Study of Energy Absorption Efficiency for a Few Thin-walled Tubes in Axial Crushing*

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Abstract

With a view to examine the axial crushing behavior of thin-walled structures, which are used as energy-absorbing components of transports under various dynamic compressive loads, some thin-walled circular and regular polygonal tubes were investigated mainly by the FEM simulation. The former examination was focused on the effects of the reinforcement with circular ribs placed at regular intervals in the axial direction and the moderation of load undulations due to corrugated pattern on the load-displacement relation. On the other hand, the latter examination was focused on the effects of the shape of cross section (the number of apexes and the thickness). The absorbed energy per unit volume and some indexes of efficiency were used for the examination of those axially crushing tubes. Especially, the effective range of the interval between ribs, the aspect ratio of rib, the dimensions of corrugation, and the number of apexes were primarily discussed from the view point of the comprehensive efficiency which laid emphasis on the absorbed energy per unit volume.

Key words : Thin-walled Tube, Dynamic Progressive Buckling, Energy Absorption

1. Introduction

Remarkable developments of transports in speed and diffusion have caused severe and many-faceted requirements for the reliability of the machinery under high-speed working conditions. The requirements for safety and environmental issues have grown increasingly and cause the advancement of the shock-absorber's performance on energy-absorbing efficiency and weight reduction. The progressive buckling of the thin-walled circular tube has been studied⁽¹⁾⁻⁽¹⁰⁾ for the understanding of the crushing behavior of shock-absorbing components. The behavior of the altered thin-walled circular tube with ribs⁽¹¹⁾⁻⁽¹³⁾, grooves^{(14),(15)}, corrugated pattern⁽¹⁶⁾⁻⁽¹⁹⁾ or foam filler⁽²⁰⁾⁻⁽²²⁾ has also been studied. Furthermore, the axial crushing behavior of the square tubes (plain one^{(23),(24)} and reinforced ones with ribs, foam filler, and so on^{(25),(26)}) and some other regular polygonal tubes^{(27),(28)} has been studied, too.

In this paper, some thin-walled tubes subjected to dynamic axial compression were investigated to contribute to the determination of design policy mainly by the FEM simulation using the explicit code LS-DYNA⁽²⁹⁾ Ver.960. First, the circular tube with some circular ribs placed at regular intervals in the axial direction was modeled with the hexahedral solid element with 8 nodes, and its axially crushing behavior was calculated with varying the interval between ribs or the dimensions of rib to evaluate the effects of ribs on the energy absorption efficiency. Second, the axially corrugated circular tube was modeled with the 3D-shell element with 4 nodes to evaluate the effects of the corrugated pattern on the moderation of load undulations on the load-displacement relation. Finally, the regular polygonal tubes were also treated to evaluate the effects of the shape of cross section (the number of apexes and the thickness) on the energy absorption efficiency and the buckling mode. In order to evaluate the performance

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of thin-walled tubes, the absorbed energy per unit volume was evaluated together with some indexes of efficiency through the simulations.

2. Indexes of performance of shock absorber

So far, some indexes for the evaluation of the shock absorber's performance have been proposed^{(3),(7),(9)}. The shock absorbing component should have high energy absorption efficiency, the flatness of the resultant load time history to avoid excessive deceleration, and high compressibility to preserve the space for the human without increasing the mass. In this study, the absorbed energy per unit volume e_V , the overshoot of the load η_O , the coefficient of variation of the load η_V and the geometric efficiency (the volumetric efficiency, the stroke efficiency) η_G are defined to evaluate the performance of the thin-walled tubes, as follows:

$$\begin{cases} e_V = E_e / V_e, \\ \eta_O = \frac{P_{\max} - \bar{P}_e}{\bar{P}_e}, \\ \eta_V = \frac{a_e}{\bar{P}_e} \text{ with } a_e = \sqrt{1/N \sum_{i=1}^N (\bar{P}_e - P_i)^2}, \\ \eta_G = \frac{\Delta L_{\lim}}{L}, \end{cases} \quad (1)$$

where E_e and V_e are the absorbed energy up to the displacement ΔL_e and the crushed-part volume at ΔL_e respectively. $P_{\max}$ and $\bar{P}_e$ are the maximum load and the mean load up to ΔL_e (see Fig.1). P_i is the i -th discrete value of the load P , which is obtained from the time history of P by sampling with the time interval t_s , and N is the sampling number up to ΔL_e . L is the initial length of the shock absorber in the axial direction, and $\Delta L_{\lim}$ is the displacement where the load abruptly increases under the densification condition.

