Development of Light-Weight Rigid Core Panels*

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Abstract

By processing triangle or square pyramid shaped indents on a flat sheet, panels with periodical indents in regular plane tiling patterns are manufactured. Highly rigid core panels are newly developed by setting this panel (as top panel) on a reversed one (as bottom panel), and gluing/welding them at the apexes of pyramids to the vertexes of the tiling patterns in the bottom panel. The basic model named Dia-Core is a panel created in the form of Octet-Truss developed by Fuller which corresponds to the space filling model consisting of a combination of two tetrahedra and one octahedron. By varying the geometric patterns that appear on the panel surfaces, all possible patterns based on geometrical considerations are devised, systematically creating various modified models with larger welding portions.

Key words: Octet-Truss, Core Structure, Plane Tiling, Space Filling

1. Introduction

Light-weight but highly rigid and strong core materials such as honeycomb cores are indispensable in the constructions of aerospace components and they contribute to further weight saving in the production of motorcar and railway vehicle components. Core materials are promising not only for strengthening member materials, but are exceptionally functional in that they are able to act as acoustic absorbers, noise insulators and heat retainers. Furthermore, the various geometric patterns created by the indents on the surface give these core materials potential to work as aesthetically pleasing architectural components as well as various engineering products, giving rich diversity in design.

Honeycomb core, a classic and major core structure, has developed by improving its materials and adhesives since its first use in an American aircraft in 1949. These cores are produced from slips of aluminum, paper or plastic glued into 3D forms[1]. But because of the use of polymer adhesive, they have disadvantages such as its dependence in strengths on the quality of it, and lack in heat resistance. Troublesome processing procedure bring raise in their cost, resulting in scarce application in a substantial market like architectural members. On the other hand, there are quite a few researches on developments of other types of core structures than the honeycomb. An example of such core models is the Zeta-Core proposed by Miura[2]. But, very few of these models have been in the production line for they can not compete with the honeycomb cores in their strength or stiffness.

In the future, consumed engineering materials are to be reduced in amount to release the environmental load and further weight reduction/cutting will be required in every field of engineering. A cheaper core material is an expected effective weight cutting material, and to achieve this, the above problems are to be cleared by developing simple core models with improved processing technology.

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The main purpose of this research is to develop such core models that are superior to honeycomb core overall: in stiffness, heat resistance and cost performance, etc. Here we propose new core panels created from 2 tongued plates produced by press working. To design the panels, we recall the plane tiling and the space filling models\(^{(3)}\) in classical geometry. The former offers surface patterns on a given core structure, and the later gives necessary condition for the third dimension when constructing the structures into 3D forms\(^{(4)}\).

The present paper consists of the following 3 parts. First, fundamental periodic plane tiling patterns are explained based on the concept of regular, semiregular and duality, and thereafter transformation method from regular to semi-regular patterns is explained while various patterns which appear during the transformations are shown. By these operations, all the geometrically possible plane tiling patterns are revealed. Next, based on the space filling model of tetrahedron and octahedron known as Octet-Truss invented by Buckminster Fuller, a new core panel named Dia-Core is proposed. This core panel is composed of two pieces of the same sheets glued/welded together, with each sheet prepared beforehand by pressing periodical indents in shapes of triangular or square pyramids. Finally, modified core panels giving wider gluing/welding portions are modeled, and some of their samples made of plastics and metal sheets are introduced.

2. Plane tilings

2.1 Regular tilings and Archimedean (semiregular) tilings

Core structures composed of prismatic or polyhedral shaped cells generally show periodical patterns on their surface. For example, the surface pattern of a honeycomb core is regular tiling of hexagons like a beehive. From point of design, the possibilities for drawing patterns that cover a plane are endless. But the number of tiling patterns composed of regular polygons only with every vertex identical is limited. We will explain the tiling patterns with regular polygons that are useful for designing new core structures.

