Reduction of Calculation Time for Load Path U^* Analysis of Structures*

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Abstract

The parameter U^* has been introduced by the authors to indicate the load paths in a structure. The most serious problem encountered when calculating U^* for actual structures is the high computation time required. In the present study, we propose two methods for reducing the time of FEM calculations required to obtain U^* distributions. In the first method, essential terms are extracted from the inverse structural stiffness matrix, and these terms are used to calculate the internal stiffness. In the second method, three independent inspection loadings are applied to each point. In this operation, multiple geometry conditions are converted to multiple mechanical boundary conditions. An actual structure is calculated using this second method, and the rate of decrease in the calculation time is over 90 percent.

Key words: Structural Analysis, Finite Element Method, Computational Mechanics, Numerical Analysis, Load Transfer, Load Path

1. Introduction

It is important to realize high stiffness, high strength, and lightweight structures for automotive vehicle bodies. To meet these requirements, it is necessary to investigate the entire structure from the viewpoint of load transfer and load paths. Load transfer in structures has generally been studied by examining the stress⁽¹⁾, strain, or strain energy. However, a structure having a circular hole shows high stress values around the hole because of stress concentration. Since it is unreasonable to conclude that the hole is effective for load transfer, the stress distribution is likely to be misleading with respect to the decision of load paths.

The parameter U^* is a new concept introduced by the authors for the expression of load transfer in structures⁽²⁾. The parameter U^* is explained as the connectivity between the loading point and an arbitrary point in a structure and is not based on the concept of stress, strain, or strain energy. Load paths are expressed from the contour lines of the U^* distribution. A comprehensive understanding of the structure is obtained from the U^* distribution and the location of the load paths.

In a usual calculation of the U^* distribution using the finite element method (FEM), it is necessary to calculate the U^* value for each node of the FE model based on the direct definition of U^* . For this reason, more than one million iterative FE calculations are required in some cases. In such cases, the calculation time becomes enormous beyond the level of the practical usage.

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In the present study, two calculation methods for the U^* distribution are introduced to reduce the FE computation time. The index U^* is formulated with the internal stiffness, the elements of which are constituted by the connectivity between a loading point and an arbitrary point. Using the formulation of U^* with the internal stiffness, the authors herein propose methods by which to obtain the U^* distribution with a single FE calculation for an entire structure.

In the first calculation method, the internal stiffness is obtained by extracting the components from the inverse matrix of the structural stiffness matrix. The second proposed calculation method is called the inspection loading method, in which the internal stiffness is obtained by applying multiple loading conditions at the U^* calculation point.

The first method cannot be applied to general-purpose FEM solvers like MSC.Nastran, in which the calculation process for the inverse matrix of the structural stiffness matrix is not included.

Conversely, the second method can be used in these solvers. Moreover, the second method in combination with the customized MSC.Nastran by DMAP allows the U^* distribution to be calculated more effectively⁽³⁾.

The computation time of the second method is compared with that of the conventional calculation method with regard to an FE model of an actual heavy-duty truck cab structure. It is shown that the inspection loading method can greatly decrease the computation time to a level of practical usage.

The present paper is based on Ref.(4) written by the present authors.

2. Expression of Load Transfer with U^*

2.1 Previous study on U^* ⁽²⁾

Figure 1(a) shows an elastic body with loading point A and supporting point B. Point C is an arbitrary point on the body. The index U^* is defined as

$$U^* \equiv 1 - \left(\frac{U'}{U} \right)^{-1} \quad (1)$$

where U is the strain energy stored in the entire body when Point C is unrestrained (Fig.1(a)), and the value U' is the stored energy when Point C is restrained (Fig.1(b)). Since the strain energy U' has a higher value when the connectivity between Points A and C is stiffer, the index U^* expresses the connectivity between the loading point and an arbitrary point.