e_V is similar to the absorbed energy per unit mass and the mean axial stress. In this study, the crushed part is defined as the domain where the equivalent plastic strain exceeds 6% of its maximum value⁽¹⁰⁾. The definition of η_O is a little different from that in the transient response and is affected by ΔL_e . $P_{\max}$ is defined as the first peak load because the violent fluctuation at the rising part of the load time history is most important for the safety at the crash, while $P_{\max}$ is sometimes defined as the second or third peak load for the real shock absorber because the first peak load is decreased by giving some initial imperfection intentionally⁽⁹⁾. a_e corresponds to the standard deviation in the statistics and depends on t_s . η_V means the relative dispersion of P_i and also depends on ΔL_e . In both the experiment and the numerical simulation, the stroke ΔL_e should be large enough to properly evaluate $\bar{P}_e$ and a_e , while t_s should be small enough for the same reason.

3. Numerical analysis

3.1. Material model

The target materials employed in this study are STKM12A (JIS G 3445: Carbon steel tubes for machine structural purposes), SPCC (JIS G 3141: Cold-reduced carbon steel sheets and strips), and annealed S20C (JIS G 4051: Carbon steel for machine structural use). Those are modeled as isotropic elastic/viscoplastic materials based on the von Mises yield criterion. The dynamic yield stress or the flow stress σ_y is represented in the form of the Johnson-Cook

Fig. 1 Schematic of load-displacement relation

model without thermal coupling⁽³⁰⁾:

$$\sigma_y = \left\{ \sigma_0 + B(\varepsilon_{eq}^p)^{n_J} \right\} \left(1 + C \ln \frac{\dot{\varepsilon}_{eq}^p}{\dot{\varepsilon}_0} \right), \quad (2)$$

where σ_0 is the initial yield stress under the reference plastic strain rate $\dot{\varepsilon}_0$, and ε_{eq}^p and $\dot{\varepsilon}_{eq}^p$ are the equivalent plastic strain and the equivalent plastic strain rate respectively. The concrete values of material constants for STKM12A, SPCC and S20C-A, which were determined by the quasi-static and the high-speed (the nominal strain rate $\dot{\varepsilon}_N \approx 10^3 \text{s}^{-1}$, by the non-coaxial Hopkinson bar method) tension tests, are shown in Table 1. The flow stress–plastic strain relations of those with $\dot{\varepsilon}_0 = 1$ are shown in Fig.2. In this study, the nominal strain rate $\dot{\varepsilon}_N$ was fixed at 0.1s^{-1} to focus the discussion on the influence of geometry, and $\dot{\varepsilon}_{eq}^p$ will reach about 10s^{-1} during the development of wrinkle. The circular tube was made of STKM12A by cutting to be a given thickness, while the polygonal tube was made of SPCC or S20C-A. SPCC tube was made by welding two bended plates together (low precision and low cost), and S20C-A tube by the electric discharge machining (high precision and high cost). The initial yield stress of STKM12A is 2~3 times as large as that of SPCC or S20C-A, and then it is difficult to compare the experimental results between the circular and the polygonal tubes.

Table 1 Material properties of STKM12A⁽¹⁰⁾, SPCC and S20C-A

Material	ρ (kg/m ³)	E (GPa)	ν	σ_0 (MPa)	B (MPa)	n_J	C
STKM12A	7860	208	0.28	345	360	0.11	0.0065
SPCC ($h = 0.5 \text{mm}$)	7860	208	0.28	186	1080	0.65	0.0408
S20C-A	7860	205	0.28	213	1028	0.529	0.0689

Fig. 2 Flow stress–plastic strain relations by Eq.(2) with material constants in Table 1

3.2. Example of numerical analysis

The experiment was conducted by the Instron universal testing machine (25T) under constant velocity condition (500mm/min), and then $\dot{\varepsilon}_N = 0.083 \text{s}^{-1} \approx 0.1 \text{s}^{-1}$ was obtained with the specimen length $L = 100 \text{mm}$. On the other hand, Fig.3(a) shows an example circular tube model of the mean radius $R = 17.65 \text{mm}$ and the thickness $h = 0.9 \text{mm}$ with 33648 elements of the hexahedral solid type. The material is STKM12A. The velocity V , which rises in the shape of sine curve of the amplitude V_0 up to $50 \mu\text{s}$ and becomes constant as shown in Fig.3(a), is applied to the upper rigid block in the $-z$ direction, while the lower end of the lower elastic block is fixed in all directions. In the explicit FEM simulation, it takes enormous time to calculate relatively low speed phenomenon, so that the simulation was conducted with the conditions $V_0 = 5 \text{m/s}$ and $\dot{\varepsilon}_0 = 500 \text{s}^{-1}$, which causes the strain-rate effect equivalent to $\dot{\varepsilon}_N = 0.1 \text{s}^{-1}$. However, it must be careful with using this technique not to cause large inertial effect. The frictional force F_C is calculated with the coefficient of friction μ in the following Coulomb's law⁽²⁹⁾:

$$F_C = \mu N_C, \quad \mu = \mu_k + (\mu_s - \mu_k) e^{-\beta |V_C|}, \quad (3)$$

where N_C is the contact force in the normal direction to the contact surface, and μ_s and μ_k are the static and kinetic coefficients of friction respectively. β is the constant to vary μ continuously with the relative velocity between the contact surfaces, V_C . Table 2 shows the concrete

values used in the simulation. Figure 3(b) shows the comparison of the load P –displacement ΔL relation between the experimental and calculated results. The experimental and calculated results agree well with each other, and both the crushed-tubes show the axisymmetric mode in the progressive buckling. h/R (≈ 0.05) is so large that the buckling occurs in fully plastic. In the case of relatively short circular tube, the plastic buckling load P_{cr}^p is obtained as follows⁽¹⁾:

$$P_{cr}^p = 2\pi R h \sigma_{cr}^p = \frac{2\pi E_t h^2}{\sqrt{3(1-\nu^2)}}, \quad (4)$$

where E_t is the tangent modulus, and σ_{cr}^p is the plastic buckling stress. E_t at the initiating buckling is evaluated to be about 19GPa at $\varepsilon^p \approx 0.001$ by Eqs.(2) and (4).

Table 2 Properties of contact conditions for steel

Type of contact condition	μ_s	μ_k	β (s/m)
Contact between specimen and block	0.14	0.10	0.5
Contact of specimen itself	0.28	0.20	0.5

Fig. 3 Comparison between experimental and calculated results (STKM12A)

4. Thin-walled circular tube with ribs

The axial crushing behavior of the thin-walled circular tube greatly depends on the ratio of thickness to radius and the initial imperfection, and the tendency to crush in the non-axisymmetric buckling mode grows with the decrease of this ratio^{(1), (2), (8)}. The control of the buckling mode is important because e_V in the non-axisymmetric mode is about 20% less than that in the axisymmetric one at the maximum^{(10), (12)}. The reduction of the thickness of component is indispensable to save the machine weight, and then the reinforcing (or reducing) method of the thin-walled component is also important to keep its strength. In this section, the effect of the circular ribs (or grooves), which are placed at regular intervals in the axial direction, was investigated. This alteration will be done by cutting the circular tube on a lathe, for example. Figure 4(a) shows the model of the base plain circular tube, and Figs.4(b) and (c) show the model of the circular tube with n_R ribs and that of the unit tube with only two ribs. h_R , L_R and L_I are the rib thickness, the rib width and the interval between two adjacent ribs. The material is STKM12A.

4.1. Numerical examination by using whole model

All the models are crushed up to 50mm, and the mean load $\bar{P}_e$ is evaluated with $\Delta L_e = 50$ mm. Figures 5(a)~(h) show P – ΔL relations for $n_R = 0 \sim 13$ with $L_R = 4$ mm and $h_R = 2$ mm. The plain circular tube ($n_R = 0$) is crushed in the non-axisymmetric mode with triangular wrinkles (h/R is smaller than that of the model in Fig.3(a)), while any other tube with more than 2 ribs is crushed in the axisymmetric one.

$\bar{P}_e$ gradually increases with n_R while $n_R \leq 9$, and $\bar{P}_e$ for $n_R = 13$ is about 4 times larger than that for any other case. The load fluctuation after the first peak also increases with n_R

Fig. 4 Models of circular tubes with n_R ribs ($L_I = \frac{L - n_R L_R}{n_R - 1}$ in the case of (b))

Fig. 5 Load-displacement relations ($L_R = 4\text{mm}$, $h_R = 2\text{mm}$, $\Delta L_e = 50\text{mm}$)

in $n_R \leq 9$, and only the tendency for $n_R = 13$ is far different from that for any other case. P for $n_R = 13$ is almost constant around 26kN up to $\Delta L \approx 26\text{mm}$ but suddenly fluctuates with an enormous amplitude after that and reaches a peak value of about 101kN. It was found by following up the calculated deformation that the original tube part of the model of $n_R = 13$ was fully crushed until about 26mm, and then the rib parts also began to crush.

