Numerical Simulation for Predicting Fatigue Damage Progress in Notched CFRP Laminates by Using Cohesive Elements*

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Abstract
This study proposes the cohesive zone model (CZM) for predicting fatigue damage growth in notched carbon-fiber-reinforced composite plastic (CFRP) cross-ply laminates. In this model, damage growth in the fracture process of cohesive elements due to cyclic loading is represented by the conventional damage mechanics model. We preliminarily investigated whether this model can appropriately express fatigue damage growth for a circular crack embedded in isotropic solid material. This investigation demonstrated that this model could reproduce the results with the well-established fracture mechanics model plus the Paris’ law by tuning adjustable parameters. We then numerically investigated the damage process in notched CFRP cross-ply laminates under tensile cyclic loading and compared the predicted damage patterns with those in experiments reported by Spearing et al. (Compos. Sci. Technol. 1992). The predicted damage patterns agreed with the experiment results, which exhibited the extension of multiple types of damage (e.g., splits, transverse cracks and delaminations) near the notches.

Key words: Composite Material, Fatigue, Delamination, Finite-element Analysis (FEA), Cohesive Zone Model (CZM)

1. Introduction
Carbon-fiber-reinforced composite plastics (CFRPs) have superior mechanical properties, including higher-specific strength and specific stiffness than metal materials(1). Recently, CFRPs have frequently been applied to structural components in the engineering field. In the aeronautical field, CFRP is the most promising candidate for structural materials utilized in next-generation airplanes. Plans are being made to apply it to primary structures. Boeing has announced that as much as 50 percent of the primary structure on the 787 will be made of composite materials(2).

Many stress-concentrated sections exist in a large-scale composite material structure. Such stress concentration in the composite structure induces complex damage(3), especially under cyclic loading. Detailed understanding of the behavior of damage under cyclic loading is required in the engineering application of CFRP.

This study proposes a new simulation to predict the fatigue damage progress in notched laminates. This model analyzes the notched laminate using the layer-wise finite-element model(4). Moreover, we introduce a concept of damage mechanics into the fatigue damage accumulated in the cohesive elements. We applied the proposed simulation to the fatigue
damage in a notched CFRP cross-ply laminate and compared the predicted results with experiment results reported in the literature\textsuperscript{5).}

2. Simulation procedure

2.1 Finite-element method with cohesive elements

This simulation uses a finite-element method based on the principle of virtual work, including the external virtual work of the cohesive traction as given by

\[
\int_V \sigma : \delta E dV + \int_S_{coh} T \cdot \delta E dS = \int_S_T f \cdot \delta u dS, \tag{1}
\]

where \( \sigma \) is stress, \( E \) is the strain tensor of the solid elements, \( T \) is the traction of the cohesive elements, \( f \) is the external force, and \( u \) is the displacement vector.

The damage analysis in this study utilizes the cohesive zone model (CZM) proposed by Geubelle et al\textsuperscript{6).} A cohesive element is inserted into the boundary between two neighboring solid elements (Fig. 1). This element acts as a nonlinear spring that links the solid elements and generates traction that resists an increasing relative displacement (\( \Delta \)) between the two solid elements. The relation between traction \( T \) and relative displacement \( \Delta \) is defined by using the parameter \( s \).

\[
T_i = s \Delta_i \tau_{max} \quad (i = n, t, b) \tag{2}
\]

The subscripts \( n \), \( t \), and \( b \) indicate the cracking modes of normal tension, in-plane shear, and out-of-plane shear, \( \tau_{max} \) is the maximum stress, and \( \Delta_i \) (\( i = n, t, b \)) is the critical relative displacement in each cracking mode. Equation (2) specifies that the parameter \( s \) defines the stiffness of the cohesive element (i.e., the parameter \( s \) represents the residual strength of the element).

\[
\Delta_{ni} = \frac{2G_{ic}}{\tau_{max} s_{ni}}, \quad \Delta_{ti} = \frac{2G_{ic}}{\tau_{max} s_{ni}}, \quad \Delta_{bi} = \frac{2G_{ic}}{\tau_{max} s_{ni}} \tag{3}
\]

Here, \( G_{ic} \) \( (i = I, II, III) \) is the critical energy release rate in each mode, and \( s_{ini} \) is the initial value of parameter \( s \) (\( s_{ini} = 0.999 \) in this study). The residual strength parameter \( s \) is calculated as a function of the normalized relative displacements \( \Delta = \{ \Delta_n/\Delta_{nc}, \Delta_t/\Delta_{tc}, \Delta_b/\Delta_{bc} \}^T \).

