Effect of Particle Fragmentation on the Stress Evolution in the Beam Impacted by a Particle Using Hybrid Method*

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Abstract
In this research, collision of a glass bead particle onto a ceramic beam is analyzed by the hybrid method that combines discrete element method (DEM) and finite element method (FEM), developed by the authors. Contact forces between the particle and the beam are analyzed by the Penalty method and Christensen failure criterion is used for dealing with the initiation and propagation of cracks in the glass bead particle. Numerical analysis is executed to examine the initiation and propagation of cracks and fragmentation appearance in the particle, behaviors of stresses of elements at the contact center of the particle and the beam, and stress evolution beneath the surface of the beam by particle impact and its fragmentation.

Key words: Crack Propagation, Fragmentation, Stress Evolution

1. Introduction
A particle impact problem is a crucial one in engineering materials, because those are subjected to impulsive loads, and in some cases suffer erosion. However, brittle solid particle impact problems are not fully understood, because difficulty in monitoring phenomena under impact loading, which occur in extremely short period. Many researchers have dealt with particle impact problems to understand impact erosion mechanisms. Raindrop erosion problems attracted many attentions of many researchers, because aluminum alloy leading edges of aircraft wings and radomes were susceptible to raindrop erosion. N. E. Wahl reviewed researches on raindrop erosion damage published during 1950 to 1965 (1). Raindrop erosion still attracts interests in the field of soil mechanics (2). Recently, S. M. Walley and J. E. Field reviewed researches on particle impact damages done in Cavendish Laboratory, Cambridge University (3). In the review on the raindrop erosion (1), N. E. Wahl highlighted the deformation of the raindrop after collision onto a target material quoting F. P. Bowden & T. Tabor’s research and H. J. Brunton’s research and explained that side flow of the raindrop after the collision could produce the difference in damages caused by raindrop impact and particle impact. On the other hand, in the review of the particle impact damage by
Walley and Field, the main attention was paid onto damage of the target material, but not particle itself. As mentioned by Wahl \(^{(1)}\), deformation or side flow of a raindrop after the collision must have a significant effect on the particle impact damage. One of the authors carried out the experimental researches on damage of ceramics by particle impact \(^{(4)}\). He also carried out research on raindrop impact onto glass fiber reinforced plastics – GFRP \(^{(5-7)}\). The damage of the material as whitening zone was detected in GFRP subjected to repeated raindrop impact. This white zone consists of micro-cracking, and debonding of the fibers and the matrix. The raindrops were imitated by nylon balls. The nylon ball was broken by each impact test.

Concerning the erosion caused by particle impact, behaviors of the particle after collision onto the target materials are crucial to understand the erosion of the target materials. Fragmentation of the particle in impact problem can change the contact time and the contact pressure between the particle and the target material. Therefore, an effect of the fragmentation of a particle on erosion of the target material may be anticipated without any suspicion. However, researches on this subject have not been carried out yet.

In this research, the hybrid method, which has been developed in the previous paper \(^{(8)}\), is applied to the simulation of a glass bead particle impacted onto a ceramic beam, where the particle is modeled by DEM with seven-element arrangement and the beam is modeled by FEM with triangular elements as shown at Fig.1. In the previous paper, the authors have examined the behavior of elements near contact surface, stress evolution in sub-surface areas and the accuracy of the numerical analysis in the case of elasto-dynamic contact problem by using the hybrid method \(^{(9)}\). In this paper, the hybrid method is used to analyze the behavior of elements near contact surfaces, initiation and propagation of cracks, and fragmentation appearance in the particle, and stress evolution beneath the surface of the target material when particle fragmentation takes place.

