Stress Analysis and Strength Evaluation of Scarf Adhesive Joints with Dissimilar Adherends Subjected to Static Tensile Loadings*

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Abstract
The interface stress distributions in scarf adhesive joints with dissimilar adherends under static tensile loadings are analyzed using two-dimensional and three-dimensional finite element calculations. The effects of the adherends and adhesive Young’s modulus, the scarf angle and the adhesive thickness on the interface stress distributions are examined. In addition, the joint strength is predicted using the interface stress distributions based on the maximum principal stress theory. It is found that when the scarf angle is around 60°, the singular stress at the edges of the interfaces is minimum in the 3-Dimensional FEM calculations. Furthermore, it is noticed that the strength of the joints with dissimilar adherends is smaller than that of the joints with similar adherends. For verification of the FEM calculations, the strains in the adherends and the joint strengths were measured in the experiments. The measured strains are in fairly good agreement with those obtained from FEM calculations. Also, the measured joint strength is fairly consistent with the calculated results.

Key words: Stress Analysis, Interface Stress Distribution, Singular Stress, FEM, Adhesive, Dissimilar Adherends, Tension, Joint Strength, Scarf Adhesive Joint

1. Introduction

Recently, as the performance of adhesive has been enhanced, adhesive joints have been used in some fields of industries such as mechanical structure, automobile, aerospace engineering, wood industry and so on. However, since it is difficult to estimate the joint strength in practice, it is necessary and important to research on the stress distributions and strength evaluation for the adhesive joints under several types of external loadings. One of the merits for adhesive joints is to join different material adherends easily. Thus, adhesive joints with dissimilar adherends are used widely as well as the joints with similar adherends. Some studies have been carried out on the stress and strength evaluation for adhesive joints with similar adherends, such as butt adhesive joints[1-10], lapped adhesive joints[11-18], and scarf adhesive joints subjected to static tensile loadings[19-23]. Suzuki et. al[21,23] have investigated on the interface stress distributions in scarf adhesive joints using two-dimensional (2-D) FEM calculations and they have reported that the singular stresses at the edges of the interfaces vanish when the scarf angle is about 52°. However no research has been carried out on the stress analysis and strength evaluation of scarf adhesive joints with dissimilar adherends. In addition, a few researches have been done on the interface...
stress distributions in the three-dimensions (3-D) for the scarf adhesive joints with dissimilar adherends except the research by Suzuki, et. al [20-23]. Scarf adhesive joints have been used in aircrafts and so on. However, it is important how to determine the optimum scarf angle and adhesive material properties for better joining. It is necessary to know the interface stress distribution and the estimation of joint strength in the scarf adhesive joints with dissimilar adherends subjected to static tensile loadings from a reliable design standpoint.

In the case of dissimilar adherends, the stresses at both the adhesive-adherend interfaces will be different and it is necessary to examine a position where the joint rupture initiates. Furthermore, much research on the stress analysis and strength estimation for adhesive joints under static loadings has been done in two-dimensional analyses. It is necessary to know the interface stress distributions of scarf adhesive joints in the three-dimensional analyses taking into account the stress distributions in the thickness direction. Suzuki et al. showed the 3-D FEM analysis [20]. However, the mesh in their analysis is relatively rough and the detail of the singular stresses at the edges in the thickness direction is not clear.

In this paper, the stress distributions in scarf adhesive joints with dissimilar adherends under static tensile loadings are analyzed using the 2-D and the 3-D finite-element method (FEM) and the difference in the interface stress distributions is shown between the 2-D and the 3-D FEM calculations. The difference in the interface stress distributions between the upper and lower interfaces is also examined. The effects of the adherends and the adhesive Young’s modulus, the scarf angle, and the adhesive thickness on the interface stress distributions are examined. In addition, the above characteristics of the joints with dissimilar adherends are compared with those of the joints with similar adherends. For verification of the FEM calculations, the strains in the joints were measured using strain gauges. Furthermore, the joint strength, indicated by the rupture tensile loading divided by the cross-sectional area of the joint, is estimated with the 3-D FEM calculations in elasto-plastic deformation range. Experiments to measure the rupture tensile loading were also carried out, and the numerical results are compared with the experimental results.