The relationship concerning the loading and deformation among Points A, B, and C is expressed in terms of the “internal stiffness” (or relative stiffness⁽²⁾) as follows:

$$\begin{Bmatrix} p_A \\ p_B \\ p_C \end{Bmatrix} = \begin{bmatrix} K_{AA} & K_{AB} & K_{AC} \\ K_{BA} & K_{BB} & K_{BC} \\ K_{CA} & K_{CB} & K_{CC} \end{bmatrix} \begin{Bmatrix} d_A \\ d_B \\ d_C \end{Bmatrix} \quad (2)$$

where each tensor K with subscripts is the internal stiffness including 3×3 scalar elements,

Fig.1 Internal stiffness

and each vector $\mathbf{p}$ or $\mathbf{d}$ with subscript is a loading vector or a displacement vector including three scalar elements. Although Eq.(2) appears to be identical to the equation of the primitive FE analysis, the meaning of the equation is completely different. Equation (2) shows the behavior with respect to the entire structure.

The index U^* in Eq.(1) can be rewritten using the internal stiffness in Eq.(2) as follows:

$$U^* = \left(1 - \frac{2U}{(\mathbf{K}_{AC}\mathbf{d}_C) \cdot \mathbf{d}_A} \right)^{-1} \quad (3)$$

where the vector $\mathbf{d}_C$ is the displacement of Point C when the displacement $\mathbf{d}_A$ is applied in Fig.1(a). The notation $(\cdot)$ indicates an inner product of vectors. The tensor and vector notation is employed instead of the matrix notation for the products. Equation (3) shows that the index U^* is described in terms of $\mathbf{K}_{AC}$, which indicates the degree of connectivity between Points A and C.

Throughout the present paper, a tensor is expressed by a bold face capital letter, such as $\mathbf{K}_{AC}$, and a vector is denoted by a bold face lower-case letter, as $\mathbf{d}_A$. On the other hand, a matrix or a column vector is denoted in brackets, such as $[\mathbf{K}_{AC}]$ or $\{\mathbf{d}_A\}$.

2.2 Distribution of U^* and load path

As an example of U^* calculation, an FE model and the calculation results are shown in Fig.2. The model is a flat plate with a circular hole. The right-hand side is restrained and the in-plane displacement is applied on the left-hand side. The calculated U^* distribution is shown with the U^* potential lines, and the orthogonal lines to these potential lines are also shown in this figure. These orthogonal lines are called the “stiffness lines”.

The stiffness line that has the smallest gradient is the ridge line of the U^* curved surface, which can be regarded as the most stiffened route from the loading point. It can be concluded the ridge line of the U^* curved surface transfers the largest loading, and we can therefore define this line as the “load path”.

In other words, if a vector $\mathbf{k}^{(-)}$ is introduced as

$$\mathbf{k}^{(-)} = -\text{grad}U^* \quad (4)$$

the successively traced line along the smallest $\mathbf{k}^{(-)}$ is the load path. The introduced vector $\mathbf{k}^{(-)}$ is called the “stiffness decay vector”, which is a conservative quantity.

In Fig.2, the potential lines of U^* are shown as thin lines, and the stiffness lines are expressed as dashed lines. In addition, the load paths are represented as boldface lines. The effect of stress concentration is excluded, and the location of the load paths agrees with our intuitive expectation.

2.3 Genetic algorithm optimization using $U^{*(5)}$

The concept of U^* is introduced in the optimization process using Genetic Algorithm (GA). A combination of U^* and the GA optimization process is applied to a simple FE model of the flat plate shown in Fig.3(a). The design parameter is the thickness of each

Fig.2 Distribution of U^* and load paths

Fig.3 Structural optimization

element of the calculation model. The objective function is determined by the following conditions: U^* values along the load paths should decay uniformly and continuously, and the load path from the loading point should be consistent with the load path from the supported point.

Note that this objective function has no relationship to the concepts of stress, strain, or strain energy density. Figure 3(b) shows the optimized model (50th generation).

