Technical Review**Integrating Sphere Theory for Measuring Optical Radiation**

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ABSTRACT

The integrating sphere is a useful apparatus for measuring optical radiation. To use or design an integrating sphere for a particular application, it is important to understand how it works in theory. The theory of the integrating sphere is based on the theory of radiation interchange within an enclosure comprised of diffuse reflecting surfaces. In this paper, the theory and error aspects of the integrating sphere are discussed.

KEYWORDS: integrating sphere, total luminous flux, reflectance, baffle, radiance, luminous intensity

1. Introduction

The integrating sphere has been in use for a long period of time in the field of photometry and colorimetry, as a tool for evaluating the total luminous flux of a light source or the diffuse reflection/transmission characteristics of materials. Historically, it is reported that a spherical integrating photometer with a diameter of 50 cm was first produced around 1900 by Ulbricht. In Japan, the study of total luminous flux measurement using an integrating sphere with a diameter of 2 m was introduced in the Electro-technical Laboratory in 1915¹⁾.

Thereafter, in accordance with the development of discharge lamps, the integrating sphere has been spread as a means for evaluating the total luminous flux and color of a light source.

As novel light sources such as LED and EL developed recently have become widespread, the technique of using an integrating sphere to measure total luminous flux and light color is again assuming significance.

The total luminous flux of a light source is significant in determining not only the output (lm), but also the luminous efficiency (lm/W) of the light source. Another method to measure the total luminous flux of a light source is with the goniophotometer²⁾ that is normally used to measure luminous intensity distribution, from which total luminous flux can be determined. This method, however, requires a rather large apparatus and a dark room for housing the apparatus.

Furthermore, since these measurements are performed successively by divided up the whole space, the time taken is long, and it requires a light

source that can emit light stably throughout the long measurement process.

On the other hand, the integrating sphere system has the advantage of fast measurement and does not require a dark room. The inside wall of the hollow sphere is coated with a substance such as a barium sulfate powder with the property to diffuse white light and a light emitting source is placed at the center of the sphere. The integrating sphere system is therefore suited for measuring total luminous flux. Since the illuminance on the inside wall of the integrating sphere is in proportion to the total luminous flux, the total luminous flux can be conveniently determined by merely measuring the illuminance at one position on the inside wall. Another advantage is that there is no need to prepare a dark room because the integrating sphere itself forms a closed space. However, the shadow formed within the integrating sphere by an internal structure such as a device for turning on the light source can be a source of error in the measurement. In this paper, the theory of an integrating sphere used for measuring the total luminous flux of a light source is described using a simple geometric optical model so as to discuss reduction in the errors that may arise.

In addition, the theory of the integrating hemisphere is described as a new technology for measuring the total luminous flux of a light source.

2. Theory of the integrating sphere**2.1 Measurement of light sources with spatial distribution of luminous intensity in 4π space**

The theory of measurement, using an integrating sphere, of the total luminous flux of a point light source

having a spatial distribution of luminous intensity in the whole 4π space, will be explained using a plane model as shown in Figure 1. It is assumed that a light source is placed at the center of an integrating sphere with a radius r so as to irradiate an infinitesimal segment A, positioned on the inside wall of the integrating sphere at an angle α from the light source, with luminous intensity $I_0(\alpha)$. The illuminance E_{a0} on the segment A is expressed by the following formula:

$$E_{a0} = I_0(\alpha)/r^2 \quad \dots (1)$$

Assuming that the diffuse reflectance of the inside wall of the integrating sphere is ρ (luminous reflectance), and that the segment A has an area dS , the flux φ_a reflected by the segment A is expressed by the following formula:

$$\varphi_a = \rho \cdot E_{a0} \cdot dS \quad \dots (2)$$

Consider another infinitesimal segment B positioned on the inside wall of the integrating sphere in a direction at an angle θ to the normal of the segment A. Assuming that the segment A is an isotropic reflecting diffuser, the luminous intensity $I_a(\theta)$ from the surface A toward the surface B is expressed by the following formula:

$$I_a(\theta) = (\varphi_a \cdot \cos \theta)/\pi \quad \dots (3)$$

Figure 1 Optical model in which a light source emits light at the center of an integrating sphere

The incident angle of $I_a(\theta)$ against the segment B is θ , and the distance between the segment A and the segment B is $2r \cdot \cos \theta$. Therefore, the illuminance E_{ab} on the segment B derived from the luminous intensity $I_a(\theta)$ is expressed by the following formulas:

$$\begin{aligned} E_{ab} &= I_a(\theta) \cdot \cos \theta / (2r \cdot \cos \theta)^2 \\ &= \varphi_a / (4\pi \cdot r^2) \\ &= \rho \cdot I_0(\alpha) \cdot dS / (4\pi \cdot r^4) \quad \dots (4) \end{aligned}$$

As is obvious from formula (4), reflected light from the segment A irradiates all the positions on the inside wall of the integrating sphere with uniform illuminance regardless of the outgoing angle θ from the segment A. Since the internal surface area of the integrating sphere is $4\pi \cdot r^2$, and the solid angle from the center of the sphere, where the light source is placed, corresponding to the whole space is 4π the area dS of the infinitesimal segment can be expressed by using an infinitesimal solid angle $d\Omega$, as follows:

$$\begin{aligned} dS &= (4\pi \cdot r^2 / 4\pi) \cdot d\Omega \\ &= r^2 \cdot d\Omega \quad \dots (5) \end{aligned}$$

