ABSTRACT

The refraction of the rays propagated into the nearly vertical direction in deep water is calculated. It is indicated that the refraction of sound rays radiated into vertical direction from a source at sea surface differ from that of rays radiated from a deep source and that the rays propagated vertically is not so sensitive to the latitude changes.

INTRODUCTION

The nonlinear parametric array has been utilized recently owing to make narrow the transmitted beam of acoustic deep fathometer. The use of sound waves in water for the transmission of intelligence between deep submersible and surface vessel has been of interest. Diffraction of ray propagated nearly horizontally in the ocean, i.e., the ray radiated at the angles less than 20° to the horizontal, has been discussed in detail in many papers. But, diffraction of the ray propagated vertically is important in case of the fathometer and the deep submersible. We calculate the paths of the ray which are radiated nearly vertically and describe the relationships between the refracted propagation and the straight line propagation of sound beams. The path of a ray is calculated by the well-known ray theory and new method to calculate the ray is not used at all. We consider that the computed results are helpful the practical applications. Diffraction and interference of the ray propagated horizontally near bottom was treated by K.V. Mackenzie in detail.

1. Fundamental equations of the ray theory

In tracing the path of any given ray through a complicated velocity gradient it is advantageous to simplify the analysis by breaking up the path into separate segments, each short enough so that the velocity gradient may be assumed constant over its length. The path of a ray through a layer of constant velocity gradient \( g_1 \) is an arc of a circle, as shown in Fig.1. \( \theta_1 \) is the angle formed by a ray and
a horizontal line at the point S. R₁ is the radius of a circle. C₁ and C₂ are the velocity at the points S and B, respectively. The radius of a circle R₁ is related to the difference in depth Δd₁ of two points S and A,

\[ Δd₁ = R₁(cosθ₂ - cosθ₁), \quad (1) \]

where \( θ₂ \) is the angle between the incident ray and the interface AB. The velocities of sound at points S and A are related by

\[ C₂ = C₁ + g₁Δd₁. \quad (2) \]

The angle \( θ₁, θ₂ \) are related to the velocities of sound \( C₁, C₂ \) by Snell's law

\[ \frac{cosθ₁}{C₁} = \frac{cosθ₂}{C₂} = \frac{cosθ₂ - cosθ₁}{C₂ - C₁}. \quad (3) \]

Combining these three equations, we obtain

\[ R = \frac{C₁}{g₁cosθ₁} \quad \text{(4)} \]

\[ cosθ₂ = cosθ₁[1 + \frac{Δd₁ g₁}{C₁}] \quad \text{(5)} \]

when \( θ₁, g₁, \) and \( C₁ \) are given, \( θ₂ \) is obtained. If this process is repeated for a number of different depths, the angle of inclination \( θ_n \) and \( θ_{n+1} \) at the interface n and n+1 will be given by the following equations,

\[ cosθ_{n+1} = (1 + \frac{Δd_n g_n}{C_n})cosθ_n \quad , \quad (n=1,2,3\cdots) \quad \text{(6)} \]

\[ cosθ_n = (1 + \frac{Δd_{n-1} g_{n-1}}{C_{n-1}})\cdots(1 + \frac{Δd_2 g_2}{C_2})(1 + \frac{Δd_1 g_1}{C_1}) \quad \text{(7)} \]

The horizontal distance traveled in each layers \( Δx_1, Δx_2, \cdots, Δx_n \) are expressed by the following equations,

\[ Δx_1 = R₁(sinθ₁ - sinθ₂) \]

\[ Δx_2 = R₂(sinθ₂ - sinθ₃) \]

\[ \cdots \]

\[ Δx_n = Rₙ(sinθₙ - sinθ_{n+1}) \quad \text{(8)} \]

From Eqs.(1) and (8), we can find the following equation,

\[ \frac{Δx_1}{Δd_1} = \cot \frac{θ₁ + θ₂}{2} \]
and similarly,
\[ \frac{\Delta x_2}{\Delta d_2} = \cot \frac{\theta_2 + \theta_3}{2} \] (9)