Figure 6 shows the formation order and the wrinkle modes of respective models. The models of $n_R = 3 \sim 5$ yield some wrinkles in each interval L_I , and the load fluctuations of the models of $n_R = 3$ and 4 are irregular to a greater or lesser extent, as shown in Figs.5(c) and (d) respectively. This is caused by the formation of the last wrinkle in the residual part of each interval, which is smaller than the length of one wrinkle yielded without the effect of ribs, L_W . L_W is obtained by the classical theory of elastic buckling⁽¹⁾ as follows:

$$L_W = \pi \left\{ \frac{2Rh}{\sqrt{3(1-\nu^2)}} \right\}^{\frac{1}{2}} \approx 10.2\text{mm}. \quad (5)$$

$L_I = 20\text{mm}$ for $n_R = 5$ yields just two wrinkles, while $L_I = 44\text{mm}$ for $n_R = 3$ and $L_I = 28\text{mm}$ for $n_R = 4$ yield short wrinkles lastly. The formation of wrinkle in the short region causes high resistance due to severe constraint condition at both the ends. Each of models of $n_R = 7 \sim 13$ yields only one wrinkle in each interval, and L_I for $n_R = 9$ and 13 are smaller than L_W .

Fig. 6 Wrinkles of circular tubes with n_R ribs ($L_R = 4\text{mm}$, $h_R = 2\text{mm}$, $\Delta L = 50\text{mm}$)

Fig. 7 Results of circular tubes with n_R ribs ($L_R = 4\text{mm}$, $h_R = 2\text{mm}$. e_V is evaluated with $\Delta L_e = 26\text{mm}$. η_O and η_V only for $n_R = 13$ are evaluated with $\Delta L_e = 26\text{mm}$, or else $\Delta L_e = 50\text{mm}$.)

Figure 7(a) shows the comparison of the P - ΔL relation among the models of $n_R = 0, 2, 7, 9$ and 13 , and Fig.7(b) shows the changes of e_V , η_O and η_V with L_I/L_W . In Fig.7(a), the overall magnitude of P and its fluctuation obviously increases with n_R in $n_R \leq 9$, and the result for $n_R = 13$ is quite different from the other results. In Fig.7(b), η_O and η_V in $n_R \leq 9$ are evaluated with $\Delta L_e = 50\text{mm}$, and those only for $n_R = 13$ are evaluated with $\Delta L_e \approx 26\text{mm}$ because this practically means the upper limit of the crushable distance ΔL_{lim} for $n_R = 13$. However, e_V is evaluated with $\Delta L_e = 26\text{mm}$ for all the models because e_V changes with ΔL_e sensitively for a small number of wrinkles. In Fig.7(b), e_V increases with decreasing L_I/L_W or with n_R ; the ratio of increase to the case of plain tube becomes about 28% at $L_I/L_W = 9.04$ ($n_R = 2$) and 520% at $L_I/L_W = 0.39$ ($n_R = 13$), and η_O decreases with L_I/L_W ; the ratio of decrease to the plain tube becomes about 20% at $L_I/L_W = 9.04$ and 89% at $L_I/L_W = 0.39$. η_V also drops about 22% while L_I/L_W changing from 9.83 to 9.04 ($n_R : 0 \rightarrow 2$), but it increases about 21% as compared with the plain tube while L_I/L_W changing from 9.04 to 1.18 ($n_R : 2 \rightarrow 7$), and again it steeply decreases with L_I/L_W (finally decreases about 61%).

For the shock-absorbing component, e_V is required to be large, and η_O and η_V to be small. The alteration of n_R from 0 to 2 improves all the indexes mentioned in Section 2 except η_G , which is required to be large but is hardly capable of estimation with the whole model ($\eta_G \approx 0.26$ only for $n_R = 13$ in Fig.5(h)). Then, the increase of n_R in $2 \leq n_R \leq 7$ improves e_V and η_O but disimproves η_V . Especially, η_V is the most disimproved at $n_R = 7$ ($L_I/L_W \approx 1.18$). Again, all the indexes except η_G are improved in $n_R \geq 7$ ($L_I/L_W \leq 1.18$) with increasing n_R . If η_V and η_G can be disregarded, the increase of n_R is always effective. If η_V is the most important factor, however, n_R should be 2 (only changing the non-axisymmetric mode to the axisymmetric one) or a certain number larger than 9 in this examination. On the other hand, the increase of n_R obviously disimproves η_G .