Fig. 1. Cohesive element linking two neighboring solid elements.
\[ s = \min \left( s_{\text{min}}, \max \left( 0, 1 - \frac{1}{\Delta} \right) \right) \]  

(4)

The above defined cohesive element acts as a penalty element during small relative displacement (i.e., \( s = s_{\text{ini}} \)), where \( s \) becomes smaller with larger relative displacement, and a cohesive element becomes a crack surface that yields no traction if \( s = 0 \).

The principle of virtual work considering the traction of the cohesive element is then transformed into the matrix form as

\[ (K + M_{\text{coh}})U = f, \]  

(5)

where \( U \) and \( f \) are the nodal vectors of displacements and external forces, \( K \) is the stiffness matrix, \( M \) denotes solid elements, and \( coh \) denotes cohesive elements. Equation (5) is solved by the direct iterative method\(^{(7)}\) for a nonlinear function of \( s \).

2.2 Fatigue evolution law for the residual strength parameters for cohesive elements

In general, fatigue crack growth follows Paris’ law given by

\[ \frac{da}{dN} = C \Delta K^{m}, \]  

(6)

where \( a \) is the crack length, \( N \) is the number of cycles, and \( \Delta K \) is the range of stress intensity factor during the unit cycle. However instead of using Paris’ law, we propose applying damage mechanics to the residual strength of the cohesive element. The damage mechanics model proposed by Kachanov\(^{(8)}\) is then introduced to decrease the residual strength parameter.

\[ \frac{ds}{dN} = -\alpha \frac{f^{\beta}}{s^{\gamma}} \]  

(7)

\[ f = \left( \frac{\tau_a}{\tau_{a\text{max}}} \right)^2 + \left( \frac{\tau_i}{\tau_{i\text{max}}} \right)^2 + \left( \frac{\tau_b}{\tau_{b\text{max}}} \right)^2 \]  

(8)

Here, \( N \) denotes the number of cycles, and the parameters \( \alpha, \beta \) and \( \gamma \) are used to reproduce the decrease of the residual strength parameter and then reproduce the crack growth rate.

2.3 Flowchart

Figure 2 presents a flowchart of the simulation. At first, \( s = s_{\text{ini}} \) is assigned for all cohesive elements. Next, the basic equation (Eq. (5)) is solved with a fixed residual strength of a cohesive element. The residual strength of a cohesive element is then updated based on the relative displacement of cohesive elements (Eq. (4)), and this procedure is iterated until convergence.

These processes are conducted every \( \Delta N \) cycles. The decrease of the residual strength parameter due to cyclic loading was approximated from the solution at the previous step \( N_i \) based on Eq. (7). Equation (7) was applied to each cohesive element with the decreased residual strength parameter (\( s < s_{\text{ini}} \)).