Therefore the total horizontal distance traveled by the ray in each layers \( X \) is expressed in the following equation
\[ X = \Delta d_1 \cot \frac{\theta_1 + \theta_2}{2} + \Delta d_2 \cot \frac{\theta_2 + \theta_3}{2} + \cdots + \Delta d_n \cot \frac{\theta_n + \theta_{n+1}}{2} \] (10)

2. Sound velocity profile in the sea water

The distribution of sound velocity in deep water is not so variable as it is in shallow water. In the majority of cases, the velocity as a function of depth has a more characteristics form. Typical sound velocity profiles for deep water fall into several categories, depending upon the latitude and the ocean.(3)

Fig. 2 shows the typical sound velocity profiles for the North Atlantic Ocean in winter. Fig. 2 A, B, and C are those of equatorial, moderate, and polar latitudes, respectively. The sound velocity profile is subject to daily, seasonal, and meteorological changes. The ray propagated in a vertical direction is not very sensitive to the vertical distribution of sound velocity. Therefore we consider mainly the rays that is nearly vertical in the sea water having the velocity structure indicated in curve B of Fig. 2. The curve A and C of Fig. 2 are used merely for the purpose of comparison with the ray propagated in the sea water of the curve B of Fig. 2.

The velocity gradient of the upper layer is positive from the surface to a depth of 450 m and a surface sound channel is formed. In the middle layer, the sound velocity gradient is negative at the depths ranging from 450 m to 1200 m. The depth of minimum velocity occurs near 1200 m, followed by a constant positive velocity gradient of 0.018 sec. To find the ray path, curve B is divided up into about 25 depth intervals in each of which the velocity is approximated by a constant gradient.
3. Calculation of the ray path

We calculate the refraction of rays that are radiated into the nearly vertical direction at the surface of the sea water having the velocity structure indicated in curve B of Fig. 2. The resulting path of the rays radiated into vertical direction at angles of $10^\circ$, $20^\circ$, $30^\circ$, $40^\circ$, $50^\circ$, $60^\circ$, $70^\circ$, $80^\circ$ are shown in Fig. 3. The ray radiated at an angle of $10^\circ$ is nearly a straight line and the horizontal distance traveled by the ray is $1803$ m at a depth of $10000$ m, $1060$ m at a depth of $6000$ m. Similarly, the rays radiated at the angles of $20^\circ$, $30^\circ$, and $40^\circ$ are a little affected by the sound velocity structure. The influence of the refraction for the ray path increase when the radiation angle is increase and the rays radiated at the angle of larger than $70^\circ$ do not reach to the bottom at the depth of $1000$ m. For example, the ray radiated at an angle of $80^\circ$ becomes horizontal at the depth of $4728$ m ($\circ$ indicat nadir points) and the horizontal distance between the source and this point is about $32$ Km and this ray reappear at a surface of the horizontal distance about $64$ Km. Many scientists discuss such a ray propagated into horizontal direction.

The ray diagram for a source at a depth of $6000$ m is shown in Fig. 4. In this figure, we assume that a radiation angle, $\theta$ is the
angle between the vertical line and the ray upgoing from a source. As shown in Fig. 3, the ray radiated into the downward direction at the angle of 80° is horizontal at the depth of 4728 m and does not reach the depth of 6000 m. But the ray radiated into the upward direction at the same angle from the source at the depth of 6000 m reaches the sea surface. The rays radiated at the angle of smaller than 90° have not a vertex point but the rays radiated at the angle of larger than 90° have a nadia point unless the ray reaches a bottom.

It is apparent from these figures that there is a little difference between the ray radiated downward from the sea surface and that radiated upward from a deep source. In the latter case, the horizontal range for the ray to reach the sea surface is the function of the source depth and the radiation angle.

Fig. 5 shows the relation between the horizontal distance and the radiation angle for various source depth. If the ray is radiated at the angle of less than 60°, the horizontal distance at the surface is less than 3000 m for the source depths of 1000 and 2000 m. But when the source depth is larger than them, the horizontal distance at the surface rapidly with increasing the radiation angle. In this figure the points marked x indicate that the rays are returned downward without a surface reflection.