2. FEM Calculations

Figure 1 shows a model for the 3-D FEM calculations of a scarf adhesive joint

![Figure 1 A model for 3-D FEM calculations of a scarf adhesive joint with dissimilar adherends under a static tensile loading](image-url)
subjected to a static tensile loading. The dimensions of the upper and lower adherends are the same while the materials are different. Young's modulus and Poisson's ratio of the upper adherend is denoted by $E_1$ and $\nu_1$, those of the lower adherend by $E_2$ and $\nu_2$, and those of the adhesive by $E_3$ and $\nu_3$. The adherend length is denoted by $h_1$ (see Fig.1), the adherend width and thickness by $w$ and $2t_1$, respectively. The adhesive length and thickness are denoted by $2l$ and $2t_n$, respectively. Half part of the joint is analyzed because the joint is symmetric with respect to $z=t_1$. Cartesian coordinates ($x$-$y$-$z$) and $s$-$n$ coordinates are employed. The origins of $x$-$y$-$z$ and $s$-$n$ coordinate systems are at the same position which is denoted by $o$ (see Fig.1). The boundary conditions applied are as follows: The lower adherend is fixed in the $y$-direction and a tensile loading is applied to the end of the upper adherend.

Figure 2(a) shows an example of mesh divisions of a scarf adhesive joint with dissimilar adherends in the 2-D FEM calculations. The FEM code employed was ANSYS. The total number of nodes and elements employed in the present study were 1891 and 1800, respectively. The smallest element size was chosen as $5\times5\mu m$ at the interfaces between the adhesive and the adherends. Figure 2(b) shows an example of mesh divisions of the scarf.
adhesive joint with dissimilar adherends in the 3-D FEM calculations. The total number of nodes was chosen as 30256 and the total number of elements as 27000. The smallest element size was chosen as 5×5×5µm at the interfaces. SS400 (JIS) mild steel and brass (C2800) were chosen as the adherend materials and epoxy (SUMITOMO 3M Co., Ltd., Scotch-Weld 1838) as the adhesive.

Figure 3 shows the effect of minimal 3-D FEM mesh size on the stress distribution near the left corner of upper interface. The ordinate is the normalized maximum principal stress $\sigma_1/\sigma_0$, where $\sigma_0$ is the average tensile stress, and the abscissa is the common logarithm of the distance $r$ to the corner. It is observed that the difference in the maximum principal stresses between the case of 2µm and the case of 5µm is smaller. Thus, the smallest element size is chosen as 5×5µm in the 2-D FEM models (Figure 2(a)) and as 5×5×5µm in the 3-D FEM models (Figure 2(b)) at the interfaces between the adhesive and the adherends.

3. Experimental Method

Figure 4 shows the dimensions of the specimens used in the experiments for measuring the strains and the joint strengths of scarf joints subjected to static tensile loadings. The adhesive thickness $t_{n}$ of the specimens was chosen as 0.1 mm, the adhesive length $l$ as 32mm and the adherend thickness $t_{1}$ as 9 mm. The material of the specimens was mild steel (SS400, JIS) for the upper and brass (C2800) for the lower adherend. The experiments were carried out after bonding and solidifying a pair of specimens with different materials for eight hours with the epoxy bond (SUMITOMO 3M Co., Ltd., Scotch-Weld™ 1838) at 60°C.

Figure 5 shows the schematic of the experimental apparatus. A compression which was applied to the test jigs was converted to a tensile loading in the experimental setup shown in Fig.5. In the experiments, strains at 3 points at the adhesive layer were measured using strain gauges with a length of 1 mm (KYOWA Electronic Instruments Co., Ltd., KFC-C1-1), and the magnitude of the applied load was measured with a load cell. The output signals were recorded with an oscilloscope through dynamic amplifiers. Furthermore, the rupture loading was also measured.

4. Numerical Results and Comparisons with Experimental Results

4.1. Results of FEM Calculations

4.1.1. Effect of Young’s modulus of adherend $E_2$ on the interface stress distributions at the upper and lower interfaces
Figure 6 shows the upper and lower interface stress distributions obtained from the 2-D FEM calculations when a scarf adhesive joint with dissimilar adherends is subjected to a static tensile loading, where the upper adherend is assumed mild steel ($E_1=209$ GPa), the lower one is assumed brass ($E_2=103$ GPa) and the adhesive is epoxy ($E_3=3.34$ GPa). The ordinate is the normalized maximum principal stress $\sigma_1/\sigma_0$, where $\sigma_0$ is applied average stress and the abscissa is the normalized distance $s/l$ ($-1<s/l<1$). It is observed that the singular stresses occur near the edges of upper and lower interfaces and that the stress singularity at the upper interfaces (the interface with higher Young’s modulus of the adherend) is larger than that at the lower interfaces. So it is assumed that the rupture initiates from the edge of upper interfaces. For this reason, the following discussion will focus on the upper interface.

