit is not distorted, the full width $2\Delta\theta_d$ as shown in Fig. 5 is expected to vary according to $1/\kappa(\lambda)$ with respect to the wavelength. Consequently, the measured quantity $\Delta\theta_d$ is seen to give the correction factor in the estimation of the pass band by use of the derivative of the wavelength calibration curve with respect to θ_p . In case that the worm wheel attached to the prism table does not rotate linearly with the worm attached to the drum scale³⁾, the relation between $d\theta_p$ and di associated with $d\lambda$ is rewritten by the relation between $d\theta_d$ and di associated with $d\lambda$ as shown in

$$di = \kappa(\lambda) \frac{d\theta_p}{d\theta_d} \cdot d\theta_d$$

$$d\lambda = \frac{C}{\kappa(\lambda)} \cdot \frac{d\theta_d(\lambda)}{d\theta_p(\lambda)} \cdot \frac{1}{d\theta_d/d\lambda} \quad \dots\dots\dots(9)$$

The measured results of $\Delta\theta_d$ are plotted in Fig. 6(a) for the various line spectra. In order to avoid the influence of the slit image distortion in these measurements, the slit height was limited to 3 mm, and an aperture of the diameter of 20 mm was placed in front of the collimator mirror. The ordinate scale of $2\Delta\theta_d$ corresponds to the necessary driving pulses to rotate the image of the slit by the angle of $(S_1 + S_3)/f = 2S/f$. Therefore, the obtained $\Delta\theta_d$ is shown in

Fig. 6 (a) Measured values of $2\Delta\theta_d$ for various line spectra and their fitting curve.

Fig. 6 (b) The apparent pass band obtained from the derivative of the wavelength calibration curve and the effective pass band after correction.

$$\Delta\theta_d \propto \frac{S_1}{f} \cdot \frac{1}{\kappa(\lambda)} \cdot \frac{d\theta_d}{d\theta_p} \quad \dots\dots\dots(10)$$

The dependence of the effective pass band on these change of $\Delta\theta_d$ is estimated by considering $G(\lambda)$ shown in Eq. (11), where normalization was made by its average $\overline{\Delta\theta_d}(\lambda)$ over the measured values of $\Delta\theta_d(\lambda)$.

$$G(\lambda) = \frac{\Delta\theta_d(\lambda)}{\overline{\Delta\theta_d}(\lambda)} \quad \dots\dots\dots(11)$$

In Fig. 6(a), the solid curve denotes the fitting curve of $2\Delta\theta_d(\lambda)$ by least squares method and the ratio of this curve to $2\overline{\Delta\theta_d}(\lambda)$ shown by a dot-dash-line is $G(\lambda)$.

By use of $G(\lambda)$, the effective pass band is given in Eq. (12) with the derivative of the wavelength scale calibration curve.

$$\Delta\lambda_{\text{eff}} \propto G(\lambda) \cdot \frac{1}{d\theta_d/d\lambda} \quad \dots\dots\dots(12)$$

Fig. 6(b) shows the apparent pass band λ_d obtained on the basis of only the derivative of $d\theta_d/d\lambda$ in case of the slit widths of 0.3 mm and the effective pass band $\Delta\lambda_{\text{eff}}$ obtained on the basis of the above considerations. It is found that there is maximum systematic difference of about $\pm 5\%$ between the apparent pass band and the effective pass

band. The residual uncertainty was estimated to be about $\pm 1\%$ by considering standard deviation between the experimental points and the fitting curve shown in Fig. 6(a).

4. Slit width correction

4.1 Equation for the slit width correction

In order to obtain the purely monochromatic radiation the slit width must be infinitely small. In the actual measurement, however, the use of the finite slit width yields a finite band width of radiation. We estimate the amount of the correction to the errors caused by this effect. As shown in Eq. (1) the measured photocurrent is affected by the radiation of not only setting wavelength λ_0 but also the wavelength λ around λ_0 . Since the dependence of $\kappa(\lambda)$ on λ can be neglected within the pass band centered at λ_0 , the slit function $t(\lambda_0, \lambda)$ is expressed as a function of the rotation angle of the prism θ_p .