Fig. 6 shows the relation between the horizontal distance at the sea surface and the source depth for various radiation angles. It is shown that when the radiation angles are less than 40° the plot of the horizontal distance versus the source depth are nearly a straight line. In this figure, three dotted lines indicate that the relation between the vertical and horizontal range of the ray which radiated from the sea surface at the angles of 20°, 40°, and 60°. There is a little difference between the solid and dotted lines for angle of 20° and 40°. But when the radiation angle is 60° there is a difference between them at the deep source.
As shown in Fig. 6, the relation between the horizontal distance and source depth is nearly a straight line and consequently, it appears that the ray is a little affected by the vertical distribution of the velocity of sound. Therefore we make another attempt to look the refraction of a ray.

A new quantity is introduced, the deviation of ray, $\Delta R$ defined as the difference between the horizontal distance of a refracted ray $R_r$ and that of a stright ray $R_s$ ($\Delta R = R_r - R_s$). Fig. 7 A and B shows the relation between the deviation of ray versus the source depth for various radiation angles. Fig. 7 A shows the case of the upgoing ray from the deep source and then the abscissa indicates the source depths. Fig. 7 B shows the case of the downgoing ray from the sea surface and then the abscissa indicates the depth traveled by the ray.
It is observed that the resulting curve crosses the axis. In Fig. 7 A, $\Delta R$ is small positive value for the depth of less than 2500 m and is large negative value for the deeper sources than 2500 m. In Fig. 7 B, on the contrary, $\Delta R$ is small negative value for the depth of less than 5000 m and is large positive value for the deeper depths than it. Therefore the downgoing rays originating from the sea surface are affected by the further refraction which causes the rays to spread more rapidly than the spherical divergence. But the upgoing rays from the deep source are affected by the fewer refraction than the spherical divergence.

Those figures also show that when the radiation angles are less than 50° the rays are not so affected by the refraction.

Fig. 8 shows the difference between the horizontal ranges of the ray leaving the source into a downward direction for the sound velocity profiles in different latitudes.
It is apparent from Fig. 7 A and B that the rays radiated at the larger angle than $50^\circ$ are affected by the refraction. Therefore, for the radiation angle of $60^\circ$, the difference between the horizontal distance traveled by the ray in the sea water having the velocity structure indicated in curve B of Fig. 2 and that in curve A and C are shown in Fig. 8.

It is apparent from this figure that the ray propagated into the vertical direction is not so affected by the latitude changes.

4. Conclusions

We may be able to make the following conclusions by the results of our calculations.

(1) The refraction of sound waves radiated into vertical direction from a source at sea surface differ from that of waves radiated from a deep source.

(2) When the rays are radiated at the radiation angle of less than $40^\circ$, the plot of the horizontal distance versus the depth is nearly a straight line.

(3) The downgoing rays radiated from the sea surface are affected by the further refraction than the spherical divergence, but the upgoing rays from the deep source are affected by the fewer refraction.

(4) The rays propagated vertically is not so sensitive to the latitude changes.

References

Profile of the authors

Tamio Kuyama

Tamio Kuyama was born in Tokyo in 1902 and appointed Underwater Acoustics Research Member of Technical Research and Development Institute of Japanese Navy in 1928. In his Navy years (1926-1945) he studied Underwater Acoustics and established the foundation of passive sonars.

Retiring Navy after the War, he passed some time (1946-1949) at the Department of Geophysics, Tokyo Imperial University, as a research member. He received his Ph.D degree from Tokyo Imperial University in 1946. In the following few years (1949-1953) he worked at the Research Institute of Marine Instrument Company, as the director.

He joined the National Defense Academy of Japan in Yokosuka, first as professor of physics (1953) and later (1962) became professor of Underwater Acoustics for the Post Graduate Course. He devoted himself mainly to the research on the sound propagation in the ocean. His last years at the National Defense Academy (1967-1971) were very busy as vice-superintendent.

He's been with the Fifth Research Center of Research and Development Institute of the Defense Agency of Japan as the research advisor since 1971.

Toshiaki Kikuchi

Toshiaki Kikuchi was born in Fukushima in 1936 and now lives in Yokosuka. He received his Bachelor's Degree in Electric Engineering from Yamagata University in 1959. He then joined the National Defense Academy as assistant of applied physics in 1959 and began to study Underwater Acoustics, especially the sound propagation in the ocean. He became assistant professor of Underwater Acoustics for the Post Graduate Course in 1969. He is a member of the Acoustical Society of America.