Figure 7 shows the effects of Young’s modulus of lower adherend $E_2$ on the interface stress distributions at the left edge of upper interfaces obtained from the 3-D FEM calculations, where the scarf angle is chosen as 60° and the adhesive thickness $2t_n$ as 0.1 mm. Young’s modulus of lower adherend $E_2$ varies as 67.8GPa, 103GPa and 209GPa, while Young’s modulus of adhesive $E_3$ and that of the upper adherends $E_1$ are kept constant at 3.34GPa and 209GPa, respectively. The ordinate is the normalized maximum principal stress $\sigma_1/\sigma_0$.

![Figure 6](image1.png)

**Fig.6 Effect of the scarf angle on the normalized upper and lower interface stress distribution obtained from 2-D FEM calculations ($E_1=209$GPa, $\nu_1=0.29$, $E_2=103$GPa, $\nu_2=0.35$, $E_3=3.34$GPa, $\theta=60^\circ$, $2t_n=0.1$mm, $2l=32$mm)**

![Figure 7](image2.png)

**Fig.7 Effect of adherend Young’s modulus on the normalized upper interface stress distribution (maximum principal stress) near the left edge ($s/l=-1$, $z/t=1$) obtained from 3-D FEM calculations ($E_1=209$GPa, $\nu_1=0.29$, $\nu_2=0.35$, $E_3=3.34$GPa, $\nu_3=0.38$, $\theta=60^\circ$, $2t_n=0.1$mm, $2l=32$mm, $t_1=4.5$mm)**

<table>
<thead>
<tr>
<th>$E_2$ (GPa)</th>
<th>2-D FEM</th>
<th>3-D FEM (Left corner)</th>
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<tr>
<td></td>
<td>Left corner</td>
<td>Right corner</td>
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<tr>
<td>67.8GPa</td>
<td>0.529</td>
<td>0.054</td>
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<tr>
<td>103GPa</td>
<td>0.615</td>
<td>0.036</td>
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<tr>
<td>209GPa</td>
<td>0.709</td>
<td>0.020</td>
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Table 1. The values of $K$ and $\lambda$ in the singular stress near the corners of the upper interface ($s/l=-1$, $z/t=1$) obtained from 2-D and 2-D FEM calculations when $E_2$ varies as 67.8GPa, 103GPa and 209GPa ($E_1=209$GPa, $\nu_1=0.29$, $\nu_2=0.35$, $E_3=3.34$GPa, $\nu_3=0.38$, $\theta=60^\circ$, $2t_n=0.1$mm, $2l=32$mm, $t_1=4.5$mm)
stress $\sigma_1/\sigma_0$ and the abscissa is the enlarged normalized distance $s/l$ (-1$<s/l$<-0.95). It is observed that singular stresses and the maximum value of the maximum principal stress occur at the edge of the interfaces. It can also be observed that the singular stresses decrease as the value $E_2$ increases. So it is assumed that the joint strength increases as the Young’s modulus ratio $E_1/E_2$ between two adherends decreases. That is, $E_1/E_2$ approaches 1.

It is found theoretically that the stress distribution at a bi-material wedge corner can be expressed by $\sigma=K\sigma_0^{-\lambda}$ [24], where $\sigma$ is a stress component or principal stress, $K$ is the stress intensity, and $\lambda$ is the index of the singular stress. Theoretically $K$ and $\lambda$ are determined by the material and geometry characteristics, however from a stress distribution obtained from FEM or experiments, the values $K$ and $\lambda$ can be determined. Table 1 shows the values $K$ and $\lambda$ of the normalized principal stress $\sigma_1/\sigma_0$ obtained from the 2-D FEM and 3-D FEM calculations when $E_2$ varies as 67.8, 103 and 209GPa. In the 3-D FEM calculations, the values $K$ and $\lambda$ are shown at the edges of the outer surface ($z=t_1$) and the middle face ($z=0$). The values $K$ and $\lambda$ at the middle face ($z=0$) in the 3-D FEM results seem to approach those at the left edge on the 2-D FEM results. It is considered that the stress singularity in the thickness direction is taken into account in the 3-D FEM calculations.

4.1.2 Effect of adhesive Young’s modulus on the interface stress distributions

Figure 8 shows the effects of the adhesive Young’s modulus $E_3$ on the stress distributions near the left edge of the upper interfaces obtained from the 3-D FEM calculations where the scarf angle is chosen as 60° and the adhesive thickness $2t_n$ as 0.1 mm. The adhesive Young’s modulus $E_3$ varies as 1.67GPa, 3.34GPa and 6.68GPa while the Young’s modulus of adherends $E_1$ and $E_2$ are held constant at 209GPa and 103GPa, respectively. The abscissa is $s/l$ (-1$<s/l$<-0.95). It can be also observed that the singular stress decreases as the adhesive Young’s modulus $E_3$ increases. This indicates that the joint strength increases as the adhesive Young’s modulus $E_3$ approaches adherend Young’s modulus $E_1$ and $E_2$.