$$t(\lambda_0, \lambda) = u(\theta_{p0}, \theta_p). \quad (13)$$

When $\Delta\theta_p$ is denoted as the rotating angle at which the image of the entrance slit is just shifted off from the position of the exit slit, the slit function $u(\theta_p, \theta_p)$ with slit widths $S_1 = S_2 = S_3$ will be triangular as depicted in Fig. 7. When the relation between λ_0 and θ_{p0} is expressed by $\lambda_0 = H(\theta_{p0})$, the wavelength λ around λ_0 is written by $\lambda = H(\theta_{p0} + \theta_p)$. Integration by the variable of the wavelength in Eq. (1) is reduced to that by the variable θ_p . When $i_{ST}(\theta_{p0})$, $j_{ST}(\theta_{p0})$, $\mathcal{T}(\theta_{p0})$, and $\mathcal{S}(\theta_{p0})$ are used corresponding to the functions $I_{ST}(\lambda_0)$, $J_{ST}(\lambda_0)$, $T(\lambda_0)$ and $S(\lambda_0)$, the integration of the Eq. (1) is carried out approximately in case of the slit function shown in Fig. 7, and the results is given by

$$\begin{aligned} I_{ST}(\lambda_0) &\equiv i_{ST}(\theta_{p0}) \\ &\cong F \cdot \left(\Delta\theta H' + \frac{\Delta\theta_p^3}{12} H''' \right) \\ &\quad + F' \cdot \left(\frac{\Delta\theta_p^3}{6} H'' \right) \\ &\quad + F'' \cdot \left(\frac{\Delta\theta_p^3}{12} H' + \frac{\Delta\theta_p^5}{60} H''' \right). \quad (14) \end{aligned}$$

Fig. 7 An illustration of triangular shape slit function.

In this equation $F(\theta_{p0})$ denotes the function,

$$\begin{aligned} F(\theta_{p0}) &= J_{ST}\{H(\theta_{p0})\} T\{H(\theta_{p0})\} S\{H(\theta_{p0})\} \\ &= j_{ST}(\theta_{p0}) \mathcal{T}(\theta_{p0}) \mathcal{S}(\theta_{p0}), \quad (15) \end{aligned}$$

and superscript ($'$, $''$, $'''$) denote the first, the second and the third derivatives at θ_{p0} , i.e. $(dH/d\theta_p)\theta_{p0}$, $(d^2H/d\theta_p^2)\theta_{p0}$, $(d^3H/d\theta_p^3)\theta_{p0}$. According to the analysis by Nonaka⁴⁾, F is transformed to Eq. (16),

$$\begin{aligned} F &= \frac{1}{\Delta\theta_p H' + \frac{\Delta\theta_p^3}{12} H'''} \left\{ i_{ST} - F' \cdot \left(\frac{\Delta\theta_p^3}{6} H' \right) \right. \\ &\quad \left. - F'' \cdot \left(\frac{\Delta\theta_p^3}{6} H'' \right) - F''' \cdot \left(\frac{\Delta\theta_p^3}{12} H' \right) \right. \\ &\quad \left. + \frac{\Delta\theta_p^5}{6} H''' \right\}, \quad (16) \end{aligned}$$

by using Eqs. (14) and (15).

When only the first order term of $\Delta\theta_p$ is considered in Eq. (16), F' and F'' are given approximately as Eqs. (17) and (18), respectively.

$$F' \cong \frac{1}{\Delta\theta_p} \left(\frac{i_{ST}' H' - i_{ST} H''}{H'^2} \right), \quad (17)$$

$$\begin{aligned} F'' &\cong \frac{1}{\Delta\theta_p H'} \left[1 - \frac{i_{ST}}{i_{ST}'} \left\{ \frac{H'''}{H'} - 2 \left(\frac{H''}{H'} \right)^2 \right\} \right. \\ &\quad \left. - 2 \frac{i_{ST}'}{i_{ST}''} \frac{H''}{H'} \right]. \quad (18) \end{aligned}$$

By substituting that F' and F'' into Eq. (14) spectral sensitivity $S(\lambda_0)$ or $\mathcal{S}(\theta_{p0})$ is obtained as shown in Eq. (19) in which the slit width correction is taken into account.