As shown in Fig. 1, tilings with a single kind of regular polygons are referred to as regular tilings: a tiling with triangle, six surrounding each vertex \(\{3,6\}\); square, four around a vertex \(\{4,4\}\); and hexagons, three around a vertex \(\{6,3\}\). Here \(\{p,q\}\) is the Schlafli notation \(p,\) the edge number of a polygon, \(q,\) the number of tiles around a vertex. The regular tiling of \(\{3,6\}\) is dual of \(\{6,3\}\), while \(\{4,4\}\) is self-dual; placing a dot at the centroid of each polygon and connecting the dots with straight lines will allow its dual to emerge.

Relaxing the condition that only one kind of regular polygon is used, but still requiring that each is surrounded identically, 8 Archimedean tilings are obtained (Fig.2). The numbers represent edge numbers of polygons which surround each vertex in the order of arrangement; \([3,6,3,6]\) shown in Fig.2 means that a given vertex is surrounded by \(T, H, T, H\) in this order. We can construct other patterns by combining these 11 tilings or dividing the tiles into smaller ones. But plane tilings composed of regular polygons only with each vertex surrounded identically are limited only to these 11 patterns.
2.2 Transformation of three regular tilings and Archimedean tilings

All Archimedean plane tilings except [3,3,3,4,4] can be produced from three regular tilings of {3,6}, {4,4} and {6,3} (the tiling of [3,3,3,4,4] is excluded for not having rotational symmetry). We will explain the composition methods of these Archimedean plane tilings by organizing transformation of plane tilings. Transformation method is a way of making a semiregular figure from a regular one by varying the shapes of each polygon and vertex little by little. At the point where all edges become equal length, Archimedean tilings emerge, but at other points of transformation, tilings include irregular polygons.

As shown in Fig.3, cut the corners of each triangle in regular tiling of {3,6} (pattern (a)). A small regular hexagon emerges on each vertex of tiling (pattern (b)). Enlarge these hexagons until they touch their neighbors. Change tiling patterns to Archimedean tiling of [3,6,3,6,3] (patterns (b)→(c)→(d)→(e)). Enlarge these hexagons and change their overlapping portions to segments. Hexagons change to dodecagons and the tiling changes to regular tiling of {6,3} through Archimedean tiling of [3,12,12] (patterns (e)→(f)→(g)). Such sequence of operation is called Truncation. Truncating a regular tiling until original polygons change to dots (no area) will enable its dual pattern to emerge. Therefore, inverse transformation from {6,3} to {3,6} is also possible. On the other hand, tiling of {4,4} changes to the same pattern with its positions of vertexes and polygons interchanged from the original one because of self-duality (patterns (h),(i),(j)).

\[ \text{Fig. 3 } \text{Transformation of regular tilings by Truncation. The regular tiling changes to its dual.} \]
\[ \{3,6\} \rightarrow [3,6,3,6,3] \rightarrow [3,12,12] \rightarrow \{6,3\}, \{4,4\} \rightarrow [4,8,8] \rightarrow \{4,4\}. \]

Truncation is a kind of transformation of regular tilings to their dual by regarding vertexes as areas that have no size and enlarging them. On the other hand, we can also construct dual patterns by reducing the areas of polygons. For a regular tiling \{p,q\}, reduce each polygon but still keep its shape and position of centroid. New \(q\)-gons emerge on each vertex and edges of original polygons change to rectangular areas (Fig.4). Continue with this operation. These \(q\)-gons touch their neighbors and the rectangular areas return to segments. At this point, all polygons of original tilings change to vertexes, and their dual tilings emerge. This operation is called Separation. Like truncation, separation to regular tiling of {3,6} is equal to reverse order of {6,3}, and {4,4} changes to the same pattern because of self-duality.

