A Method to Eliminate Measurement Error Caused by Residual Strain of Grating in Moiré Interferometry*

Shinichi SUZUKI**, Tomotaka MIYASHITA***, Hideyuki KIMURA**** and Shouhei NISHIKITA**

**Toyohashi University of Technology, Tempaku-cho, Toyohashi, Aichi, 441-8580, Japan, E-mail: shinichi@mech.tut.ac.jp
***YEC, Nishikaizuka, Iwata, Shizuoka, 438-0026, Japan
****JFE Steel Co. Koukan-cho, Fukuyama, Hiroshima, 721-0931, Japan

Abstract
Moiré interferometry is a very sensitive optical method to measure the in-plane displacement of a solid specimen, and is useful to measure small deformation of solids. But, there often appears measurement error caused by the residual strain of the grating that is pasted on a specimen in Moiré interferometry. The present paper applies Moiré interferometry to the measurement of opening displacement of a notch in a plate specimen, and proposes a method to eliminate the error brought by the residual strain of the grating. The proposed method measures the change of the order of Moiré interference fringes with varying the tensile force applied to the specimen, and obtain the residual displacement of the grating and the true opening displacement of the notch. The experimental results show that the opening displacement given by the proposed method is proportional to the square root of the distance from the notch tip. This fact says that the proposed method is successful in eliminating the measurement error caused by the residual strain of grating, and can measure the true COD without any phase-shifting methods.

Key words: Optical Measurement, Moiré Interferometry, Grating, Residual Strain, Crack Opening Displacement, Stress Intensity Factor

1. Introduction
Moiré interferometry is a very sensitive optical method to measure the in-plane displacement and strain of a solid specimen, hence, it is useful to measure small deformation of solids. Recently, Suzuki and Miyashita applied the Moiré interferometry to measure the opening displacement of bifurcated notches. The purpose of their study was associated with rapid crack bifurcation of fast propagating cracks, as follows.

When the crack speed of a fast propagating crack is high enough, the crack bifurcates into two cracks suddenly. In order to figure out the bifurcation mechanism, they measured crack opening displacement (COD), and obtained the energy release rate before and after bifurcation. The COD method to obtain the energy release rate was a well established method for single cracks, but not for bifurcated cracks. Accordingly, Suzuki and Miyashita performed static experiments with bifurcated notches, and proved that the COD method is valid also for bifurcated cracks.

In their static experiments, they successfully measured opening displacement of notches by Moiré interferometry. But, Moiré interferometry often brings measurement error that is
caused by residual strain of the grating pasted on a specimen surface, and the error prevents accurate measurement of displacement.

In Moiré interferometry, a grating is pasted on the specimen surface in the measurement region. Figure 1 shows the process to make a specimen with a grating around a notch tip, which process is the same as that used by Suzuki and Miyashita\(^{(5)}\). First, the specimen with the notch is made (Fig. 1(a)). Next, the very thin grating without any strain is made by aluminum (Al) evaporation onto the mold, as shown in Fig. 1(b), which mold is a phase hologram made by interference of two laser beams. Third, the aluminum grating on the mold is pasted on the specimen surface around the notch tip with liquid adhesive (Fig.1(c)). Lastly, the mold is taken off the aluminum grating, and the grating and adhesive in the notch is removed.\(^{(1)}\) In the last process, strain arises in the aluminum grating on the specimen, which is called residual strain of the grating in the present paper. The residual strain brings the error into the COD measurement by Moiré interferometry.

![Fig. 1 The process to make a specimen with the grating around a notch tip.](image)

When the applied force to the specimen is large enough, the strain of the specimen is much larger than the residual strain of the grating. In that case, Moiré interference fringes give approximately the true strain. But more accurate method is required, when, for example, one tries to confirm the validity of a new theory.\(^{(5)}\)

The present paper proposes a method to eliminate the error on COD measurement of a notch caused by the residual strain of the grating, and shows that the method can measure the COD accurately. The method was used in the study of Ref. (5).