Accordingly, the formula (4) can be converted to the following formula (6):

$$E_{ab} = \rho \cdot I_0(\alpha) \cdot d\Omega / (4\pi \cdot r^2) \quad \dots (6)$$

Since the total luminous flux Φ of the light source is obtained as the integral of $I_0(\alpha) \cdot d\Omega$ of the formula (6) with respect to the whole space, the illuminance E_{b1} on the segment B, derived from primary reflected light of the flux emitted from the light source and reflected by the whole inside wall of the integrating sphere is expressed by the following formula (7):

$$E_{b1} = \rho \cdot \Phi / (4\pi \cdot r^2) \quad \dots (7)$$

As described above, the illuminance on the inside wall derived from the primary reflected light of the flux emitted from the light source and reflected by the whole inside wall of the integrating sphere is constant regardless of position. Hence, the illuminance E_{a1} on the segment A derived from the primary reflected light is equal to the illuminance E_{b1} on the segment B.

The illuminance E_{b1} on the segment B causes a secondary reflection with reflectance ρ is further from the segment B.

Assuming that the segment B has an area dS' , the flux φ_{b2} reflected by it is expressed by the following formula:

$$\varphi_{b2} = \rho \cdot E_{b1} \cdot dS' \quad \dots (8)$$

Next, irradiation of another infinitesimal segment C located on the inside wall of the same integrating sphere, with the secondary reflected light reflected by the segment B, will be examined. Assuming that the segment B is an isotropic reflecting diffuser, flux φ_{b2} reflected by the segment B is expressed by using reflected luminous intensity I_{B0} in the vertical direction of the segment B as follows:

$$\varphi_{b2} = \pi \cdot I_{B0} \quad \dots (9)$$

Assuming that the angle between the segment C and the segment B is θ' the illuminance E_{BC2} on the segment C derived from the secondary reflected light reflected by the segment B is expressed by the following formulas:

$$\begin{aligned} E_{BC2} &= I_{B0} \cdot \cos \theta' \cdot \cos \theta' / (2r \cdot \cos \theta')^2 \\ &= \varphi_{b2} / (4\pi \cdot r^2) \quad \dots (10) \end{aligned}$$

In other words, regardless of the outgoing angle θ' from the segment B, the secondary reflected light from the segment B irradiates all positions on the inside wall of the integrating sphere uniformly. The flux φ_{b2} reflected by the segment B is expressed on the basis of the formulas (7) and (8) as:

$$\begin{aligned} \varphi_{b2} &= \rho \cdot E_{b1} \cdot dS' \\ &= \rho \cdot (\rho \cdot \Phi / (4\pi \cdot r^2)) \cdot dS' \quad \dots (11) \end{aligned}$$

Since the integral of the area dS' with respect to the whole inside wall of the integrating sphere corresponds to the total surface area of the inside wall of the integrating sphere, the illuminance E_{c2} on the segment C, derived from the secondary reflected light from the whole inside wall of the integrated sphere is expressed by the following formula:

$$E_{c2} = \rho^2 \cdot \Phi / (4\pi \cdot r^2) \quad \dots (12)$$

Since, as described earlier, the illuminance on the inside wall, derived from the secondary reflected light of the flux emitted from the light source and reflected by the whole inside wall of the integrating sphere, is constant regardless of the position, the illuminance E_{a2} on the segment A derived from the secondary reflected light is equal to the illuminance E_{c2} .

Similarly, in consideration of higher-order reflected light, illuminance E_a on the segment A is expressed by the following formulas:

Figure 2 Structure of actual integrating sphere (spherical photometer)

$$\begin{aligned} E_a &= I_0(\alpha)/r^2 + \rho \cdot \Phi / (4\pi \cdot r^2) \\ &\quad + \rho^2 \cdot \Phi / (4\pi \cdot r^2) \\ &\quad + \rho^3 \cdot \Phi / (4\pi \cdot r^2) \\ &\quad \dots \\ &= I_0(\alpha)/r^2 + \rho \cdot \Phi / \{(1 - \rho) \cdot 4\pi \cdot r^2\} \quad \dots (13) \end{aligned}$$

Accordingly, when a photometer for measuring illuminance is provided on the inside wall of the integrating sphere, and a baffle for blocking luminous flux $I_0(\alpha)$ from the light source is provided between the light source and the photo-detector as illustrated in Figure 2, the term $I_0(\alpha)/r^2$ in the formula (13) is eliminated, resulting in the photometer producing an output in proportion to the total luminous flux Φ of the light source. It is therefore possible to determine the total luminous flux of a test light source by measuring the test light source and also a total luminous flux standard light source that has been calibrated for total luminous flux, using the same integrating sphere. Here, it is necessary that the difference in luminous intensity distribution between the test light source and the total luminous flux standard light source is not large.