4.1.3 Effect of scarf angle on the interface stress distributions

Figure 9 shows the effect of scarf angle on the upper interface stress distributions near the both edges of the interfaces obtained from the 2-D FEM calculations. From the results, it is found that the singular stresses near the edges of the interfaces vanish when the scarf
Figure 10 shows the effect of scarf angle on the upper interface stress distributions near the edges of the interfaces obtained from the 3-D FEM calculations. It is observed from the 3-D FEM calculations that the singular stresses at the edges of interfaces are not vanished when the scarf angle is around 51.86°, the maximum principal stresses at the left edge of the upper interface are the smallest when the scarf angle is around 60° for the joints. So it is assumed that the strength of the joints with dissimilar material adherends subjected to the tensile loading is maximum when the scarf angle is around 60°.

Figure 11 shows the interface stress distribution in the adherend thickness direction (the z direction in Fig. 1) near the edges of the upper and the lower interfaces obtained from 3-D FEM calculations. It is seen that the singular stresses near the edges of the interfaces in the adherend thickness direction. So the singular stresses near the edges of the interfaces obtained from the 3-D FEM calculations are larger than those obtained from the 2-D FEM calculations.

Figure 9 Effect of the scarf angle on the upper interface stress distribution near the edge of the interfaces obtained from 2-D FEM calculations ($E_1=209$ GPa, $v_1=0.29$, $E_2=103$ GPa, $v_2=0.35$, $E_3=3.34$ GPa, $v_3=0.38$, $2t_n=0.1$ mm, $2l=32$ mm)

(a) near the left corner ($s/l=-1$, $z/t=1$) (b) near the right corner ($s/l=1$, $z/t=1$)

Fig. 9 Effect of the scarf angle on the upper interface stress distribution near the edge of the interfaces obtained from 2-D FEM calculations ($E_1=209$ GPa, $v_1=0.29$, $E_2=103$ GPa, $v_2=0.35$, $E_3=3.34$ GPa, $v_3=0.38$, $2t_n=0.1$ mm, $2l=32$ mm)

(a) near the left corner ($s/l=-1$, $z/t=1$) (b) near the right corner ($s/l=1$, $z/t=1$)

Fig. 10 Effect of the scarf angle on the upper interface stress distributions near the edges of the interfaces obtained from 3-D FEM calculations ($E_1=209$ GPa, $v_1=0.29$, $E_2=103$ GPa, $v_2=0.35$, $E_3=3.34$ GPa, $v_3=0.38$, $2t_n=0.1$ mm, $2l=32$ mm, $t_1=4.5$ mm)

(a) near the left corner ($s/l=-1$, $z/t=1$) (b) near the right corner ($s/l=1$, $z/t=1$)
Figure 12 shows the comparison of the maximum principal stress near the right edge of the upper interface between the result obtained from the 3-D FEM calculations when $z=0$ and the result obtained from 2-D FEM calculations (plain strain). It is found that the difference between the two results is not so large. From the comparison of the results between Fig.9 and Fig.10, it can be observed that the singular stress predicted from the 3-D FEM is much larger than that predicted from the 2-D FEM. It is considered that the stress singularity in the $z$ direction is taken account in the 3-D FEM calculations.
4.1.4 Effect of adhesive thickness on the interface stress distributions

Figure 13 shows the effect of the adhesive thickness $2t_n$ on the upper interface stress distributions near the left edge of the interface obtained from the 3-D FEM calculations when the scarf angle is 60°. The adhesive thickness $2t_n$ varies as 0.05, 0.1 and 0.15 mm. The stress singularity at the edge of the interface increases with an increase of the thickness $t_1$. As a result, it is assumed that the rupture tensile loadings increase as the adhesive thickness decreases. This result is the same as the scarf joint with similar adherends under static tensile loadings [19].