![Fig.1 Hybrid method](image)

### 2. Christensen Failure Criterion

The failure criterion, which is bridging the gap between the failure criterion for ductile and brittle materials, has been developed by Christensen \(^{(9)}\) and \(^{(10)}\). This criterion has a quadratic form and it is intended that the criterion should be applicable to a wide range of materials. The stress-based Christensen’s criterion states that failure occurs when the sum of three normalized principal stresses and the normalized octahedral shear stress exceeds the value of one. For general materials, Christensen failure criterion is written in term of principal stresses as follows:

\[
\left(\frac{1}{\sigma_t} - \frac{1}{\sigma_c}\right)(\sigma_1 + \sigma_2 + \sigma_3) + \frac{1}{\sigma_c \sigma_t} \left\{\frac{1}{2} [\sigma_1 - \sigma_2]^2 + [\sigma_2 - \sigma_3]^2 + [\sigma_3 - \sigma_1]^2\right\} \geq 1
\]

where \(\sigma_t\) and \(\sigma_c\) are the tensile and the compressive strength of the material. For brittle materials, an additional criterion should be included, that there is no principal stress shall exceed the tensile strength \(\sigma_t\) and the magnitude of the tensile strength \(\sigma_t\) should be equal or less than a half of the compressive strength \(\sigma_c\). For two-dimensional plane stress brittle materials, Christensen failure criterion is shown in Eqs. (2) to (4).
3. Determination of springs to be broken

Based on the cross-sectional stress calculation method by Gaeni et al. (11), the principal stresses and its directions can be calculated by Eqs. (5) and (6).

\[
\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}
\]  

(5)

\[
\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}
\]  

(6)

Transformation to the principal directions is illustrated in Fig. 2.

Application of the Christensen failure criterion to the DEM model is to break spring connection associated with the criterion. In this case, the springs connecting one element to other elements are six. To determine springs to be broken, the directions of the maximum principal stress \(\sigma_1\) is computed and is compared with the directions of connecting springs. The maximum principal stress direction is used for determining the broken springs. The springs located close to the principal stress direction are selected to be broken, as depicted in Fig. 3.
To determine which spring should be broken, a spring connecting the element ② or ⑤, principal stresses are calculated for the elements ② and ⑤. If the maximum principal stress in the element ② is greater than that in the element ⑤, the spring connecting the element ② is broken. It is postulated that the maximum principal stresses in the center element and the elements ② and ⑤ are not remarkably different in the magnitude and the direction.

4. Material Properties

A glass particle and a ceramic beam are used for the numerical analysis. Mechanical properties of both glass and ceramic materials are summarized in Table 1. Two different types of glass materials, the high and the low strength of glass particles are used in the analysis to examine an effect of particle strength on stress evolution near an impact point.

<table>
<thead>
<tr>
<th></th>
<th>Glass</th>
<th>Ceramic</th>
</tr>
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<tbody>
<tr>
<td>Density (kg/mm³)</td>
<td>2.5E-06</td>
<td>3.2E-06</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>731</td>
<td>290</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.17</td>
<td>0.28</td>
</tr>
<tr>
<td>σc – high strength (MPa)</td>
<td>1200</td>
<td>-</td>
</tr>
<tr>
<td>σt – high strength (MPa)</td>
<td>120</td>
<td>-</td>
</tr>
<tr>
<td>σc – low strength (MPa)</td>
<td>600</td>
<td>-</td>
</tr>
<tr>
<td>σt – low strength (MPa)</td>
<td>60</td>
<td>-</td>
</tr>
</tbody>
</table>

5. Results and discussion

5.1 Model

A problem to be analyzed is a beam with 40.0 mm in length and 4.0 mm in height impacted by a disk particle with 2.0 mm in radius. The beam is discretized by triangular finite elements, and the particle is discretized by discrete elements. The beam consists of 3118 triangular elements and the particle consists of 745 elements with the radius of 0.07 mm. The smallest element size in the subsurface of the beam is 10 μm.

Fig.4 Hybrid method model (DEM for particle and FEM for beam)

Overview of the model for the hybrid method and the close-up view of the element arrangement of DEM model near the contact area are shown in Fig.4. As seen in the close-up view, the boundary of the disk particle cannot be smoothly represented by the arrangement of the discrete elements. This is one of the disadvantages in DEM hexagonal element model. Although the number of elements is increased and the radius of the element is decreased, DEM still encounters difficulty to realize the phenomenon that the contact area gradually increases from the point contact. In this work, three elements are initially contacted with the surface of the beam. The initial contact area is taken to be approximately contact area at the maximum contact pressure (8), which is calculated by Hertz’s formula.
In this impact simulation, the model shown in Fig.4 is numerically solved using 0.001 μsec time step increment. Stresses in the particle are calculated by using the cross-sectional stress calculation method \(^{(11)}\). The discrete element code used in the present paper is basically the same as that developed by a part of the authors. This code can provide considerably precise numerical results as indicated in the previous paper \(^{(12),(13)}\). Behavior of stresses in elements at the contact center of the particle and the beam, initiation and propagation of cracks and fragmentation appearance of the particle, and stress evolution beneath the surface of the beam with the particle fragmentation are numerically analyzed by using the hybrid method.