2.2 Position of the light source within the integrating sphere

The influence of the position of the light source inside the integrating sphere is discussed. It is assumed in Figure 3 that there is a point light source with no size, at a position away from the center O of an integrating sphere with inside wall of reflectance ρ , and that a point A on the inside wall

Figure 3 Optical model in which a light source emits light at a position shifted from the center of an integrating sphere

of the integrating sphere is irradiated with luminous intensity I_0 . The resulting illuminance E_a at the point A is expressed by the following formula:

$$E_a = I_0 / (r + a)^2 \quad \dots (14)$$

Since the solid angle from the center of a sphere $(r + a)$ corresponding to the whole inside wall of the sphere is 4π , the area S occupied by a unit solid angle at a distance $(r + a)$ is expressed by the following formula:

$$S = 4\pi \cdot (r + a)^2 / 4\pi = (r + a)^2 \quad \dots (15)$$

Figure 4 Shadow of a light source formed on the inside wall of an integrating sphere by reflected light from a point A when the light source has a non-zero size

Accordingly, there is, at the point A, a secondary light source with area S and luminous intensity $\rho \cdot I_0$ in the vertical direction regardless of the distance from the light source. It may therefore be regarded that the whole inside wall of the integrating sphere is irradiated with this secondary light source. In other words, the light source with the luminous intensity I_0 irradiates the inside wall of the integrating sphere with constant illuminance regardless of its position within the integrated sphere.

Incidentally, it is assumed in the model used in this case that the size of the light source is negligible, and that the flux emitted from the light source can be measured regardless of where the light source is positioned within the integrating sphere. In reality, the light source has a given size as illustrated in Figure 4. A shadow of the light source is projected by the reflected irradiation from the point A on the inside wall of the integrating sphere onto a portion of the inside wall of the integrating sphere on the opposite side. As the light source is moved farther from the center of the integrating sphere and closer to the point A, the influence of the shadow of the light source formed by the reflected light becomes larger. Although the error of the integrating sphere is smaller when the shadow is smaller, when the light source is disposed at the center of the integrating sphere the shadow of the light source is generated uniformly on the inside wall of the integrating sphere and its influence is minimized.

Figure 5 Model of shadow of a spherical light source disposed at the center of an integrating sphere, formed by light reflected by a point A disposed on the inside wall of the integrating sphere

2.3 Diameter of the integrating sphere and size of the light source

The size of the integrating sphere should be large enough to ensure that any measurement error due to effects of self-absorption by the light source is not significant. For this examination, the size of the shadow of the light source itself, projected onto the inside wall of

the integrating sphere by light reflected by the inside wall of the integrating sphere described in the previous section. To simplify analysis, a model is assumed in which a spherical light source with radius r_s is set up at the center of the light source itself, projected onto the inside wall of the integrating sphere by light reflected by the inside wall of the integrating sphere. To simplify analysis, a model is assumed in which a spherical light source with radius r_s is set up at the center of an integrating sphere with radius r , as illustrated in Figure 5. The point A in the figure is the point on the inside wall of the integrating sphere where the perpendicular line from the center of the spherical light source intersects the wall. The shadow of the spherical light source, formed by light reflected from the point A, is projected onto the inside wall of the integrating sphere in the shape of a centroclinal spherical cap. Assuming that the vertical angle of an edge of the light source with respect to the point A is θ , a solid angle ω_s from the center of the integrating sphere, corresponding to the shadow of the light source in the shape of a centroclinal spherical cap, is obtained by the following formula:

$$\omega_s = 2\pi \cdot (1 - \cos 2\theta) \quad \dots (16)$$

Figure 6 Relationship between the ratio of the radius r_s of a spherical light source to the radius r of the integrating sphere, and the lowering of illuminance caused on the inside wall of the integrating sphere by the shadow of the light source.

Since the total solid angle from the center of the integrating sphere is 4π , the ratio of the shadow of the light source to the internal surface area of the integrating sphere is $\omega_s/4\pi$. Figure 6 shows the relationship between $\sin \theta$, that is, the ratio of the radius r_s of the spherical light source to the radius r of the integrating sphere, and the ratio of the solid angle corresponding to the shadow of the light source to the solid angle 2π of the

reflected light reflected diffusely by the point A. When the radius r_s of the spherical light source is decreased while keeping the radius r of the integrating sphere constant, the proportion of the reflected light blocked by the spherical light source also decreases. When the light source is placed at the center of the integrating sphere, the shadow formed by light reflected from the inside wall in all directions of the light source occupies a constant ratio. Assuming that the ratio of the surface area without the shadow to the internal area of the integrating sphere is k , the illuminance E_{b1} on the segment B derived from primary reflected the luminous flux emitted from the light source and reflected by the whole inside wall of the integrating sphere is expressed by the following formulas:

$$E_{b1} = \rho \cdot k \cdot \Phi / (4\pi \cdot r^2)$$

$$k = (4\pi - \omega_s) / 4\pi \quad \dots (17)$$

Similarly, in consideration of third- or higher-order reflected light, illuminance E_a at the segment A is expressed by the following formula:

$$E_a = I_0(\alpha)/r^2 + \rho \cdot k \cdot \Phi / (4\pi \cdot r^2) + \rho^2 \cdot k^2 \cdot \Phi / (4\pi \cdot r^2) + \rho^3 \cdot k^3 \cdot \Phi / (4\pi \cdot r^2) \dots$$

$$= I_0(\alpha)/r^2 + \rho \cdot k \cdot \Phi / \{(1 - \rho \cdot k) \cdot 4\pi \cdot r^2\} \quad \dots (18)$$

Since the shadow of the light source is projected by the each order of reflected light, the illuminance E_a on the segment A decreases with $\rho \cdot k$ in formula (18).