3. Verification

This section applies the proposed fatigue evolution law to a fatigue crack growth that could be analyzed by linear fracture mechanics. We assumed a circular crack with radius \( a \) in an isotropic elastic solid. An initial crack with an 8-mm radius was considered. The region within radius \( b \) was loaded by a constant pressure (\( \sigma_0 \)). The pressure distribution \( \sigma(r) \) was \( \sigma_0 \) for \( 0 \leq r \leq b \) and zero for \( b < r \leq a \). The material properties were assumed to be a Young’s modulus \( E = 70 \text{GPa} \) and Poisson’s ratio \( \nu = 0.3 \). The cracking mode was then pure mode I, and the stress intensity factor \( (K_i) \) was theoretically obtained from the following equation.
Paris’ law ($da/dN = C \Delta K^m$) was also considered for this material ($C = 2.5 \times 10^{-6}$ and $m = 2$). The finite-element model is depicted in Fig. 3. The numerical model was divided by four-node axisymmetric elements, and four-node cohesive elements were inserted into the middle surface. The strength of the cohesive elements was chosen based on the condition that the model shown in Fig. 3 fails at 100 MPa. Moreover, their critical energy release rate $G_{lc}$ was assumed to be 1 kJ/m$^2$. In order to consider the crack growth in pure mode I, large values were assigned to the maximum stress ($t_{\text{max}}$) and the critical energy release rate ($G_{lc}$). Figure 4 plots the growth of a theoretical with initial length $a = 8$ mm when the region within $r \leq 4$ mm was subjected to a maximum load of $\sigma_{\text{max}} = 500$ MPa. This crack growth was then numerically calculated by the proposed simulation. Simulated results of the crack growth are plotted in Fig. 4, along with the theoretical result. The simulated result with $\alpha = 4 \times 10^{-5}$ and $\beta = \gamma = 1.8$ agreed well with the theoretical result. Thus, the crack growth following Paris’ law can be reproduced by the cohesive elements with tuned parameters ($\alpha$, $\beta$, and $\gamma$). In general, if there are curved or multiple cracks in the composite laminates, it is difficult to obtain their stress intensity factor. However, the proposed approach enables us to analyze fatigue crack growth easily.

4. Fatigue damage progress in a notched CFRP laminate

This section predicts the damage progress in a notched CFRP cross-ply laminate subjected to tensile cyclic loading. The predicted progress is then compared to the experiment results reported by Spearing et al.(5). Figure 5(a) illustrates the schematic figure of the notched CFRP cross-ply laminate. Spearing et al. demonstrated that the fatigue damage included splits, multiple transverse cracks, and delamination at the notch tip, similar to static tensile loading. They also concluded that the damage pattern in the notched laminate could be related to the split length.

**Fig. 2. Flowchart for predicting fatigue damage progress.**
Figure 3. Finite-element model for an isotropic solid with a circular crack.

Figure 4. Comparison of theoretical and simulated fatigue crack growth.

Figure 5(b) illustrates the layer-wise finite-element model\(^{(4)}\) for the notched cross-ply laminate, assuming symmetry. The model was 20mm long and 12mm wide. The stacking configuration was [90/0]_S, and each layer was 0.125mm thick. The notched length was set to 4mm.

In this analysis, four-node Mindlin plate elements are used for 0-degree and 90-degree plies. These plate elements are bonded with cohesive elements. Thus, these cohesive elements are used to reproduce the delamination. Moreover, cohesive elements are embedded to simulate the splitting in the 0-degree ply in the x direction at the tip of the notch, and multiple transverse cracks in the 90-degree ply surround the notch. The material properties (Table 1) are derived from Spearing et al\(^{(5)}\). Thermal residual stress due to temperature change in the curing process was also considered.
4.1 Static loading

The damage progress under static loading was simulated. The simulation was conducted with many combinations of strength parameters for the cohesive elements, and the parameters that could explain the experimental result of those simulations were chosen (Table 2). We confirmed that the used values (Table 2) were consistent with the values reported in our previous study. The experiment data presented in a reference is only the relationship between split length and the applied tensile stress in static loading. In addition, splitting is a very important factor for predicting the whole damage in a laminate since it causes delamination. Therefore, we compared the simulated split length with the experiment data. Figure 6 plots the split length versus the applied tensile stress in static loading. The predicted extension of splits approximately agreed with the experiment results.

4.2 Fatigue loading test

The fatigue was simulated with the strength parameters for the cohesive elements found in the static-loading test. The edge along the x-direction was uniformly loaded by the tensile stress of 300MPa as the maximum loading. The incremental number of cyclic loading tests $\Delta N$ was set to 1000.

First, the effects of parameters $\beta$ and $\gamma$ in Eq. (7) on the damage progress were numerically investigated. Figures 7 and 8 present the simulated extension of the splits for various sets of parameters. We can observe a growth rate decrease with increasing $\beta$ in Fig. 7 and growth rate acceleration with increasing $\gamma$ in Fig. 8. In contrast, as seen in Eq. (7), the damage parameter $\alpha$ does not affect the change of growth rate. This result indicates that adjusting $\beta$ and $\gamma$ is useful in expressing the appropriate fatigue damage progress. Hereafter, the three parameters in Eq. (7) were assumed as $\alpha = 1.5 \times 10^{-4}$, $\beta = 7.0$ and $\gamma = 1.2$.