5.2 Cracking and fragmentation pattern in the particle

In order to understand further the cracking or fragmentation appearance in the particle, the cracking and fragmentation patterns at several steps of the simulation are depicted in Fig.5. The figure at each time step is captured for the impact velocity of 50 m/sec with the low strength of particle. The cracks are initiated at \(t = 0.025\) μsec in the bottom layer of discrete elements of the particle. Then, the cracks grow along whole surroundings of the elements, and at \(t = 0.25\) μsec, three elements in the bottom layer are separated from the second layer from the bottom. The second layer of discrete elements of the particle collides onto the beam surface at \(t = 2.7\) μsec after the elements in the bottom layer have been fragmented. At \(t = 3.0\) μsec, cracks are initiated in the center of the particle. The cracking takes place by reflected waves from the free top of the particle. The fragmentation propagates further to the center of the particle and reaches the top edge of the particle. Finally, the particle is fragmented into many parts.

In Fig.6, the cracking patterns at several time steps are shown for the impact velocity of 50 m/sec and the high strength of particle. Initiation of cracks takes place at \(t = 0.05\) μsec in the bottom layer of the particle. In the high strength of particle, cracking or fragmentation propagates slower than in the low strength of particle. It takes 5.0 μsec for the cracking to reach the center of the high strength of particle as shown in Fig.6. While for the low strength of particle, the cracking propagates to the top edge of the particle within the same time, 5.0 μsec as depicted in Fig.5. The high
strength of particle is also fragmented at $t = 10 \mu$sec. However, the shapes of the main halves of the particle keep clear as compared with those for the low strength of particle.

As shown in the figure, the fragmentation pattern is not symmetric with respect to the vertical centerline of the particle. The asymmetric fragmentation results from the fact that the initial elements in the particle are not exactly located at the symmetric positions with respect to the vertical centerline of the particle.

**5.3 Evolution of the contact stress on the beam by particle fragmentation**

Cracking or fragmentation in the particle during the impact influences the behavior of stress at the contact center of the beam. Stresses at the contact center of the beam with particle fragmentation are shown for impact velocity of 50 m/sec in Fig.7.
Obviously, the fragmentation of the particle can change the contact stress and contact duration between the particle and the beam. As seen in the figure, the stresses at the contact center change drastically by the particle fragmentation. The magnitude of the peak stress is reduced and the stress vibrates according to progress in the particle fragmentation while the particle and the beam are in contact. The magnitude of the peak stress is small when the lower strength of particle is used.

In Fig.8, the displacement at the contact point of the beam is shown as a function of the time. The displacement at the contact point with the particle fragmentation is significantly smaller than that without the particle fragmentation. The lower the strength of the particle, the smaller the displacement at the contact center of the beam, and the smaller the indentation as well. However, the beam still obeys bending though the deflection is small in the magnitude.

In Fig.9, contact stresses on the beam surface are shown at times of 1 and 3 µsec for the high strength of particle. When the particle is not fragmented, the contact stress has the peak of 7650 MPa at the contact center and at the time of 1 µsec. On the other hand, when the particle is fragmented, contact area is extended to 900 µm and the contact stress has the maximum value of 4000 MPa near the contact boundary.
5.4 Stress evolution beneath surface of the beam by particle fragmentation

Cracking or fragmentation in the particle during impact affects not only stress variations at the contact center, but also the stresses beneath the beam surface. Stress components at the position \( \sigma \) beneath the beam surface are shown for with and without the particle fragmentation in the Fig.10. (See Fig.4 for the x and y directions)

Without the particle fragmentation, the stress component \( \sigma_x \) beneath the beam surface is compressive while the stress component \( \sigma_y \) is tensile, but very small. When the particle fragmentation takes place, the stress component \( \sigma_x \) first fluctuates into compression and then suddenly increases up to a high magnitude in tension. The stress \( \sigma_y \) rises into higher tension magnitude than that without the particle fragmentation. Two particles with different strengths are used for the impact analysis with the particle fragmentation. The stresses beneath the beam surface in both the x and y directions are shown in Fig.11.