When the radius of the light source is, for example, 1/10 of the radius of the integrating sphere, approximately 1 % of the reflected luminous flux is lost if the light source has high absorptance. This means that when the area dS' of the infinitesimal segment positioned on the inside wall of the integrating sphere is integrated with respect to the whole inside wall of the integrating sphere as described the formula (13) in the previous section, the obtained value corresponds to 99 % of the internal area of the integrating sphere due to the shadow of the light source, and the average reflectance on the inside wall of the integrating sphere is lowered by 1 %. Actually, the absorptance of the light source is not so high, and the total luminous flux measurement with an integrating sphere is carried out as a comparative measurement with a total luminous flux standard lamp. Also, the error in a measured value due to lowering of the reflectance may be further reduced by appropriately selecting a sample light source and a standard lamp of similar sizes and similar luminous intensity distributions.

Incidentally, the illuminance lowering by the shadow of the light source may be reduced to some extent by self-absorption correction³⁾ of the light source. But, since it may not be corrected sufficiently, depending upon the irradiation conditions of the light source used for self-absorption correction, the size of the integrating sphere is generally decided by roughly limiting the maximum diameter of the light source to not more than 1/10 of the diameter of the integrating sphere. According to the CIE 84 1989: The Measurement of Luminous Flux⁴⁾, it is recommended that the maximum size of a compact light source in the shape of a bulb is 1/10 or less in diameter than the integrating sphere.

Also, the JIS, Japanese Industrial Standards, provides no precise guidelines for the size of the integrating sphere, but merely mentions that the size of the integrating sphere is selected in consideration of the temperature increase caused by heat generated by the light source⁵⁾. In this standard, a compact light source is assumed to be one with large heat generation, such as a light bulb or a high-pressure discharge lamp, and it is recommended that an integrating sphere with a diameter of at least 1m be used for a light bulb of 200 W or smaller⁶⁾. This is because the integrating sphere is a thermally closed space that confines heat within itself.

2.4 In the emitting light from the light source placed on the inside wall of the integrating sphere

With most surface illuminants such as a display and a backlight for the display, emitted light is distributed merely across the solid angle space of 2π (i.e., a hemispherical space). Such types of light sources are placed at the aperture on the wall of the integrating sphere, and the total luminous flux measured. Figure 7 illustrates, by employing a plane model, the theory of measuring the total luminous flux of a surface illuminant by using an integrating sphere.

The illuminance E_{a0} on a infinitesimal segment A positioned on the inside wall of an integrating sphere, in a direction at an angle α from a surface illuminant having a total luminous flux Φ , is obtained by the following formula using the luminous intensity $I(\alpha)$ in the direction at the angle α from the light source:

$$\begin{aligned} E_{a0} &= I(\alpha) \cdot \cos \alpha / (2r \cdot \cos \alpha)^2 \\ &= I(\alpha) / (4r^2 \cdot \cos \alpha) \end{aligned} \quad \dots (19)$$

Assuming that the inside wall of the integrating sphere is uniformly diffuse and reflects light with reflectance ρ , and that the segment A has an area dS , the flux φ_a reflected by the segment A is expressed by the following formula:

$$\varphi_a = \rho \cdot E_{a0} \cdot dS \quad \dots (20)$$

Another infinitesimal segment B is positioned on the inside surface of the integrating sphere in a direction at angle θ to the normal of the segment A. Assuming that the segment A is an isotropic reflecting diffuser, the luminous intensity $I_a(\theta)$ from the segment A toward the segment B is expressed by the following formula:

$$I_a(\theta) = (\varphi_a \cdot \cos \theta) / \pi \quad \dots (21)$$

The incident angle of $I_a(\theta)$ against the segment B is θ , and the distance between the segment A and the segment B is $2r \cdot \cos \theta$. Therefore, the illuminance E_{ab} on the surface B derived from the luminous intensity $I_a(\theta)$ is expressed by the following formula:

$$\begin{aligned} E_{ab} &= I_a(\theta) \cdot \cos \theta / (2r \cdot \cos \theta)^2 \\ &= \varphi_a / (4\pi \cdot r^2) \\ &= \rho \cdot I(\alpha) \cdot dS / (4\pi \cdot r^4) \end{aligned} \quad \dots (22)$$

Figure 7 Optical model in which a surface illuminant emits light on the inside-wall of an integrating sphere

As is obvious from the formula (22), reflected light from the segment A irradiates all of the portions of the inside wall of the integrating sphere with uniform illuminance regardless of the outgoing angle θ from the segment A. The internal area of the integrating sphere is $4\pi \cdot r^2$ the solid angle from the inside wall, where the light source is set up, corresponding to the whole space is 2π and the light source is set up in the direction at the angle α to the

normal of the segment A. Therefore, the area dS of the segment A is expressed by using an infinitesimal solid angle $d\Omega$ from the light source, by the following formula:

$$\begin{aligned}\cos \alpha \cdot dS &= (4\pi \cdot r^2 / 2\pi) \cdot d\Omega \\ &= 2r^2 \cdot d\Omega\end{aligned}\quad \dots (23)$$