2. Theory of Measurement

2.1 Virtual crack tip and coordinate system

The present study regards a notch as a crack. Linear elastic fracture mechanics assumes that the radius of curvature of a crack tip is zero. On the other hand, the radius of curvature of a notch tip is not zero but finite as shown in Fig. 2. As the result, when one
regards a notch as a crack, the crack tip, which is called virtual crack tip in the present study, exists at the position slightly distant from the notch tip.\(^{(15),(16)}\)

When the single notch is regarded as a crack, the virtual crack tip is on the center line of the notch and is \(\rho/2\) distant from the notch tip, where \(\rho\) is the radius of curvature of the notch tip. In other words, the virtual crack tip exists at the middle point between the notch tip and the center of the curvature of the notch tip.

![Single notch and virtual crack tip](image)

Fig. 2  The position of the virtual crack tip of a single notch. The origin of the coordinate system is at the virtual crack tip.

It is defined that the origin of the coordinate system that describes the displacement field exists at the virtual crack tip. The \(x\)-axis is parallel to the notch, and the \(y\)-axis is perpendicular to it. The symbol \(r\) denotes the distance from the origin of the coordinate system, and \(\theta\) the azimuthal angle.

### 2.2 Moiré interferometry

Figure 3 shows the principle of Moiré interferometry.\(^{(1)}\) The grating is pasted on the specimen surface. The direction of the lines of the grating is perpendicular to the paper plane (in \(x\)-direction). Moiré interferometry measures the displacement of the grating in \(y\)-direction.

Figure 3(a) represents the diffraction of light on the grating. Tensile force is not applied to the specimen, thus there is no strain in the specimen. And the grating has no residual strain. Therefore the grating is not deformed at all.

Two parallel light beams A and B from a laser fall onto the grating obliquely. The angle of the incidence, \(\alpha\), of the two beams are given by the following equation,

\[
\sin \alpha = \frac{\lambda}{f_0},
\]

where \(\lambda\) is the wavelength of the laser beam, and \(f_0\) is the spatial frequency of the grating.

![Moiré interferometry principle](image)

Fig. 3  Principle of Moiré interferometry.
The incident beams A and B are diffracted by the grating. When the angle of incidence $\alpha$ satisfy Eq. (1), the first order diffraction beam $A_d(+1)$ of incident beam A propagates perpendicularly to the grating surface, and the minus first order diffraction beam $B_d(-1)$ of incident beam B also propagates perpendicularly to the grating surface. The two diffraction beams $A_d(+1)$ and $B_d(-1)$ are therefore parallel to each other. Thus the brightness on the grating is uniform and there appears no interference fringe.

Figure 3(b) shows the diffraction of light on the deformed grating. When the tensile force is applied to the specimen, and/or when the residual strain exists in the grating, the spatial frequency of the grating changes to $f_0 + \Delta f'$. Then the two beams $A_d(+1)$ and $B_d(-1)$ are diffracted obliquely to the grating surface. They are not parallel with each other, as the result, there appear interference fringes on the specimen surface. The interference fringes represent the contours of the displacement $v_G$ of the grating in $y$ direction. The displacement $v_G$ is given by the following equation,

$$v_G = \frac{N}{2f_0},$$

where $N$ is the fringe order of the interference fringes. Measuring $N$ at a certain position gives the displacement $v_G$ there.

If there is no residual strain in the grating, the displacement $v_G$ of the grating is equal to the displacement of the specimen. In that case, the interference fringes represent the contours of the displacement of the specimen in $y$-direction.

But, when the grating has residual strain, the displacement $v_G$ of the grating is the sum of the displacement $v$ of specimen and the displacement $v_{GO}$ caused by the residual strain of the grating,

$$v_G = v + v_{GO}.$$  \hspace{1cm} (3)

The present paper calls $v_{GO}$ the residual displacement of the grating. Measuring fringe order $N$ of Moiré interference fringes gives $v_G$ in Eq. (3), not $v$.