For comparison, a conventional optimization process using a sensitivity analysis to make the strain energy density uniform is also applied to the same model. Figure 3(c) shows the optimized model obtained from the sensitivity analysis. Obvious similarities are observed in the two optimized structures shown in Figs.3(b) and 3(c). However, the purpose of this demonstration is not to confirm the efficiency of the optimization using U^* . The intention of this comparison is to show the propriety of the concept of U^* .

The results described in §2 were reported in previous papers by the present authors.

3. Calculation of U^* Using Conventional Method

In this section, a conventional FE calculation method of U^* is presented using its original definition, given by Eq.(1). In the FE displacement method for a linear static problem, the equation to be solved is as follows:

$$\begin{Bmatrix} p_1 \\ p_2 \\ \vdots \\ p_A \\ p_B \\ p_C \\ \vdots \\ p_n \end{Bmatrix} = \begin{bmatrix} (11) & (12) & \cdots & (1n) \\ (21) & (22) & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & & & (AA) & (AB) & (AC) \\ \vdots & & & (BA) & (BB) & (BC) \\ \vdots & & & (CA) & (CB) & (CC) \\ \vdots & & & & \ddots & \vdots \\ (n1) & \cdots & \cdots & \cdots & \cdots & (nn) \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ \vdots \\ d_A \\ d_B \\ d_C \\ \vdots \\ d_n \end{Bmatrix} \quad (5)$$

where the elements of the left-hand side are the loading vectors at the nodes, and the elements in the column vector of the right-hand side are the displacement vectors at the nodes. The matrix shows the structural stiffness matrix, and n indicates the number of nodes. Equations (2) and (5) can be regarded as the two types of expressions that show the overall behavior of a structure.

For the calculation of the U^* distribution, it is necessary to give a zero deformation condition for each node, as shown in Fig.1(b). In the FE displacement method, the overall calculation of Eq.(5) is required for each deformation condition. This means that the overall calculation of Eq.(5) is necessary for each calculation point. In conclusion, for the U^* calculation of every point, the calculation time becomes enormous. Actually, the U^* distribution calculation of an FE model that is constituted of one million nodes requires more than ten months when we employ a conventional personal computer.

In contrast, when we introduce the new methods described below, single calculation of Eq.(5) is sufficient to obtain the U^* distribution.

4. Calculation of U^* Using Inverse Structural Stiffness Matrix (Method-1)

4.1 Calculation method

Although the behavior of the structure in Fig.1(a) is expressed by Eq.(5), the elements that relate to a rigid translation remain in this equation. Since Point B is attached to the foundation, the elements that are related to Point B should be removed in this equation, and so we have

$$\begin{Bmatrix} p_1 \\ p_2 \\ \vdots \\ p_A \\ p_C \\ \vdots \\ p_n \end{Bmatrix} = \begin{bmatrix} (11) & (12) & \dots & (1n) \\ (21) & (22) & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & & & (AA) & (AC) \\ \vdots & & & (CA) & (CC) \\ \vdots & & & & \ddots \\ (n1) & \dots & \dots & \dots & (nn) \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ \vdots \\ d_A \\ d_C \\ \vdots \\ d_n \end{Bmatrix} \quad (6)$$

where the elements that relate to Point B are not included. Rewriting the above equation using the inverse matrix, we have

$$\begin{Bmatrix} d_1 \\ d_2 \\ \vdots \\ d_A \\ d_C \\ \vdots \\ d_n \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{21} & C_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & & & C_{AA} & C_{AC} \\ \vdots & & & C_{CA} & C_{CC} \\ \vdots & & & & \ddots \\ C_{n1} & \dots & \dots & \dots & C_{nn} \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \\ \vdots \\ p_A \\ p_C \\ \vdots \\ p_n \end{Bmatrix} \quad (7)$$

where new matrices C with subscripts represent the flexibility matrices that include 3×3 scalar elements. Remembering that the external force is not applied to every element except Points A and C, we extract the following equation from the above equation:

$$\begin{Bmatrix} d_A \\ d_C \end{Bmatrix} = \begin{bmatrix} C_{AA} & C_{AC} \\ C_{CA} & C_{CC} \end{bmatrix} \begin{Bmatrix} p_A \\ p_C \end{Bmatrix} \quad (8)$$