Accordingly, the formula (22) is converted to the following formula (24):

$$E_{ab} = \rho \cdot I(\alpha) \cdot d\Omega / (2\pi \cdot r^2 \cdot \cos \alpha) \quad \dots (24)$$

Since $\Phi/2$ is obtained as the integral of $\{I(\alpha)/\cos \alpha\} \cdot d\Omega$ of the formula (24) with respect to the whole space (2π), the illuminance E_{b1} on the segment B, derived from the primary reflected luminous flux by the whole inside wall of the integrating sphere, is expressed by the following formula (25):

$$E_{b1} = \rho \cdot \Phi / (4\pi \cdot r^2) \quad \dots (25)$$

Since, as described above, the illuminance on the inside wall derived from the primary reflected luminous flux from the whole inside wall of the integrating sphere is constant regardless of the position, the illuminance E_{a1} on the surface A derived from the primary reflected light is equal to the illuminance E_{b1} on the surface B. The illuminance E_{b1} on the segment B causes a secondary reflection with reflectance ρ from the segment B. Assuming that the segment B has an area dS' , the flux φ_{b2} reflected by it is expressed by the following formula:

$$\varphi_{b2} = \rho \cdot E_{b1} \cdot dS' \quad \dots (26)$$

Next, the irradiation of yet another infinitesimal segment C positioned on the inside wall of the same integrating sphere with the secondary reflected light from the segment B is examined. Assuming that the segment B is an isotropic reflecting diffuser, the flux φ_{b2} reflected by the surface B is expressed using luminous intensity I_{B0} in the vertical direction of the segment B, as follows:

$$\varphi_{b2} = \pi \cdot I_{B0} \quad \dots (27)$$

Figure 8 The shadow of a light source formed on the inside wall of an integrating sphere when the light source having luminous intensity distribution in 4π space emits light on the inside wall of the integrating sphere

Assuming that the angle between the segment C and the segment B is θ' , the illuminance E_{BC2} on the segment C derived from the secondary reflected light from the segment B is expressed by the following formula:

$$\begin{aligned}E_{BC2} &= I_{B0} \cdot \cos \theta' \cdot \cos \theta' / (2r \cdot \cos \theta')^2 \\ &= \varphi_{b2} / (4\pi \cdot r^2)\end{aligned}\quad \dots (28)$$

In other words, the secondary reflected light from the segment B irradiates all the positions on the inside wall of the integrating sphere with uniform illuminance regardless of the outgoing angle θ' from the segment B. The flux φ_{b2} reflected by the segment B is expressed on the basis of the formulas (26) and (27) by the following formula:

$$\begin{aligned}\varphi_{b2} &= \rho \cdot E_{b1} \cdot dS' \\ &= \rho \cdot \{ \rho \cdot \Phi / (4\pi \cdot r^2) \} \cdot dS'\end{aligned}\quad \dots (29)$$

Accordingly, the illuminance E_{c2} on the infinitesimal segment C derived from the secondary reflected light from the whole inside wall of the integrating sphere is expressed by the following formula:

$$E_{c2} = \rho^2 \cdot \Phi / (4\pi \cdot r^2) \quad \dots (30)$$

As described above, the illuminance on the inside wall derived from the secondary reflected light is constant regardless of the position. The illuminance E_{a2} on the surface A derived from the secondary reflected light is therefore equal to the illuminance E_{c2} .

Similarly, in consideration of third- or higher-order reflected light, illuminance E_a on the segment A is expressed by the following formula:

$$\begin{aligned} E_a &= I_0(\alpha)/r^2 + \rho \cdot \Phi/(4\pi \cdot r^2) \\ &\quad + \rho^2 \cdot \Phi/(4\pi \cdot r^2) \\ &\quad + \rho^3 \cdot \Phi/(4\pi \cdot r^2) \\ &\quad \cdot \cdot \cdot \cdot \\ &= I_0(\alpha)/r^2 + \rho \cdot \Phi/\{(1 - \rho) \cdot 4\pi \cdot r^2\} \\ &\quad \cdot \cdot \cdot \cdot \quad (31) \end{aligned}$$

Accordingly, when a photometer for measuring illuminance is provided on the inside wall of the integrating sphere, and a baffle (Figure 8) for blocking direct luminous flux $I_0(\alpha)$ from the light source is placed between the light source and the photometer in the same manner as described, the term $I_0(\alpha)/r^2$ of the formula (31) is eliminated, resulting in an output of the photometer in proportion to the total luminous flux Φ of the light source.

Also in this case, the total luminous flux is obtained through comparative measurement with a total luminous flux standard light source. This total luminous flux standard light source should be a surface illuminant having the luminous intensity distribution in the space range of 2π as described. The generally available total luminous flux standard light source is an incandescent lamp, having wide luminous intensity distribution in the 4π space. When such a standard light source is placed on the inside wall of an integrating sphere, a large shadow of the light source is projected by the light from a point A near the light source on the inside wall of the integrating sphere, as illustrated in Figure 8, and this causes an error in the measurement of total luminous flux.