Now, we can transform regular tilings in two different ways, and also use these at the same time. For regular tilings, reduce the areas of the polygons but maintaining the shapes and positions of centroids, cut their corners as shown in Fig.5. Next, connect the corners if

\[ \text{Fig. 4 } \text{The tilings emerge from three regular tilings by Separation. Original polygons become smaller.} \]
\[ \text{New hexagons, squares, and triangles emerge on each vertex of \{3,6\}, \{4,4\}, and \{6,3\}. Edges change to rectangles.} \]
they surround the same vertex of the original tiling. Triangles, squares and hexagons of \{3,6\}, \{4,4\} and \{6,3\} change to the twice-number edged polygons which are hexagons, octagons and dodecagon respectively. Thus, dodecagons, octagons and hexagons newly emerge at the positions of original vertexes and the edges of original tilings form rectangles. During this operation, Archimedean tiling of \{4,6,12\} and \{4,8,8\} emerge when the lengths of all segments become equal.

Fig. 5  The tiling patterns transformed by *Truncation* and *Separation*. All edges of original tilings change to rectangles. (n) Triangles of \{3,6\} become hexagons, and new dodecagons emerge on each vertex. (o) Squares of \{4,4\} become octagons, and new octagons at vertex. (q) Hexagons of \{6,3\} change to dodecagons, and new hexagons at vertex.

By the sequence of *separation* from regular tiling \{p,q\} (Fig.4(k)(l)(m)), \(p\)-gons (changed from original tiles) and \(q\)-gons (emerged on original vertexes) connect at the corners. Punch out the rectangular areas displayed as gray areas in Fig.6 and allow the polygons (displayed by white) to rotate around the connecting corners. The polygons which changed from original tiles can be rotated in the opposite direction from those which newly emerged on the vertexes. The punched out rectangular areas become parallelograms as shown in Fig.6. Continue with this rotation to the point where these parallelogram can be divided into two regular triangles, Archimedean tilings of \[3,3,3,3,6\] and \[3,3,4,3,4\] emerge from \{3,6\} or \{6,3\} and \{4,4\} respectively. These tilings are called *Snub tilings*.

Fig. 6  *Snub* tiling; rectangular areas of Fig.4 (k), (l) and (m) change to parallelograms, and Archimedean tilings \[3,3,3,3,6\] and \[3,3,4,3,4\] emerge when they can be divided into two regular triangles.

As mentioned above, the Archimedean tiling \[3,6,3,6\] is obtained by the *truncation* of the regular tiling of \{3,6\} or \{6,3\}. But there is another way to obtain this tiling pattern. As shown in Fig.7, \{3,6\} consists of three groups of straight lines. Each line is parallel with other lines within their group, and crosses with lines in the other groups by 120 or 60 degrees. Maintain the cross angles but shift the parallel lines of the horizontal group to its perpendicular direction. Then, the triangles which point toward up (triangle (a)) reduce their size but the other triangles which point toward down (triangle (b)) change to hexagons. When these hexagons become regular, Archimedean tiling \[3,6,3,6\] appears. This transformation is a different kind from the others mentioned above because not all polygons of the original regular tilings change equally.

Now we can construct 7 Archimedean plane tilings. The remaining is Archimedean tiling of \[3,3,3,4,4\], but as already mentioned, it has no rotational symmetry so it cannot be

Fig. 7  *Shifting* of horizontal parallel lines to transform to Archimedean tiling from regular tiling of \{3,6\} of \[3,6,3,6\]. Triangles (a) become smaller, and triangles (b) change to hexagons.
created by transforming polygons. Note that these transformations are continuous and that there are general tilings of regular polygons mixed with irregular ones between these 10 tilings.

3. Fundamental models of new core panels

3-1 Space filling models

As regular polygons can tile plane, there are ways to fill space by polyhedra without gaps, known as space filling models. For example, only cubes within platonic polyhedra stack to completely fill space. On the other hand, neither tetrahedron nor octahedron can fill up space without gap by itself regardless of the simplicity in their shapes. But as shown in Fig.8, if we use both of them at the same time, mounting two tetrahedra on two opposing sides of an octahedron, we get the rhombohedron which fills space. More patterns of space fillings are known with the use of semiregular polyhedra. For example, only truncated octahedron fills space by itself, and the rhombic dodecahedron, the dual of the cuboctahedron, can also fill space by itself\(^3\)\(^4\). When we design new core structures, the concept of space filling models is a good guideline to design effective core models, and we must select the best one taking simplicity in forming work and rich stability in structure into account.