On the other hand, in Eq.(2), the elements related to fixed Point B can be removed, and we have

$$\begin{Bmatrix} p_A \\ p_C \end{Bmatrix} = \begin{bmatrix} K_{AA} & K_{AC} \\ K_{CA} & K_{CC} \end{bmatrix} \begin{Bmatrix} d_A \\ d_C \end{Bmatrix} \quad (9)$$

where the matrix elements in the right-hand side show the internal stiffness. Note that the internal stiffness in Eq.(9) is not the same quantity as the entire structural stiffness shown in Eq.(5). On the other hand, the matrices in Eq.(8) are the elements of an entire structural flexibility matrix. Thus, the types of matrices in Eqs.(8) and (9) are different. Notwithstanding, we find that Eqs.(8) and (9) show the same relation between the deformation and loading. Comparing Eqs.(8) and (9), we can obtain the following relation:

$$\begin{bmatrix} K_{AA} & K_{AC} \\ K_{CA} & K_{CC} \end{bmatrix} = \begin{bmatrix} C_{AA} & C_{AC} \\ C_{CA} & C_{CC} \end{bmatrix}^{-1} \quad (10)$$

The flexibility matrix of the entire structure in Eq.(7) can commonly be determined in a usual FE calculation process. The same is true for the right-hand side of Eq.(10).

The purpose of the present discussion is to obtain the value of each element in the

left-hand side of Eq.(10). We can determine these elements from the right-hand side of Eq.(10). Using the element K_{AC} in the left-hand side of Eq.(10), the value of U^* can easily be obtained from Eq.(3).

Since the above mentioned process can be used for the calculation of every other nodal point without the additional calculation of the structural stiffness matrix, we can expect a notable reduction in the U^* calculation time.

4.2 Reduction of computation time

Using the above method, an FEM code is programmed by the authors. The distribution of U^* for a simple two-dimensional FE model of a flat panel is calculated on every nodal point.

The calculation time of the present method is compared with that of the conventional method mentioned in §3. The results are shown in Fig.4. It is indicated that a significant reduction of the computation time is realized by the use of the present U^* calculation method, and the greater the number of nodes becomes, the more significant the effect of the reduction becomes.

5. Inspection Loading Method (Calculation Method-2)

5.1 Calculation method

Although the newly introduced calculation method described above (Method-1) is highly effective, it requires the inverse matrix calculation of the structural stiffness matrix in Eq.(6). However, in almost every general-purpose FEM solver, such as MSC.Nastran, the inverse matrix calculation is not used to solve Eq.(6).

Since the inverse matrix calculation of the structural stiffness matrix is not necessary in the second method described in the present section, the second method can be applied to the large-scale general-purpose FEM solvers.

As mentioned above, in the FE displacement method, each structural stiffness matrix calculation is necessary whenever the geometrical boundary condition changes. In contrast, a single calculation of the structural stiffness matrix is sufficient to obtain solutions for multiple mechanical boundary conditions. In other words, if it would be possible to replace multiple geometrical boundary conditions in multiple mechanical conditions, the computation time could be reduced.

The purpose of the present method is to convert multiple geometrical boundary conditions into multiple mechanical boundary conditions for the calculation of the internal stiffness K_{AC} .

Points A and B are restrained and an inspection loading is applied to Point C, as shown in Fig.5. Under the present situation, from Eq.(2), the reaction force at Point A is expressed as

$$\{p_A\} = [K_{AC}]\{d_C\} \quad (11)$$

Fig.4 Calculation time for U^* (Method-1)

where $\{d_C\}$ is the column vector of the displacement of Point C that is caused by the inspection loading.