Fig.10 Stress evolution at the position \( \sigma \) in the sub-surface for the low strength of particle

Fig.11 Effects of particle strength to the stress evolution in the sub-surface of the beam at position \( \sigma \)
Close examination to the stress evolution in the sub-surface of the beam shown in the Fig.11 reveals two different profiles of the stress component $\sigma_x$ at position $\textcircled{1}$. $\sigma_x$ changes rapidly from compression to tension for the high strength particle, while $\sigma_x$ gradually increases with time and then steeply increases to the peak tensile stress for the low strength of particle. The final fragmentation appearances of both the particles with high and low strength after impact are shown also in Fig.11.

Fig.12 Stress evolution in the sub-surface in the different depth from the top surface

To examine an effect of the depth from the beam surface on the stress condition, the stress components at the positions $\textcircled{1}$ and $\textcircled{2}$ are shown in Fig.12. At the position $\textcircled{1}$, both the stress components $\sigma_y$ and $\sigma_x$ are tensile while at the position $\textcircled{2}$, both the stress components $\sigma_y$ and $\sigma_x$ are compressive.

Fig.13 Stress along the depth from the surface

In Fig.13, the stress components in x and y directions are plotted as a function of the depth from beam surface along the line connecting the position $\textcircled{1}$ and $\textcircled{2}$ at $t=4.8 \mu\text{sec}$. It is seen that the stress component $\sigma_x$ changes from tension to compression along the depth.
Search of the position and the time at which the stress component $\sigma_x$ on the beam surface has the maximum value was done. Search result revealed that the stress component $\sigma_x$ attains the maximum value, 2600 MPa, at 200 $\mu$m from the contact center and at the time of 3 $\mu$sec after the impact in case of no fragmentation of the particle. On the other hand, in case of the particle fragmentation, the stress component $\sigma_x$ attained the maximum value of 2900 MPa at the position around 750 $\mu$m distant and from the contact center and at the time of 9 $\mu$sec after the impact in case of fragmentation of the particle. In Figs. 14 and 15, stress components $\sigma_x$ on the beam surface are plotted as a function of the distance from the contact center when the stress component $\sigma_x$ attains the maximum value for both the cases. It should be noted that the stress component in x direction on the beam surface reaches the larger maximum value in case of the particle fragmentation than in case of no fragmentation of the particle.

It is well known that the tensile stress component $\sigma_x$ in x direction may cause ring cracks near the impact surface. The results shown in Figs. 14 and 15 suggest that ring cracks may be initiated at the lower impact velocity in case of the particle fragmentation than in case of no fragmentation of the particle. After the ring cracks are
initiated near the surface, the tensile stress component $\sigma_y$ shown in Fig.13 can bend the initiated cracks to the parallel direction to the surface at the sub-surface. Eventually, material removal is made by coalescence of the bent cracks. This result can provide a good explanation for the experimental result on fracture of a ceramic plate by high speed particle impact (14). The experiment results showed that impact by a glass particle with fragmentation caused the fracture of the ceramic plate, while impact of a steel particle without fragmentation did not cause the fracture of the ceramic plate, when both impacts of the glass and steel particles generated the same peak strain on the back surface.

6. Conclusions

A hybrid method utilizing DEM and FEM is applied to a dynamic contact problem in which a glass bead particle collides onto a ceramic beam at the velocity of 50 m/sec. Contact stresses, contact durations between the particle and the beam, and cracking and fragmentation patterns of the particle are analyzed by this hybrid method. The following conclusions are obtained

- Developed hybrid method can analyze problems on the particle impact onto the beam with particle fragmentation, precisely.
- Cracking and fragmentation of the particle reduce the maximum contact stress between the particle and the beam.
- Particle fragmentation can cause large tensile stress component in the beam axis direction on the beam surface.
- Eventually, the particle fragmentation has a harmful effect on the surface damage of brittle materials such as ceramics.

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References