### 2.3 Opening displacement of notch and grating

The present study regards a notch as a crack, then, the opening displacement of the notch is denoted by $COD$. Suppose that there are two points P and Q that are at the same $x$-position but on the opposite side of the notch, as shown in Fig. 4. Then Eq.(3) gives the following equations,

$$v_{GP} = v_P + v_{GOP},$$  \hspace{1cm} (4)

$$v_{GQ} = v_Q + v_{GOQ},$$  \hspace{1cm} (5)

where $v_{GP}$ and $v_{GQ}$ are the displacement of the grating at point P and Q, $v_P$ and $v_Q$ are the displacement of the specimen at point P and Q, and $v_{GOP}$ and $v_{GOQ}$ are the residual displacement of the grating at point P and Q, respectively.

---

**Fig. 4** Measurement of opening displacement of notch from Moiré interference fringes.
Subtracting Eq. (5) from Eq. (4), one can obtain the following equations,

\[ \text{COD}_G = \text{COD} + \text{COD}_{GO}, \]  
\[ \text{COD}_G = v_{GP} - v_{GQ}, \]  
\[ \text{COD} = v_P - v_Q, \]  
\[ \text{COD}_{GO} = v_{GOP} - v_{GOQ}, \]

where COD is the true opening displacement of the notch, COD$_G$ is the opening displacement of the grating, and the COD$_{GO}$ is the opening displacement caused by the residual strain of the grating. The present paper calls COD$_{GO}$ the residual opening displacement of the grating. The opening displacement of the grating is the sum of the opening displacement of the specimen and the residual opening displacement of the grating.

### 2.4 Measurement of COD of notch

Substituting Eq. (2) into Eq. (7), one can obtain the following equations on COD$_G$,

\[ \text{COD}_G = v_{GP} - v_{GQ} = \frac{N_P - N_Q}{2f_0} = \frac{\Delta N}{2f_0}, \]  
\[ \Delta N = N_P - N_Q, \]

where $N_P$ and $N_Q$ are the fringe order of the interference fringes at point P and Q. The COD$_G$ of the grating is given through Eq. (10) by measuring the difference $\Delta N$ of the fringe order between point P and Q; see Fig. 4.

The COD of the specimen is proportional to the external force $P$ applied to the specimen,

\[ \text{COD} = aP, \]

where $a$ is a proportional constant. Substituting Eq. (10) and (12) into Eq. (6), one obtains the following equation,

\[ \frac{\Delta N}{2f_0} = aP + \text{COD}_{GO}. \]

Here, $a$ and COD$_{GO}$ are unknowns. The difference of fringe order $\Delta N$ is measured in experiments with varying the applied force $P$, that is, $\Delta N$ is measured as a function of $P$. Then, the unknowns, $a$ and COD$_{GO}$, are determined with the least square method.

The COD can be obtained through the following equation,

\[ \text{COD} = aP = \text{COD}_G - \text{COD}_{GO}. \]

By measuring $P$ and COD$_G$ at various positions along a notch, one can know the true opening displacement along the notch.

### 2.5 Opening displacement and stress intensity factor

The theory of linear elastic fracture mechanics says that the singular stress field of plane stress exists near the crack tip. In the singular stress field of plane stress, the crack opening displacement, COD, is described by the following equation:

\[ \text{COD} = \frac{8}{\pi} \frac{K_I}{\mu} \frac{1}{1 + \eta} \sqrt{r}, \]

where $K_I$ is the stress intensity factor, $\mu$ is the modulus of rigidity, and $\eta$ is the Poisson’s ratio, respectively. When COD at a position $r$ is measured, the stress intensity factor $K_I$ is given by Eq.(15).

Equation (15) says that if COD is measured in the singular stress field of plane stress, the COD is proportional to $\sqrt{r}$. If the COD of the notch that is obtained through the procedure described in the previous sections is proportional to $\sqrt{r}$, one can say that,
(1) The method of COD measurement described in the previous sections is correct,
(2) The measurement was carried out within the singular stress field.

Equation (15) also says that the stress intensity factor, $K_I$, given by the COD measurement must be constant as long as COD is measured in the singular stress field. Accordingly, if the $K_I$ given by the COD measurement of the previous sections is constant, one can again say the two things mentioned above.