Accordingly, in the method described at this point in which light is emitted from the light source placed on the inside wall of the integrating sphere, it is necessary to prepare a total luminous flux standard light source of a surface illuminant having been calibrated by the goniophotometer or by any other way. This method does not require a support structure for the light source within the integrating sphere and, hence, absorption error derived from such a support structure can be avoided.

On the other hand, as the area of the surface illuminant provided on the inside wall of the integrating sphere is relatively large, losses caused by absorption of light by the lamp itself within the sphere become too large to ignore. In order to estimate a critical area, a model as illustrated in Figure 9 is assumed. There is a disc-like light source

with radius r_s on the inside wall of the integrating sphere of radius r . This model is used for calculating the ratio of the perpendicular to the light source from a point A positioned on the inside wall of the integrating sphere to 2π .

A solid angle ω'_s , taken from the center of the integrating sphere and corresponding to the proportion of the inside wall of the integrating sphere covered by the disc-like light source, is obtained by the following formula:

$$\omega'_s = 2\pi \cdot (1 - \cos \theta) \quad \cdot \cdot \cdot (32)$$

Since the total solid angle from the center of the integrating sphere is 4π , the ratio of the shadow of the light source in the shape of a spherical cap to the internal surface area of the integrating sphere is $\omega'_s/4\pi$. Figure 10 shows the relationship between $\sin \theta$, that is, the ratio of the radius r_s of the spherical light source to the radius r of the integrating sphere, and the ratio of the solid angle corresponding to the shadow of the light source to the solid angle 2π of the reflected light diffuse reflected by the point A. As a result, when the radius of the light source corresponds to 30 % of the radius of the integrating sphere, approximately 3 % of reflected light is lost to the absorption of light by the lamp itself in the sphere.

Figure 9 Model used for calculating the ratio, to the internal area of an integrating sphere, of an area where a disc-like light source having a radius r_s and disposed on the inside wall of the integrating sphere is irradiated with diffusely reflected light reflected by a point A positioned on the inside wall of the integrating sphere

This means that when the area dS' of the infinitesimal segment disposed on the inside wall of the integrating sphere is integrated with respect to the whole inside wall

of the integrating sphere as described with reference to the formula (11) in the previous section, the obtained value corresponds to 97 % of the internal area of the integrating sphere due to the shadow of the light source, and the illuminance on the inside wall of the integrating sphere (namely, the output of the integrating sphere) is lowered by 3 %. In reality, the absorbance of the light source is not so high, and the total luminous flux measurement with the integrating sphere is carried out as a comparative measurement with a total luminous flux standard lamp. Therefore, error in the measured value derived from the lowering of the illuminance may be further reduced by appropriately selecting a sample light source and a standard lamp with similar sizes and similar luminous intensity distributions.

No standard has been found for directly prescribing the size of a light source to emit light at the inside wall of an integrating sphere. But, according to the LM-79⁵⁾, “the size of an opening of an integrating sphere is preferably determined so that the area of the opening corresponds to 2 % or less of the whole area of the inside wall of the integrating sphere”. However, if employing a sample light source largely different in luminous intensity distribution from a standard light source — such as a beam light source having extremely narrow luminous intensity distribution — the loss of the primary reflected light on the inside wall of the integrating sphere (namely, the loss due to the absorbance of the light source itself) is large due to the theory described above. When reduction in illuminance on the inside wall of the integrating sphere is limited to 1 % or less as in the CIE recommendation⁴⁾ mentioned above, it would appear that the radius of the light source should preferably not exceed 15 % of the radius of the integrating sphere.

Figure 10 Relationship between the ratio of the radius r_s of the spherical light source to the radius r of an integrating sphere, and the ratio of the area of the light source to the internal area of the integrating sphere

3. Theory of the integrating hemispheres

3.1 Theory and structure of the integrating hemispheres

An integrating hemisphere has been proposed for the measurement of a surface illuminant^{7,8)}. As illustrated in Figure 11, this is an integrating photometer comprising a plane mirror and an integrating hemisphere whose inside wall is coated or processed with a material such as barium sulfate that perfectly diffuse reflects the radiation to be measured. The light source whose total luminous flux is to be measured is mounted on the plane of the plane mirror (that is, a front surface mirror), at a position corresponding to the center of curvature of the integrating hemisphere. In this case, an integrating sphere is formed optically by the hemisphere, and its virtual image formed by the front surface mirror.

The theory of this integrating photometer is described with reference to Figure 11. The light source to be measured is assumed to be a surface illuminant having a uniform diffused luminous intensity distribution as described above. The angle between the normal to the light source and a line extending from a point corresponding to the center of the light source to an infinitesimal segment A on the inside surface of the hemisphere is α . The center of the light source is at the center of curvature of the integrating hemisphere. The total luminous flux of the light source is Φ , and the luminous intensity in a direction at the angle α to the normal of the light source is $I_0(\alpha)$.