$$\begin{aligned} S(\lambda_0) &\equiv \mathcal{S}(\theta_{p0}) \\ &= \frac{i_{ST}(\theta_{p0})}{j_{ST}(\theta_{p0}) \mathcal{T}(\theta_{p0}) \Delta\theta_p H'(\theta_{p0})} \left\{ 1 \right. \\ &\quad \left. - \frac{i_{ST}''(\theta_{p0})}{i_{ST}'(\theta_{p0})} \left(\frac{\Delta\theta_p^2}{12 + \Delta\theta_p^2 \frac{H'''(\theta_{p0})}{H'(\theta_{p0})}} \right) \right\}. \quad (19) \end{aligned}$$

In this equation the second term in the bracket shows the required correction. The second derivative of the photocurrent with respect to θ_{p0} in Eq. (19) is preferable to be transformed by the second derivative of the photocurrent with respect to the setting wavelength λ_0 according to the consideration on the relation between θ_a and θ_p as described in Sec. 3.2. On the other hand, the derivatives of $H(\theta_{p0})$ are the instrumental functions given directly by the measured quantities which are the derivative of the wavelength calibration curve $d\theta_a/d\lambda$ and the correction factor of $\Delta\theta_a/\Delta\theta_d$. Thus, the spectral sensitivity distribution of $\mathcal{S}(\lambda_0) \equiv \mathcal{S}(\theta_{p0})$ is expressed by

$$\begin{aligned} S(\lambda_0) &= \frac{I_{ST}(\lambda_0)}{J_{ST}(\lambda_0) T(\lambda_0) \Delta\theta_p H'(\theta_{p0})} \left\{ 1 \right. \\ &\quad - \frac{I_{ST}''(\lambda_0)}{I_{ST}'(\lambda_0)} \frac{(\Delta\theta_p \cdot H'(\theta_{p0}))^2}{12 + \Delta\theta_p^2 \frac{H'''(\theta_{p0})}{H'(\theta_{p0})}} \left(1 \right. \\ &\quad \left. \left. + \frac{I_{ST}'(\lambda_0)}{I_{ST}''(\lambda_0)} \frac{H''(\theta_{p0})}{H'(\theta_{p0})^2} \right) \right\}, \quad (20) \end{aligned}$$

where all the quantities are obtainable through the measurement.

4.2 Experimental results

amount of corrections were evaluated on the basis of the experimental results of photodetectors with two different types of spectral sensitivity distribution as examples. $I_{ST}'(\lambda_0)$ and $I_{ST}''(\lambda_0)$ were calculated on the basis of the measured photocurrent according to the numerical computation formula for the first and the second differentiation by use

Fig. 8 (a)

Fig. 8 (b)

Examples of the photocurrents, the first and the second derivatives required for the slit width correction;
(a) for S-20 type and (b) for S-11 type detectors. the ordinate scales of the first and the second deriuatives are $\mu\text{A}/\text{nm}$ and $\mu\text{A}/\text{nm}^2$ respectively.

Fig. 9 The first, the second and the third derivatives of $H(\theta_{p0})$.

of the Lagrange interpolation method⁵⁾. Fig. 8(a) and (b) are the examples of I_{ST} , I_{ST}' and I_{ST}'' for the two different detectors. $H'(\theta_{p0})$, and $H''(\theta_{p0})$ $H(\theta_{p0})$ are the instrumental functions given direct tives of the wavelength calibration curve and the correction factor of the rotation angle of the prism. Fig. 9 shows the calculated results of the derivatives of $H(\theta_{p0})$. When $S_0(\lambda_0)$, the sensitivity of no slit width correction, is denoted as Eq. (21),

$$S_0(\lambda) = \frac{I_{ST}(\lambda)}{J_{ST}(\lambda)T(\lambda)\Delta\theta_p H'(\theta_p)}, \quad \dots\dots\dots(21)$$

the errors due to the employment of the finite slit width is estimated by

$$E(\lambda) = \frac{S(\lambda) - S_0(\lambda)}{S(\lambda)} \times 100 (\%). \quad \dots\dots\dots(22)$$

The computed results of $E(\lambda)$ are shown in Fig. 10 for the detectors of the spectral sensitivity distribution type of S-20 and S-11 respectively. While the S-20 type showed only the errors less than 0.3% over the wavelength region between 260 nm and 780 nm, the S-11 type showed 1% to 4% in the wavelength region longer than 600 nm. The considerable amount of errors may be originated especially in the wavelength region of the small dispersion in the prism monochromator.