4.2. Numerical examination with using unit model

Here, the model of the unit circular tube with only two ribs, which corresponds to one span of the whole circular tube with n_R ribs as shown in Figs.4(b) and (c) ($R = 17.45\text{mm}$, $h = 0.5\text{mm}$, $L_R = 4\text{mm}$, $h_R = 2\text{mm}$), was treated to discuss the effects of L_I/L_W on the energy absorbing efficiency. In Fig.4(c), the rib part is assumed to hardly deform as compared with the tube part, and at both the end surfaces the coefficient of friction μ is taken to be 0. As

Fig. 8 Results of unit tubes ($R = 17.45\text{mm}$, $h = 0.5\text{mm}$, $L_R = 4\text{mm}$, $h_R = 2\text{mm}$)

mentioned in the subsection 4.1, η_V has the peak value at $L_I/L_W \approx 1.18$ (≈ 1) in Fig.7(b), and the secondary and following peaks of P in Fig.5(f) or (g) show generally the same level close to that at the first peak. Those figures imply that the buckling behavior of all the succeeding crushing spans will approach to that of the first crushed one as L_I/L_W decreases in $L_I/L_W \leq 1$. The calculation of the whole model with small ribs costs too many computer resources, and calculating one span is enough to study the effects of L_I/L_W in $L_I/L_W \leq 1$.

Figure 8(a) shows $P-\Delta L$ relations at some L_I/L_W in $L_I/L_W \leq 1$ (L_I changes from 3mm to 10mm at intervals of 1mm), and the peak value of P rises about 19% as L_I/L_W decreases. ΔL_{lim} decreases from 7.04mm to 1.77mm together with L_I/L_W . Meanwhile, the elastic buckling load increases and approaches to that calculated for the Euler buckling of elemental strip⁽¹⁾:

$$P_{cr}^e = 2\pi R \left(\frac{\pi^2 D}{L_I^2} \right) = \frac{\pi^3 E R h^3}{6(1-\nu^2) L_I^2} \quad \text{with} \quad D = \frac{E h^3}{12(1-\nu^2)}, \quad (6)$$

where D is the flexural rigidity of shell. P_{cr}^e changes from 25.4kN ($L_I = 10\text{mm}$) to 283kN ($L_I = 3\text{mm}$), while the simulated buckling load changes from 29.2kN to 34.8kN. From Eq.(2), the load at the initial yield becomes $\sigma_y \times 2\pi R h \approx 26.6\text{kN}$ for $\varepsilon^p = 0$ and $\dot{\varepsilon}^p/\dot{\varepsilon}_0 \approx 0.083$. Then, the buckling occurs in fully plastic region at the following load⁽¹⁾:

$$P_{cr}^p = \begin{cases} \frac{\pi^3 E_I R h^3}{6(1-\nu^2) L_I^2} = 2\pi R h \times \frac{\pi^2 E_I}{12(1-\nu^2)} \left(\frac{h}{L_I} \right)^2 & \text{if } L_I \leq L_H \\ \frac{2\pi E_I h^2}{\sqrt{3(1-\nu^2)}} = 2\pi R h \times \frac{E_I}{\sqrt{3(1-\nu^2)}} \left(\frac{h}{R} \right) & \text{otherwise} \end{cases} \quad (7.1)$$

L_H is the threshold value of L_I for the bifurcation between two buckling modes and is obtained by equating the right side of Eq.(7.1) with that of Eq.(7.2). Then,

$$L_H = \frac{\pi}{2} \left\{ \frac{R h}{\sqrt{3(1-\nu^2)}} \right\}^{\frac{1}{2}} = \frac{L_W}{2\sqrt{2}} \approx 3.60\text{mm}. \quad (8)$$

Thus, the plastic buckling stress $\sigma_{cr}^p = P_{cr}^p/(2\pi R h)$ is the function of h/L_I if $L_I \leq L_H$. In Fig.8(a), P_{cr}^p is almost constant in $6\text{mm} \leq L_I \leq 10\text{mm}$ ($0.59 \leq L_I/L_W \leq 0.98$), so that E_I is evaluated to be about 31GPa in that range by Eq.(7.2) and 26GPa for $L_I = 3\text{mm}$ ($L_I/L_W = 0.29$) by Eq.(7.1). In Figs.8(b) and (c), another set of indexes e'_V , η'_O , η'_V , and η'_G for the fully crushing of unit model are introduced as follows:

$$e'_V = \frac{E_{\text{lim}}}{V_I}, \quad \eta'_O = \frac{P_{\text{max}} - \bar{P}_{\text{lim}}}{\bar{P}_{\text{lim}}}, \quad \eta'_V = \frac{a_{\text{lim}}}{\bar{P}_{\text{lim}}} \quad \text{and} \quad \eta'_G = \frac{\Delta L_{\text{lim}}}{L_I}, \quad (9)$$