![Tetra Octa Rhombohedron](attachment:image)

**Fig. 8**  Space filling model of octahedron and tetrahedron. Rhombohedron (the second right) is made of two tetrahedra and one octahedron. Rhombohedron can fill space (right).

3-2 Fundamental models of new core panel

The frame of the space filling model by octahedra and tetrahedra is known as the Octet-truss invented by Fuller. Octet-truss is a very rigid structure and has been widely used in architectures\(^5\). We will develop new core models based on this space filling models. Figure 9 shows the fundamental models of core panels. Type I panel of the figure has periodical tetrahedral shaped indents on every other tile on a triangular grid. This panel itself is unstable because tetrahedron can not fill space alone. By setting this panel (as top panel) on a reversed one (as bottom panel), and gluing the apexes of pyramids to vertices of the tiling patterns of the bottom panel, octahedral spaces are formed periodically between the two panels. As shown in Fig.8, they fill space without gaps and two panels make up a rigid structure. On the other hand, the panel of Type II in the figure has half of octahedral indents, and by gluing two pieces, hollow tetrahedra emerge.

Each tongued panel can be manufactured by press working and “panelized” by welding. Before panelizing using two pieces, they can be stacked up in a smaller space, for easier transportation. We call Type I “Triangular Dia-Core”, and Type II “Square Dia-Core”.

![Glue/weld](attachment:image)

**Fig. 9** Setting a panel on a reversed panel (bottom one) and gluing at the apexes of pyramids to vertexes of the tiling in bottom panel produce highly rigid core panels. (Type I ; having tetrahedral shaped indents on a triangular grid. Type II ; having half of octahedral shaped indents on a square grid.)
4. Modified models of Dia-Core

4.1 Modified models based on the transformation of the plane tilings

In the preceding section, basic models of Dia-Core consisting of tetrahedral/octahedral indents are introduced. Applying the transformation of plane tiling patterns to these basic models gives a variety of modified models with diversity in design, some of which contribute to easier plastic forming especially at the singular points of the apexes and the indents.

Both top and bottom surfaces of fundamental Dia-Cores have the same pattern of \{3,6\} or \{4,4\}, and the apexes of the pyramids of the bottom panel position at the vertexes of the tiling patterns of the top panel; each vertex of the pattern positions at the centroid of each triangle or square.

As mentioned in §2, regular tilings can be transformed into various kinds of patterns. These transformations change the shape of the original polygons, and new small polygons are produced; the original polygons and the new polygons coexist in the pattern. Transform the patterns of both top and bottom surfaces of triangular Dia-Core at the same time as shown in Fig.10. The tetrahedral indents (dotted line) of Dia-Core change to truncated ones in which all ridges are trimmed. Like in this example, generally the base of an indent changes to another polygon, while a new polygon emerges at the apex on the top panel. Figure 10 shows the initial stage of the truncation from \{3,6\} to \{6,3\}.

![Fig. 10 Transformation of a core and the corresponding changes in the surface patterns. During the transformation, the bottom faces of the indents change from triangles to hexagons, and the top faces of the indents newly emerge on the vertexes as hexagons.](image)

This operation can be applied to other transformed plane tiling patterns mentioned in §2. We will explain possible variation models of Dia-Core, by using these tiling patterns where the tiling patterns labeled (a)~(s) correspond to the surface patterns of core models labeled the same but in capital letters ((A)~(S)).