Taking the radius of curvature of the integrating hemisphere as r , the illuminance E_{a0} on the segment A is expressed by the following formula (33):

$$E_{a0} = I_0(\alpha)/r^2 \quad \dots (33)$$

Assuming that the inside wall of the integrating hemisphere is isotropic diffused and reflects light with reflectance ρ and that the infinitesimal segment A has an area dS , the flux φ_a reflected by the infinitesimal segment A is expressed by the following formula:

$$\varphi_a = \rho \cdot E_{a0} \cdot dS \quad \dots (34)$$

Consider another infinitesimal segment B positioned on the inside wall of the integrating hemisphere at an angle θ to the normal of the segment A. Also, let the virtual image of the surface B formed by the front surface mirror be designated as the infinitesimal segment B'. Assuming that the surface A is an isotropic reflecting diffuser, the luminous intensity $I_a(\theta)$ from the surface A toward the surface B' is expressed by the following formula:

$$I_a(\theta) = \varphi_a \cdot \cos \theta / \pi \quad \dots (35)$$

Figure 11 Plane model of the integrating hemisphere

Since the surface B' is on the inside wall of the virtual image of the integrating hemisphere, the incident angle of $I_a(\theta)$ against the surface B' is θ , and the distance between the surface A and the surface B is $2r \cdot \cos \theta$. Therefore, the illuminance $E_{ab'}$ on the surface B', derived from the luminous intensity $I_a(\theta)$, is expressed by the following formula:

$$\begin{aligned} E_{ab'} &= \rho_M \cdot I_a(\theta) \cdot \cos \theta / (2r \cdot \cos \theta)^2 \\ &= \rho_M \cdot \varphi_a / (4\pi \cdot r^2) \\ &= \rho_M \cdot \rho \cdot I_0(\alpha) \cdot dS / (4\pi \cdot r^4) \end{aligned} \quad \dots (36)$$

In this formula, ρ_M is the reflectance of the plane mirror. As is obvious from the formula (36), the reflected light from the surface A irradiates all the positions on the inside wall of the integrating hemisphere with uniform illuminance regardless of the outgoing angle θ from the surface A. Since the internal surface area of the integrating hemisphere is $2\pi \cdot r^2$, the solid angle from the wall of the sphere, where the light source is placed, corresponding to the whole space is 2π . The area dS of the infinitesimal segment can therefore be expressed using the micro solid angle $d\Omega$ as follows:

$$\begin{aligned} dS &= (2\pi \cdot r^2 / 2\pi) \cdot d\Omega \\ &= r^2 \cdot d\Omega \end{aligned} \quad \dots (37)$$

Accordingly, the formula (36) can be converted to the following formula:

$$E_{ab'} = \rho_M \cdot \rho \cdot I_0(\alpha) \cdot d\Omega / (4\pi \cdot r^2) \quad \dots (38)$$

Since the total luminous flux Φ of the light source is obtained as the integral of $I_0(\alpha) \cdot d\Omega$ of the formula (38) with respect to the whole space, the illuminance $E_{b'1}$ on the surface B', derived from primary reflected light emitted from the light source and reflected by the whole inside wall of the integrating sphere, is expressed by the following formula:

$$E_{b'1} = \rho_M \cdot \rho \cdot \Phi / (4\pi \cdot r^2) \quad \dots (39)$$

Since the surface B' is the virtual image of the surface B formed by the plane mirror, the illuminance E_{B1} on the surface B derived from the primary reflected light of the flux emitted from the light source and reflected by the whole inside wall of the integrating sphere is a sum of the illuminance E_{B1} attained by the actual image of the integrating hemisphere and the illuminance $E_{b'1}$ attained by the virtual image. For the radiation by the actual image of the integrating hemisphere, a model in which the infinitesimal segment B is directly irradiated from the infinitesimal segment A of Figure 11 is considered. Then the illuminance E_{b1} is expressed by the following formula, as described the formula (7) in the previous section:

$$E_{b1} = \rho \cdot \Phi / (4\pi \cdot r^2) \quad \dots (40)$$

Accordingly, the illuminance E_{B1} on the segment B, derived from the primary reflected light of the flux emitted by the light source and reflected by the whole inside wall of the integrating sphere, is expressed by the following formula:

$$\begin{aligned} E_{B1} &= E_{b1} + E_{b'1} \\ &= (1 + \rho_M) \cdot \rho \cdot \Phi / (4\pi \cdot r^2) \end{aligned} \quad \dots (41)$$

Similar to the formula (13) mentioned in Section 2, in consideration of third- or higher-order reflected light, illuminance E_a on the surface A is expressed by the following formulas:

$$\begin{aligned} E_a &= I_0(\alpha) / r^2 + \rho \cdot \Phi / (4\pi \cdot r^2) + \rho_M \cdot \rho \cdot \Phi / (4\pi \cdot r^2) \\ &\quad + \rho^2 \cdot \Phi / (4\pi \cdot r^2) + \rho_M^2 \cdot \rho^2 \cdot \Phi / (4\pi \cdot r^2) \\ &\quad + \rho^3 \cdot \Phi / (4\pi \cdot r^2) + \rho_M^3 \cdot \rho^3 \cdot \Phi / (4\pi \cdot r^2) \\ &\quad \dots \\ &= I_0(\alpha) / r^2 \\ &\quad + \Phi / (4\pi \cdot r^2) \{ \rho / (1 - \rho) + \rho_M \cdot \rho / (1 - \rho_M \cdot \rho) \} \end{aligned} \quad \dots (42)$$

Figure 12 Structure of the actual integrating hemispheres

Figure 12 is the structure of the actual integrating hemisphere. Specifically, when a photometer for measuring illuminance is provided on the inside wall of the integrating sphere and a baffle for blocking direct rays from the light source is provided between the light source and the photometer, the term $I_0(\alpha)/r^2$ of the formula (42) is eliminated, resulting in an output of the photodetector in proportion to approximately twice the total luminous flux Φ of the light source if the reflectance ρ_M of the plane mirror is high.