Applying the three independent inspection loadings to Point C, we rewrite Eq.(11) for each case:

$$\{p_A\}_1 = [K_{AC}]\{d_C\}_1, \{p_A\}_2 = [K_{AC}]\{d_C\}_2, \{p_A\}_3 = [K_{AC}]\{d_C\}_3 \quad (12)$$

and the contraction form of these three equations is expressed as

$$[P_A] = [K_{AC}][D_C] \quad (13)$$

where the abbreviated forms

$$[P_A] = [\{p_A\}_1 \{p_A\}_2 \{p_A\}_3], [D_C] = [\{d_C\}_1 \{d_C\}_2 \{d_C\}_3] \quad (14)$$

are used. Finally, from Eq.(13),

$$[K_{AC}] = [P_A][D_C]^{-1} \quad (15)$$

is obtained. The component K_{AC} in Eq.(2) is calculated thusly, and substitution of these values into Eq.(3) yields the value of U^* .

Since the geometrical boundary condition does not change during the process of the three loading conditions, a single calculation of the structural stiffness matrix is sufficient to obtain the solutions.

In the following calculation concerning every other Point C, the geometrical boundary condition remains unchanged. Since the geometrical boundary condition is unchanged throughout the U^* calculation of the entire structure, we conclude that only the single calculation of the structural stiffness matrix is required. Using the above method, we can expect a notable reduction of the U^* calculation time. We call the present method the "inspection loading method".

5.2 Computation time of inspection loading method

The calculation time of the present inspection loading method is compared with that of the conventional method mentioned in §3. The same two-dimensional model of a flat panel as that presented in 4.2 is calculated using MSC.Nastran for the U^* value of every nodal point.

The results are shown in Fig.6. It is shown that the computation time is reduced

Fig.5 Boundary condition for inspection loading method

Fig.6 Calculation time for U^* (Method-2)

Fig.7 Calculation time of truck FE model (Method-2)

Fig.8 U^* distribution of truck FE model

significantly by the use of the present inspection loading method. Moreover, the maximum size of the hard disk is reduced to approximately 1/5.

As another example of the comparison of the computation time, an FE model of an actual heavy-duty truck cab structure⁽⁶⁾ is calculated. The FE model has approximately 68,000 nodes. The truck cab is supported at the upper roof-rails, and the loading is applied to the cab mount bracket.

The calculation time of the present inspection loading method is compared with that of the conventional method. The results are shown in Fig.7.

When we select 200 nodes from the 68,000 nodes for the calculation of U^* , Fig.7 shows that the computation time is reduced to almost 1/13 by the use of the present method.

The calculation results of the conventional method and the inspection loading method are depicted in Figs.8(a) and 8(b), respectively. The differences of the U^* values are at most 0.02%, and these figures show almost the same U^* distribution despite the outstanding improvement in the computation time.

In U^* calculations, general-purpose FEM solvers such as MSC.Nastran are commonly used. In these cases, it is possible to specialize the algorithm for only U^* calculation. Examples⁽³⁾ of this modified process can be seen in the calculation using DMAP for MSC.Nastran. In this case, the calculation time is reduced to almost 1/4 in addition to the effect of the inspection loading method.

6. Conclusions

The parameter U^* is a new concept introduced by the authors for the expression of load transfer and load paths in structures. In a usual calculation of the U^* distribution using FEM, the calculation time greatly exceeds the level of the practical usage. Two calculation methods for the U^* distribution are introduced to reduce the FE computation time. The results of the present study are summarized as follows:

- (1) A new U^* calculation method (Method-1) is introduced. The internal stiffness K_{AC} is obtained from the extracted components of the structural flexibility matrix. Since the

element of K_{AC} of every nodal point can be extracted from the single structural flexibility matrix, the calculation time is significantly reduced.

- (2) The second proposed method (Method-2) is called the inspection loading method. Multiple geometrical boundary conditions are converted to multiple mechanical boundary conditions. This conversion provides a significant benefit to the calculation time in the FE displacement method.
- (3) The inspection loading method of U^* can be used in general-purpose FEM solvers such as MSC.Nastran. Applying the inspection loading method, an actual heavy-duty truck model is calculated. The calculation time is reduced to almost 1/13 by use of the present inspection loading method.

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