![Layer-wise finite-element model with cohesive elements for a notched cross-ply laminate.](image)

Eight-node cohesive elements for delamination were inserted into all 90°/0° layer interfaces.
Table 1  Material properties of the CFRP laminate.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Young's modulus (GPa)</td>
<td>135</td>
</tr>
<tr>
<td>Transverse Young's modulus (GPa)</td>
<td>9.6</td>
</tr>
<tr>
<td>In-plane shear modulus (GPa)</td>
<td>5.8</td>
</tr>
<tr>
<td>Out-of-plane shear modulus (GPa)</td>
<td>3.5</td>
</tr>
<tr>
<td>In-plane Poisson's ratio</td>
<td>0.31</td>
</tr>
<tr>
<td>Out-of-plane Poisson's ratio</td>
<td>0.49</td>
</tr>
<tr>
<td>Longitudinal thermal expansion coefficient (×10^4K^-1)</td>
<td>0.9</td>
</tr>
<tr>
<td>Transverse thermal expansion coefficient (×10^4K^-1)</td>
<td>36</td>
</tr>
</tbody>
</table>

Table 2  Properties for the cohesive elements.

For splits and transverse cracks

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength (Mode I, MPa)</td>
<td>60</td>
</tr>
<tr>
<td>Shear strength (Mode II and III, MPa)</td>
<td>100</td>
</tr>
<tr>
<td>Critical energy release rate (Mode I, J/m²)</td>
<td>210</td>
</tr>
<tr>
<td>Critical energy release rate (Mode II and III, J/m²)</td>
<td>360</td>
</tr>
</tbody>
</table>

For delamination

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile strength (Mode I, MPa)</td>
<td>30</td>
</tr>
<tr>
<td>Shear strength (Mode II and III, MPa)</td>
<td>68</td>
</tr>
<tr>
<td>Critical energy release rate (Mode I, J/m²)</td>
<td>210</td>
</tr>
<tr>
<td>Critical energy release rate (Mode II and III, J/m²)</td>
<td>600</td>
</tr>
</tbody>
</table>

Fig. 6.  Extension of the splits versus the applied stress in the static tensile tests.

Fig. 7.  Effect of parameter β on the split extension.
Figure 9 plots the split length versus the number of cycles for cyclic loading. The predicted extension of splits approximately agreed with the experiment. The corresponding damage progress is depicted in Fig. 10 for cyclic loading. We can observe the following characteristics in the predicted damage patterns.

1. The splits were generated at the notch tips and extended with an increasing number of loading cycles.
2. Transverse cracks were generated near a notch and were extended as the number of loading cycles increased.
3. Delamination was generated at the notch tips, and extended along the splits in triangular shapes with an increasing number of cycles.

This damage progress agreed well with the experiment result reported by Spearing et al.\(^\text{[5]}\), confirming the validity of the proposed simulation procedure for predicting the fatigue damage progress in CFRP laminates.

In this simulation, the strength parameters for the cohesive elements and the parameters for fatigue damage growth are adjusted to reproduce the damage seen in experiments. Further analysis is required to confirm the validity of these parameters. For example, future study should investigate whether this model can be applied to a laminate with a different stacking sequence or with a composite made of a different type of matrix. Moreover, the fracture mode of fatigue damage is also a very important factor in understanding fatigue damage behavior. These are our future tasks.
5. Conclusion

This study proposed a new simulation for predicting fatigue damage in composite laminates by using the CZM. We introduced a damage-mechanics concept directly into the decrease of residual strength in the cohesive elements. This improvement, along with using a layer-wise finite-element model, enabled us to conduct the simulation and to reduce the computation costs.

In this paper, the simulation was verified by predicting fatigue crack growth in an isotropic solid that can be analyzed by linear fracture mechanics. The damage progress in the notched CFRP laminate was then numerically investigated and was compared with the reported experiment results. The simulated results agreed well with the experiment results reported by Spearing et al., confirming the validity of the proposed simulation procedure for predicting the fatigue damage progress in CFRP laminates.

References


(2) Boeing Company "About the 787 Family". Boeing Company Web site (online), available