In this hemispherical integrating photometer, light is emitted from the light source and also from its virtual image within the optically integrating sphere comprising real and virtual hemispheres. No support structure is therefore required for the light source within the integrating space. This is advantageous in that absorption due to such a support structure is avoided. Furthermore, even when the luminous intensity distribution is biased toward the lower hemisphere of the integrating sphere, as with a surface illuminant or a light source emitting light only in forward directions, the luminous intensity distribution is combined with that of the virtual image of the light source and, hence, the primary term of ρ may be regarded as derived from reflection on a wide portion of the inside wall of the integrating sphere.

This hemispherical integrating photometer is applicable to a light source with the luminous intensity distribution of 4π different from the integrating sphere where a light source is placed on the inside wall. In the description given in this paper, the reflectance of the plane mirror has not been considered. The reflectance of the plane mirror is equivalent to that obtained by reducing the reflectance on the inside wall of the virtual image portion of the integrating sphere correspondingly with the reflectance of the plane mirror. Since the light within the integrating sphere is repeatedly reflected and absorbed by the inside wall, anisotropy such as polarization attributed to the mirror never arises. Even in a general integrating sphere formed as a complete sphere, the lower hemisphere becomes dirty with time, which causes a difference in reflectance between the upper and lower hemispheres. Such a difference does not largely affect

the measurement of the total luminous flux of a sample light source in comparison with a standard light source, as long as the reflectance of the plane mirror is sufficiently high.

3.2 Relationship between maximum radius of a spherical light source and radius of the hemisphere

In employing an integrating hemisphere, if the cross-section of the sample light source occupies a large part of the cross-section of the integrating hemisphere, the setup cannot function as an integrating sphere because the flux cannot reach any virtual image portions of the hemisphere. As illustrated in Figure 13 and as described in Section 2, when a surface illuminant is used as the light source, the largest shadow is formed as a shadow S by reflected light having been reflected by the point A that is directly (in the vertical direction) under the light source. The shadow S' is formed by reflected light having been reflected by a point A' disposed at an angle β to the vertical direction from the light source. The shadow S is $S \cdot \cos \beta$. θ is the vertical angle of the edge of the light source with respect to the point A. The solid angle ω_s from the center of the integrating hemisphere, corresponding to the shadow of the light source in the shape of a spherical cap, is obtained by the following formula:

$$\omega_s = 2\pi \cdot (1 - \cos 2\theta) \cdot \cos \beta \quad \dots (43)$$

Since the total solid angle from the center of the integrating hemisphere is 4π , the ratio $R(\beta)$ of the area of the spherical cap corresponding to the shadow (formed by reflected light from the point A') of the light source to the internal area of the integrating hemisphere is $\omega_s/4\pi$.

$$R(\beta) = (\omega_s/4\pi) \cdot \cos \beta \quad \dots (44)$$

Since the inside wall diffusely reflects light, the luminous intensity distribution of the reflected light is perfectly diffused. Therefore, assuming that the points A and A' are irradiated with the same reflected light from the light source, the ratio of the area of the shadow formed by the light source to the total internal area of the inside wall of the integrating sphere is obtained by integrating $R(\beta)$ with β . Figure 14 illustrates the relationship between the ratio of the radius r_s of the spherical light source to the radius r of the integrating sphere, and the ratio of the shadow area of the light source to the internal area of sphere. If the lowering of illuminance on the inside wall of the integrating hemisphere is to be limited to no more than 1 % as in the CIE recommendation⁴⁾, it would appear that the radius of the light source should

Figure 13 Model of a shadow of a disc-like light source with a radius r_s placed on the center of an integrating hemisphere formed by light from a point A placed on the inside wall of an integrating hemisphere

Figure 14 Relationship between the ratio of radius r_s of a spherical light source to the radius r of an integrating hemisphere and the ratio of internal area of integrating hemisphere to the shadow of the light source

preferably be no more than 1/3 of the radius of the integrating hemisphere.

In addition, an integrating hemisphere can measure forward emitting light sources of twice the radius or four times the surface area compared to a full sphere of the same diameter.

4. Conclusion

The theory of, and the problems arising in, measurement of total luminous flux using an integrating sphere and an integrating hemisphere have been discussed. In the actual use of an integrating sphere, it is

necessary to also examine the reflectance of its inside wall and the self absorption of the light source to be measured.

As a method for determining the total luminous flux of a light source, the goniophotometer which is a basic apparatus for measuring total luminous flux and which has high measurement accuracy, is generally employed. A spherical integrating photometer is to be employed merely as a simple method for measuring total luminous flux. However the spherical integrating photometer is advantageous because it is simple and easy to employ. Therefore, when an integrating sphere is optimally designed with respect to the aforementioned requisites in consideration of the luminous intensity distribution and other features of the light source to be measured, its measurement accuracy may be maintained at a practical level.

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