where E_{lim} is the absorbed energy until P rises abruptly, $V_I (= 2\pi R h L_I)$ is the volume of span, and $a_{\text{lim}} = \sqrt{1/N \sum_{i=1}^N (\bar{P}_{\text{lim}} - P_i)^2}$. In $L_I/L_W \leq 1$, all these indexes except η'_G improve as L_I/L_W decreases, that is, e'_V increases as L_I/L_W decreases, and η'_O and η'_V decrease together with L_I/L_W . As compared with the whole model under the same condition of $L_I/L_W = 0.39$, the absorbed energy per unit volume e'_V ($\approx 214 \times 10^3 \text{kJ/m}^3$) is a little smaller than e_V ($\approx 259 \times 10^3 \text{kJ/m}^3$) of the whole model. This is primarily caused by relatively rough mesh of the whole model and the imperfection of symmetric condition of the unit model. It is difficult to eliminate these problems, and especially the latter must be treated carefully.

Fig. 9 Effect of L_R and h_R on buckling mode ($R = 17.45\text{mm}$, $h = 0.5\text{mm}$, $L_I = 11\text{mm}$)

Fig. 10 Effects of L_R and h_R on e'_V and $\overline{\Delta R}/R$ ($R = 17.45\text{mm}$, $h = 0.5\text{mm}$, $L_I = 11\text{mm}$)

η'_G is required to be large, while it gradually decreases with L_I/L_W . In Fig.8(c), the largest value of η'_G (≈ 0.78) is very close to the semiempirical geometric efficiency η_{GA} (≈ 0.80), which was proposed by Abramowicz and Jones⁽⁶⁾ and is given by the following equation:

$$\eta_{GA} = 1 - \frac{2x_m}{L_W^p} - \frac{h}{L_W^p}, \quad L_W^p = 2 \left(\frac{\pi R h}{\sqrt{3}} \right)^{\frac{1}{2}}, \quad (10)$$

where L_W^p is the length of wrinkle obtained by Alexander's theory, and $L_W/L_W^p \approx 1.28$ for $\nu = 0.28$. x_m is half of the residual length of wrinkle, and Abramowicz and Jones determined $x_m \approx 0.07L_W^p$ from the observation. In Fig.8(c), the decrease of η'_G reaches about 24%. On the other hand, for the whole model of $n_R = 13$ ($L_I/L_W \approx 0.39$), $\eta_G \approx 0.26$ is considerably smaller than η_{GA} , however, the net ratio of the crushed distance to the total length of tube parts except ribs is about 0.54 and is rather close to $\eta'_G \approx 0.65$. It is apparently important for η_G to reduce the total length of ribs.

4.3. Numerical examination on the geometric effects of rib (unit model)

The geometric effects of rib are discussed by changing L_R and h_R (0.4, 0.8, 1.2, and 2mm respectively) with $L_I = 11\text{mm}$, which is long enough to yield one wrinkle. Figure 9 shows the effect of L_R/L_W and h_R/L_W on the buckling mode. The original circular tube crushes in the non-axisymmetric mode as shown in Fig.5(a), so that the unit model with the smallest rib of $L_R = h_R = 0.4\text{mm}$ ($L_R/L_W = h_R/L_W \approx 0.04$) also crushes in the same mode. For the models of $(L_R, h_R) = (2\text{mm}, 0.4\text{mm})$, $(2\text{mm}, 0.8\text{mm})$, the symmetric condition with respect to the middle cross-section of rib breaks with the progress of deformation, and they crush with the behavior of extending outward. The symmetric condition for the model crushing in the non-axisymmetric mode is also not exactly realized, but even the smallest rib of $L_R = h_R = 0.4\text{mm}$ hardly shows inward or outward deformation at its end surfaces until $\Delta L \approx 3.4\text{mm}$, and it keeps overall flatness of the surfaces until $\Delta L_{lim} = 8\text{mm}$ as shown in Fig.9. Then, we compared the calculated results for all the models except those of $(L_R, h_R) = (2\text{mm}, 0.4\text{mm})$, $(2\text{mm}, 0.8\text{mm})$ with each other to examine the effects of the rib geometry. The effect of L_R on the buckling mode is very limited, and all the models of $h_R = 0.4\text{mm}$ ($h_R/L_W \approx 0.04$) crush in the non-axisymmetric mode. However, h_R greatly affects the buckling mode, and the model of $h_R = 1.2\text{mm}$ ($h_R/L_W \approx 0.12$) always crushes in the axisymmetric mode, and that of $h_R = 0.8\text{mm}$ ($h_R/L_W \approx 0.08$) also crushes in the axisymmetric mode if $L_R \geq 0.8\text{mm}$.