5. Summary

It was shown that the pass band of the double prism monochromator might be estimated with the errors of about $\pm 5\%$ if it was obtained simply from the derivatives of the wavelength calibration curve of the monochromator. This may cause the serious error in the measurement of the relative spectral sensitivity distribution by two-tandem-monochromator method. The investigation was also made on the influence of the employment of finite slit widths in the monochromator on the measurement of relative spectral sensitivity distribution. Equation for the correction was given. The errors were estimated on the basis of the measured data for

Fig. 10 Examples of the estimated errors due to the finite slit width (a) for S-20 type and (b) for S-11 type photodetectors, respectively.

two kinds of detectors whose spectral sensitivity distribution were S-20 and S-11.

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Appendix

Relation between d_i and $d\theta_p$

The angles in the following description are labeled in Fig. A-1. From the Snell's law a ray of wavelength

Fig. A-1 Pass of ray in the prism

λ passes through the first prism according to the relations given by

$$\frac{\sin i}{\sin r} = n, \text{ and } \frac{\sin \omega}{\sin (2\beta - r)} = n. \quad \text{.....(A1)}$$

A ray of wavelength $\lambda + d\lambda$ undergoes the slight change of the angles dr and $d\omega$ as shown by

$$dr = -\frac{dn}{n} \tan r, \text{ and}$$

$$d\omega = \frac{dn}{n} \tan \omega \left(1 + \frac{\tan r}{\tan (2\beta - r)} \right), \quad \text{.....(A2)}$$

where n and dn are the refractive index of the prism for the wavelength λ and the change of index due to the slight change of the wavelength $d\lambda$, respectively. Since the additive dispersion type monochromator gives the incident angle of $\omega + d\omega$, and exiting angle $i + di$ for the ray of the wavelength of $\lambda + d\lambda$ at the second prism, it follows that

$$\frac{\sin (\omega + d\omega)}{\sin \{2\beta - (r + dr)\}} = n + dn, \text{ and}$$

$$\frac{\sin (i + di)}{\sin (r + dr)} = n + dn. \quad \text{.....(A3)}$$

When only the first term are considered in the expansion of Eq. (A3) the equation is reduced approximately as follows:

$$\frac{dr}{\tan (2\beta - r)} - \frac{d\omega}{\tan \omega} = \frac{dn}{n}, \text{ and} \quad \text{.....(A4)}$$

$$-\frac{dr}{\tan r} + \frac{di}{\tan i} = \frac{dn}{n}. \quad \text{.....(A5)}$$

The elimination of dr from the relation (A4) and (A5) and the further elimination of $d\omega$ by use of (A2) yield

$$di = \frac{dn}{n} \cdot 2 \tan i \left(1 + \frac{\tan (2\beta - r)}{\tan r} \right). \quad \text{.....(A6)}$$

When the prism is rotated by the angle $d\theta_p$, the ray of the wavelength $\lambda + d\lambda$ follows the path given by the relations,

$$\frac{\sin (i + d\theta_p)}{\sin (r + dr')} = n + dn, \text{ and}$$

$$\frac{\sin (\omega + d\theta_p)}{\sin \{2\beta - (r + dr')\}} = n + dn. \quad \text{.....(A7)}$$

Similar expansions of Eq. (A7) as stated above give the relation of

$$d\theta_p = \frac{\tan r + \tan (2\beta - r)}{\frac{\tan r}{\tan i} + \frac{\sin (2\beta - r)}{\tan \omega}}. \quad \text{.....(A8)}$$

Accordingly, the relation between di and $d\theta_p$ is obtained from Eqs. (A6) and (A8) as

$$di = 2 \left\{ 1 + \frac{\tan i}{\tan r} \frac{\tan (2\beta - r)}{\tan \omega} \right\} d\theta_p. \quad \text{.....(A9)}$$

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