3. Experimental Method

3.1 Specimen

Figure 5(a) illustrates the specimen used in the present study. The specimen is 250mm long, 250mm wide and 3mm thick, and is made of PMMA which is the same as that used in Ref. (5). The modulus of rigidity of PMMA is 1200MPa, and the Poisson’s ratio is 0.34. There is a notch from the left boundary toward the center of the specimen. The notch length is 125mm, and the width of the notch is 0.3mm (Fig. 5(b)). The radius $\rho$ of the curvature of the notch tip is 0.15mm, and then $\rho/2$ is 75$\mu$m. A pair of tensile forces is applied at the holes near the upper and lower boundaries of the specimen. The magnitude of the applied force is 108N, 206N, 304N or 402N. The notches were made with a CO$_2$ laser processor.

A diffraction grating is pasted around the notch tip to measure the opening displacement of the notch by Moiré interferometry. The spatial frequency $f_0$ of the grating is 488 lines/mm in the present study.

PMMA is not a perfectly elastic material, but a viscoelastic material. Strain of PMMA increases with time very slowly even if the applied force is kept constant. In order to minimize the effect of the viscoelasticity, all the measurements and photographing are finished within 30 seconds after loading the tensile force to the specimens. This is also the same as Ref. (5).
3.2 Optical system for Moiré Interferometry

Figure 6 illustrates the optical system for Moiré interferometry used in the present study. The light beam emitted from the He-Ne laser passes through lens L1 and L2, and becomes the parallel light beam. The parallel beam is divided into two beams by beam splitter BS. The light beam reflected by the beam splitter becomes collimated beam A, and the light beam transmitted through the beam splitter becomes collimated beam B. The collimated beam A and B in Fig. 6 are the same as those in Fig. 3. The first order diffraction beam of collimated beam A and the minus first order diffraction beam of collimated beam B enter into the camera, which takes the photographs of interference fringes on the specimen. This optical system is the same as that in Ref. [5].

![Optical system for Moiré interferometry](image_url)

4. Results

4.1 Moiré interference fringes

Figure 7 shows examples of Moiré interference fringes around a notch. Figure 7(a) is the photograph when the tensile force \( P \) is zero, no load. The theory of Moiré interferometry says that there appears no interference fringe, and the brightness of the specimen surface must be uniform when \( P=0 \). But, in the photograph, there appear interference fringes, that show the residual strain of the grating pasted on the specimen surface. The residual strain can’t be given only by the photograph of (a), because the orders of the interference fringes are not determined by the photograph only.

Figure 7(b) - (e) are the Moiré interference fringes of the specimen to which the tensile force \( P \) is applied. The magnitude of the tensile force is (b) 108N, (c) 206N, (d) 304N and (e) 402N. The number of interference fringes increases with the increase of the applied force. When the applied force is sufficiently large, the displacement of the specimen is much greater than the residual displacement of the grating. In that case, it is expected that the interference fringes approximately give the true displacement of the specimen.

In Fig. 7(b) – (e), the interference fringes are bent near the notch. This is the effect of the residual strain of the grating shown in Fig. 7(a). The method of the present paper can eliminate the error caused by the bend of the fringes. The present paper measures the difference \( \Delta N \) of the fringe order at the positions 0.5mm distant from the notch.
Fig. 7  Moiré interference fringes around the notch.
4.2 COD measurement

Figure 8 shows the opening displacement $COD_G$ of the grating obtained through Eq. (10) from $\Delta N$ that is the differences of fringe order $N$ of Moiré interference fringes and is measured on the photographs in Fig. 7(b)-(e). The horizontal axis is the distance from the virtual crack tip illustrated in Fig.2, and the vertical axis is $COD_G$. The open circles denote the measured $COD_G$.

![Graph showing CODG versus r](image)

Fig. 8 $COD_G$ versus $r$.