Figures 10(a), (b) and (c) show the effects of L_R and h_R on e'_V and the average change of R at the end surfaces at $\Delta L_{lim} = 8\text{mm}$, $\overline{\Delta R}$. In Fig.10(a), e'_V increases together with L_R or h_R for all the models except those of $(L_R, h_R) = (2\text{mm}, 0.4\text{mm})$, $(2\text{mm}, 0.8\text{mm})$ and becomes

Fig. 11 Models of corrugated circular tubes ($R = 17.5\text{mm}$, $h = 0.5\text{mm}$, $L = 100\text{mm}$)

almost constant in $L_R/L_W \geq 0.12$ for each h_R . In Figs.10(b) and (c), $\overline{\Delta R}$ are evaluated only for a few valid cases of the axisymmetric mode. $\overline{\Delta R}/R$ has a minus value for small L_R/L_W or h_R/L_W and then increases with those. Thus, the rib is deformable for small L_R or h_R and its rigidity increases as they increase. To insure the rigidity of rib in this examination, it should be satisfied with $L_R/L_W \geq 0.12$ and $h_R/L_W \geq 0.12$. If n_R is large enough for the whole model,

$$\eta_G = \eta'_G \frac{(n_R-1)L_I}{L} \simeq \frac{\eta'_G}{1+k\left(\frac{L_W}{L_I}\right)}, \quad k = \frac{L_R}{L_W}. \quad (11)$$

By using $L_I/L_W \simeq 0.39$, $\eta'_G \simeq 0.54$ (for the model of $n_R = 13$ in the subsection 4.1) and $k \simeq 0.12$ (that is, $L_R = 1.2\text{mm}$ and $L = 63.6\text{mm}$), η_G becomes about 0.41, which is about 60% larger than $\eta_G \simeq 0.26$ for $L_R = 4\text{mm}$ and $L = 100\text{mm}$.

5. Corrugated circular tube

To improve η_O and η_V , it is also effective to corrugate the tube. Singace and El-Sobky experimentally studied the buckling behavior of corrugated circular tubes of aluminum alloy (HT-30) and PVC⁽¹⁶⁾. They found that annealing the corrugated tube decreased the load fluctuation but also decreased the overall energy absorption. Filling the corrugated tube with the polyurethane foam enhanced the absorbed energy, while increasing the corrugation amplitude decreased it. On the other hand, the effects of the wavelength of corrugation on the buckling mode and the energy absorption were not discussed. In this section, the effects of the amplitude a_w and the wavelength l_w of the sinusoidal corrugation were investigated by using the 3D-shell model, which can represent the complex geometry without enlarging the total number of elements. However, η_G is not treated because the 3D-shell element can't exactly reproduce the extreme deformation of complex geometry with the contact problem such as absolutely crushed tube. a_w was varied in the narrow range of $0.6\text{mm} \leq a_w \leq 1.4\text{mm}$ to examine its effect on the buckling mode with the least reduction of absorbed energy. Figure 11(a) shows an example model of the corrugated circular tubes. The mean radius of corrugated circular tube $R_c(z)$ is given as follows ($z = 0$ at the bottom of tube):

$$R_c(z) = R + a_w \left\{ 1 - \cos\left(\frac{2\pi}{l_w}z\right) \right\}, \quad (12)$$

where R is the mean radius of the original circular tube and is 17.5mm . The thickness h is 0.5mm . If l_w is rather smaller than L_W ($\simeq 10.2\text{mm}$), η_G will disimprove in the same way as mentioned in the previous section, while larger l_w needs large a_w , which leads to the decrease of energy absorption, to affect the buckling mode. Then, l_w was varied around L_W so that $R_c(L) \simeq R$. The simulation was conducted under the conditions $V_0 = 1\text{m/s}$ and $\dot{\epsilon}_0 = 100\text{s}^{-1}$, which causes the strain-rate effect equivalent to $\dot{\epsilon}_N = 0.1\text{s}^{-1}$. l_w and a_w of the numerical models are shown in Table 3. Here, e_V is evaluated with $\Delta L_e = 40\text{mm}$. Figure 11(c) shows the comparison of the P - ΔL relation between the corrugated and the straight tubes. Corrugating the original circular tube with $l_w = 9\text{mm}$ and $a_w = 1\text{mm}$ changes the buckling mode from non-axisymmetric to axisymmetric and considerably decreases the load fluctuation, while the absorbed energy of the corrugated tube is only about half of that of the straight one. This

implies the importance of corrugating the tube with the smallest amplitude which is effective enough to improve η_O and η_V .