When we transform surface patterns of Type I according to the pattern (a)$\rightarrow$(b) in Fig.3, the indent shape changes from triangular pyramid model (A) to truncated hexagonal pyramid (model (B) in Fig.11). By this transformation, ridge lines of triangular pyramidal indents change to rectangles. Transform the model according to Fig.3(b)$\rightarrow$(c)$\rightarrow$(d), the shape of the indent changes from truncated hexagonal pyramid (B) to regular hexagonal prism like in honeycomb cores ((C)), then to truncated hexagonal pyramid (D). Continue truncation according to Fig.3(e)$\rightarrow$(f), and the bottom and the top of the indents change from regular hexagons and regular triangles to regular dodecagons and triangles respectively (model (E), (F) in Fig.11). Note that the bottom and the top are interchanged in the models (D), (E), and (F) because the bottom polygons become smaller than the top ones.

At the final stage of truncation, the indent shape changes to the hexagonal pyramid (model (G)), and its surface pattern becomes regular tiling of \{6,3\}. This model has regular tiling on its surface, so we can regard it as the third “regular” (hexagonal) Dia-Core. In fact, we can make the modified models of (A)$\sim$(F) shown in Fig.11, by truncating model (G).

Truncating square Dia-Core (model (H)) becomes truncated octagonal pyramid (model (I) in Fig.11). This model has jointing areas on indents like the triangular model (B) has. By transforming surface patterns according to Fig.3(i)$\rightarrow$(j), the pyramids of indents change to squared prism.
Core models (K), (L) and (M) in Fig. 12 are obtained based on the patterns in Fig. 4. By using the patterns (n), (o) and (p) in Fig. 5, the modified models (N), (O) and (P) newly emerge (Fig. 12 lower left). The core models (Q1), (R) and (Q2) in Fig. 12 are obtained as models corresponding to Snub tilings in Fig. 6.

When we transform the triangular Dia-Core by the method shown in Fig. 7, two patterns of the new models are gained according to which groups of the triangles (labeled (a) or (b)) are chosen as the indents. As shown in Fig. 13, if triangles (a) of Fig. 7 are chosen for the position of indents, the core models with truncated pyramidal indents (S)-a emerge. On the other hand, choose triangles (b) as the indent’s base, models (S)-b emerge. In this model, bottom and top polygons of the indents are hexagons and regular triangles.

5. Sample panels made by PET and Aluminum

Core samples made of plastics and Al alloy sheets were manufactured by press working. Figure 14(a) shows a manufactured PET-made core panel of the model (A) of Fig. 11,
which is the fundamental Dia-Core with tetrahedral indents, and Fig.14(b) is its enlarged photo, showing triangular tiling pattern on its surface. Figure 14(c) is a photo of truncated model (B) in Fig.11 with trimmed ridges, and Fig.14(d) is its enlarged photo. Figure 14(e) shows a sample model with pits on top of the truncated hexagonal pyramid for easier fitting. Figure 14 (f) is a PET-made test panel of the model (D) of Fig.11 and Fig.14(g) shows two of them being glued together. Figure 14(h) is a PET-made test panel of model (L) of Fig.11. These samples are made by vacuum forming. Fig.14 (i) shows the same model of (h) made of aluminum. Figure 14(j) is the model (S)-a shown in Fig.13 made by 0.5mm thick Al alloy sheet. Both samples are made by cold working.

The formation of plastic sheet is very easy because of its good formability under a relatively low heat. On the other hand, because the cold formability of metal is not so good especially in a very thin sheet (~0.05mm), the formation of thin panel is now tried by hot working using superplastic aluminums.

### 6. Conclusion

Based on the space filling model of tetrahedra and octahedra, strong but light weight core panels named Dia-Core were newly devised, and a variety of the modified Dia-Core models were developed by combining the concepts of plane tiling and space filling in classical geometry. Some samples were manufactured using plastics and thin metal sheets.

Compared to current major honeycomb core, Dia-Core has such advantageous properties as,

1. good resistance to high temperature derived from no use of polymer adhesives,
2. high shear resistance with fundamental structure composed of tetrahedra,
3. sufficient rigidity without gluing additional surface plates, and
4. good cost performance due to easy production procedure by pressing machine.

Therefore, these panels are expected to become very competitive light-weight core materials which can be compete with honeycombs overall, when they are in the production line.

### References