The lines in Fig. 8 were determined from the measured $COD_G$s through the least square method by assuming that $COD_G$ is proportional to $r^\kappa$. When the tensile force $P$ is 108N, the exponent $\kappa$ is 0.663 that deviates from 0.5 of the theoretical value of linear elastic fracture mechanics. The exponent $\kappa$ decreases with the increase of applied force $P$, and is 0.526 when $P$ is 402N. This clearly shows that the exponent $\kappa$ approaches the theoretical value of 0.5 with increasing the applied force $P$.

The above mentioned fact is in agreement with the theoretical prediction that the effect of the residual strain of grating becomes smaller with increasing the applied force $P$, because the deformation of specimen becomes much larger than the residual deformation of the grating in the case of large $P$.

![Graph showing CODG versus applied force P](image)

Fig. 9 $COD_G$ versus applied force $P$. 
Figure 9 shows $COD_G$s as a function of tensile force $P$ at some of the measurement points. The solid circles denote the measured values of $COD_G$. The lines are expressed by Eq. (13), and the unknown coefficients $a$ and $COD_{GO}$ in the equation were determined by the least square method from the measured values of $P$ and $COD_G$.

Figure 9 indicates that Eq. (13) is correct. The $COD_G$ at $P=0$ gives the residual opening displacement $COD_{GO}$ of the grating.

Figure 10 shows the residual opening displacement $COD_{GO}$ of the grating, which brings error to the $COD$ measurement. The colored solid circles denote the positions of $r$ described in Fig. 9. Figure 10 says that the absolute values of $COD_{GO}$ are roughly between 2 and 6$\mu$m on this specimen, and the maximum value of 6$\mu$m is at $r=12$mm.

![Fig. 10](image)

Fig. 10  The error, $COD_{GO}$, brought by the residual strain of grating.

Figure 9 says that the value of $COD_G$ at $r=12$mm is about 22$\mu$m when the tensile force $P$ is 108N. The ratio of $COD_{GO}$ to $COD_G$ is about 27%, which is not a small error. On the other hand, when the tensile force $P$ is 402N, the value of $COD_G$ at $r=12$mm is 97$\mu$m. The ratio of $COD_{GO}$ to $COD_G$ is about 6%. The value of 6% is one fourth as small as that in the case of $P=108$N, however, it may not be negligible in accurate measurement.

![Fig. 11](image)

Fig. 11  The true opening displacement, COD, of a notch versus $r$. 
Figure 11 expresses the true opening displacement, COD, of the notch given by Eq. (14). The horizontal axis is the distance \( r \) from the virtual crack tip, and the vertical axis is COD. The open circles denote the CODs given by Eq. (14), and the lines were obtained by the least square method with assuming that COD is proportional to \( r^\kappa \). The values of exponent \( \kappa \) are constant even if the applied tensile force \( P \) varies, and they are approximately 0.483. The value of 0.483 is very close to the theoretical value of \( \kappa \) of 0.5 given by the linear elastic fracture mechanics. This fact means that

1. The COD measurement was carried out in the region of singular stress field of plane stress, and
2. The method of the present study is correct and useful to eliminate the error brought by the residual strain of the grating and to obtain the true opening displacement, COD.

### 4.3 Stress intensity factor

Figure 12(a) shows the stress intensity factor, \( K_I \), obtained from the CODs of Fig. 11 through Eq. (15). The open circles indicate the measurement values of \( K_I \) and the horizontal lines are the means of the measured \( K_I \) values on each tensile force. The stress intensity factor is nearly constant within the measurement region, 5mm<\( r \)<26mm, under every tensile force. This fact says again that the COD measurement was carried out in the region of singular stress field of plane stress and that the method of present study is available to eliminate the errors of grating’s residual strain and to obtain the true \( K_I \) values.

For comparison, Fig. 12(b) shows the stress intensity factor obtained from CODG. The open circles denote the \( K_I \)s obtained from CODG in Fig. 7, and the lines are the mean values of \( K_I \)s in Fig. 12(a). The measured values of \( K_I \) are not constant but depend on the position \( r \). In particular, when the applied force is the smallest of 108N, the difference of \( K_I \) values between \( r=5\)mm and \( r=26\)mm is more than 20%. It can therefore be said that the two graphs in Fig. 12 clearly show the usefulness of the method proposed by the present study.