4.2 Comparison of Strain Between the Numerical and the Experiment Results

Figure 14 shows the comparison of the strains between the FEM results and the experimental results in the scarf adhesive joint with dissimilar adherends (mild steel and brass) subjected to static tensile loadings. The ordinate is the strain $\varepsilon_y$ in the $y$-direction. The abscissa is the coordinate $s$ (see Fig.1). The black circles ● show the measured strain. The solid line shows the strain distribution along adhesive layer obtained from the 3-D FEM calculation. The configuration of the scarf joint used for comparison of strains between the numerical calculation and the experiment is as follows. The scarf angle was chosen as 52°. The material constants of adherends $E_1$, $\nu_1$, $E_2$ and $\nu_2$ were chosen as 209 GPa, 0.29, 103 GPa and 0.35, respectively. The material constants of the adhesive layer, $E_3$ and $\nu_3$, were chosen as 3.34 GPa and 0.38, respectively. The adhesive thickness was 0.1 mm. In the experiments, the strain ($\varepsilon_y$) was measured at three positions along the adhesive layer (as shown in Fig.14), while in the 3-D FEM calculation, the values of $\varepsilon_y$ were averaged at the corresponding area of the glued strain guages along the adhesive layer. It is found that the 3-D FEM results of the strain are in fairly good agreements with the experimental results.

4.3 Joint Strength

In this study, the value of the exerted tensile stress when the maximum principle stress in the joint reaches the rupture strength of the adhesive is determined as the rupture stress. The joint strength is prospected in this way with the 3-D FEM calculations for several scarf joints with different scarf angles. Then, the experiments to measure the joint strength (the rupture stress) were done for the scarf joints. Figure 15 shows the measured stress-strain curve of the adhesive. The fracture stress of the adhesive is 50.96MPa. The dotted line in

\[ \frac{\text{Fracture stress: 50.96[MPa]}}{\text{Yield stress: 48.07[MPa]}} \]

\[ E=3.34[\text{GPa}] \]

\[ C=0.0439[\text{GPa}] \]

\[ \text{Fig.15 The measured stress-strain curve of epoxy adhesive used in the present study} \]

\[ \text{Fig.16 Comparison of the rupture bending moments between the measured results and the FEM calculated results with the maximum principal stress failure criterion} \]
Fig.15 shows the bi-material model of the adhesive used in the FEM calculations. Figure 16 shows the joint strengths obtained from the 3-D FEM calculations and from the experiments. For each scarf angle, rupture loads for 20 specimens were measured. The scatter in the strength is shown in Fig.16 with a vertical line. While the averaged value of the joint strength for each scarf angle is denoted by the symbol ●. The symbol □ represents the values of joint strength obtained from the 3-D FEM calculations based on the maximum principle stress failure theory. It is seen that the values of the joint strength obtained from the 3-D FEM calculations are a little bit conservative with those measured by the experiments. However, the joint strengths can be estimated more safely using the 3-D FEM calculations.

5. Conclusions

In this paper, the interface stress distributions in scarf adhesive joints with dissimilar adherends subjected to static tensile loadings were calculated using 2-D and 3-D FEM and the joint strengths were estimated in elasto-plastic deformation range. In addition, the effects of some factors were examined on the interface stress distributions. The following results were obtained.

1. The effect of dissimilarity of the adherends on the interfaces stress distribution was investigated. It was found that the singular stress at the edges of the interfaces increased as Young’s modulus of adherends with the higher value increased.

2. The effect of scarf angle of the adherend on the interfaces stress distribution was examined using 2-D and 3-D FEM calculations. The results showed that the singular stress did not vanish when the scarf angle was about 51.86˚ in the 3-D FEM calculations while it was vanished in the 2-D FEM calculations. In addition, the maximum value of the maximum principal stress $\sigma_1$ was the smallest for the present joint when the scarf angle was 60˚ in the 3-D FEM calculation. So it was assumed that the joint strength was maximum for a scarf angle of approximately 60˚. 

3. The effects of Young's modulus of the adhesive, the thickness of the adhesive layer on the interface stress distribution were examined using the 3-D FEM. The results showed that the singular stress at the edge of the interface decreased as the adhesive Young's modulus increased and as the adhesive thickness decreased.

4. From the 3-D FEM results, it was found that the singular stress occurred at the edges of the interfaces in the thickness direction as well as at the edge of the interfaces in the width direction, consequently the singular stress obtained from the 3-D FEM calculations was larger than that obtained from the 2-D FEM calculations.

5. The experiments to measure the strains and the joint strengths were carried out for verification of the FEM calculations. The 3-D FEM results were well consistent with the experimental results.

6. The joint strength was estimated using the elasto-plastic 3-D FEM calculations. The estimated joint strengths were conservative with the experimental results. It was noticed that the joint strength could be predicted based on the maximum principal stress theory. It was also found that the rupture stress was maximum when the scarf angle was around 60˚ in the present scarf adhesive joints.

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References