### 5. Discussions

#### 5.1 Comparison with phase-shifting method

Phase-shifting Moiré interferometry\(^{(2),(3)}\) is a powerful method to measure in-plane...
displacement of specimen, and is another method to measure the residual strain of the grating. Phase-shifting Moiré interferometry usually uses four pictures of interference fringes that have four different phases. It therefore requires to install the phase shifting devices in the optical system of Moiré interferometry.

On the other hand, the method described in the present paper changes the tensile force applied to the specimen, and obtains the residual strain of the grating and the true $CODs$. Thus the method of the present study can measure the true $COD$ of a notch without any phase shifting devices.

5.2 Accuracy of mold of grating and optical system

There are two other causes that make interference fringes like those in Fig. 7(a), besides the residual strain of the grating.

1. Distortion of the mold of the grating.
2. Distortion of the collimated beam A or B caused by the distortion of optical elements, mirrors or lenses for example, in the optical system of Moiré interferometry.

In order to determine the cause that produced the interference fringes in Fig. 7(a), the present study carried out an additional experiment, in which the mold of the grating is placed at the position of the specimen in the optical system in Fig. 6, instead of the specimen. As the result, no interference fringes appeared on the mold. This means that.

1. Both the mold of the grating and the optical system for Moiré interferometry are accurate enough, and
2. The interference fringes in Fig. 7(a) are certainly caused by the residual strain of the grating pasted on the specimen.

In the present study, the mold of the grating and the optical system was made accurately enough that there appear no interference fringe caused by the distortion of the mold or of the optical elements. However, the method proposed by the present paper can also eliminate the errors caused by the distortions of the mold or of the optical elements, as well as the error due to the residual strain of the grating.

5.3 Effect of thermal deformation of specimen caused by CO$_2$ laser processing

The notch of the specimen was made with CO$_2$ laser in the present study. During the processing, thermal deformation arises in the vicinity of the notch in the specimen. However, the grating is pasted on the specimen with adhesive after making the notch, as illustrated in Fig. 1. Hence, the thermal deformation of the specimen does not generate any distortion on the grating. This was confirmed by another experiment.

5.4 Slight deviation of $\kappa$ from 0.5

Figure 11 shows that the $COD$ is proportional to the 0.483th power of $r$. The measured value of 0.483 is very close to the theoretical one of 0.5, accordingly, it can be said that all the experiments were carried out correctly. But the measured value of 0.483 is slightly smaller than the theoretical value of 0.5. This might be the effect of viscoelasticity of PMMA.

The viscoelastic behavior brings small additional increase, say $\delta$, of $COD$. If $\delta$ were nearly constant along a crack, the exponent $\kappa$ obtained from the $COD$ would become smaller than 0.5 of the theoretical value. The small deviation of $\kappa$ from 0.5 in Fig. 11 might therefore be caused by the viscoelasticity of PMMA. Further details of this problem are discussed in Ref. (5).

6. Conclusions

1. The present paper proposes a method to eliminate the error in the displacement measurement by Moiré interferometry, which error is brought by the residual strain of the grating pasted on a specimen.
2. The proposed method, which measures the change of the order of Moiré interference
fringes with varying the tensile force applied to the specimen, is successful in measurement of the true opening displacement of a notch and to obtain the stress intensity factor of the notch.

(3) The method can measure the true COD without any phase-shifting devices.

Acknowledgement

The authors express our gratitude to Grant in Aid for Scientific Research (14550076) of Ministry of Education, Culture, Sports, Science and Technology, Japan.

References

(12) Suzuki, S., Sakaue, K., Simultaneous photographing of rapidly bifurcating cracks on both surfaces of plate specimens with pulsed holographic microscopy, The 25th International Congress on High-Speed Photography and Photonics, SPIE Vol. 4948 (2003), pp.170-175, SPIE.