Figure 12(a) shows the effects of a_w/L_W on e_V , η_O and η_V . Both η_O and η_V monotonically decrease (or improve) with increasing a_w/L_W . As a_w/L_W increases, the buckling mode changes from non-axisymmetric (triangular) to axisymmetric at $a_w/L_W \approx 0.098$ ($a_w = 1\text{mm}$). e_V decreases with increasing a_w/L_W for each mode and has the maximum value at $a_w/L_W \approx 0.098$ within the treated range of a_w . Figure 12(b) shows the effects of l_w/L_W on e_V , η_O and η_V . In this case, both η_O and η_V disimprove together with l_w/L_W . On the other hand, e_V increases together with l_w/L_W for each mode. e_V of the axisymmetric mode is larger than that of the non-axisymmetric (elliptic) one and has the maximum value at $l_w/L_W \approx 0.88$ ($l_w = 9\text{mm}$). The axisymmetric mode occurs within the narrow range including $0.82 \leq l_w/L_W \leq 0.88$ for $a_w = 1\text{mm}$.

With increasing a_w or decreasing l_w , η_O and η_V improve but e_V disimproves. Thus, simple corrugation is effective only for improving η_O and η_V . Then, e_V is tentatively improved by inclining the amplitude a_w in the axial direction (see Fig.11(b)) as follows:

$$a_w(z) = \frac{a_{wt} - a_{wb}}{L} z + a_{wb}, \quad (13)$$

where a_{wt} and a_{wb} are the amplitudes at the top and the bottom of tube. The magnitude of the bending moment at the outer edge of wrinkle (hinge line) is in proportion to the distance $2a_w(z)$ under the same load. Therefore, the inclined corrugated tube starts to crush at the bottom end ($a_{wb} \geq a_{wt}$), and the crushed region spreads from the bottom to the top. Figure 13(a) shows the comparison of the $P-\Delta L$ relation between the inclined corrugated (Model11) and the plain corrugated (Model3) tubes. At first, P of the inclined corrugated tube is as small as that of the plain corrugated tube but increases as the crushed region goes up. The inclination is surely effective, but the non-axisymmetric mode will occur if the ratio $(a_{wb} - a_{wt})/a_{wb}$ is too large (Model10). e_V increases about 6% at the maximum (Model11) as compared with Model3. Thus, the inclined corrugation is very effective in the design of the crushing behavior such as the generating order of wrinkle and the load history, and e_V also increases a little. On the other hand, η_G is affected by the geometric elements such as the radius of curvature around the hinge line and the thickness h and will be approximated to $\eta'_{GA} = 1 - 2x_m/l_w - h/l_w$ by replacing L_W^p with l_w in Eq.(10) if $l_w \geq L_W$ and the mode is axisymmetric.

6. Regular polygonal tubes

The axial crushing behavior of the regular polygonal tube is affected by the restraint at the ridgeline, and the number of apexes n_a besides the transverse dimension of cross section and the thickness becomes another important parameter^{(27), (28)} for the buckling load of the axially crushing tube. The radius of curvature at the ridgeline moderates the stress concentration⁽²⁷⁾ so that it also affects the behavior but was not treated here to simplify the problem. Firstly, the axial crushing behavior of SPCC tube, which was made by welding two bended plates together, was experimentally investigated. The length of side a at the cross section of tube is determined so that the cross-sectional area A is identical with that of the base circular tube for the specification of the thickness h and n_a , as shown in Fig.14. In this experiment, the internal diameter of the base circular tube is 34.5mm and $h = 0.5$ or 1.2mm ($R = 17.5$ or 17.85mm) so that $A \approx 55$ or 135mm^2 . Then, a is determined by n_a for each regular polygonal tube. The crosshead speed of the testing machine was 500mm/min, and the testing strain rate $\dot{\epsilon}_N$ was about 0.083s^{-1} .

Figures 15(a)~(c) show the deformations of tubes of $h = 1.2\text{mm}$ at $\Delta L = 70\text{mm}$, and Figs.16(a) and (b) show $P-\Delta L$ relations and e_V-n_a relation for the tubes of $h = 1.2\text{mm}$. In Fig.15(a), one can confirm the buckling mode of square tube, but in Figs.15(b) and (c) both the tubes of $n_a = 5$ and 6 didn't yield any clear buckling mode because the accuracy of the bending process in out-of-square angle was low. The absolute error in a is less than 0.05mm for $n_a = 4$ but reaches about 0.38mm for $n_a = 5$ and 0.30mm for $n_a = 6$ at the maximum.

Table 3 Details of corrugated tubes for evaluation of energy absorption efficiency
([†]straight tube: $R = 17.5\text{mm}$, $h = 0.5\text{mm}$, $L = 100\text{mm}$, [‡]Model3: base model)

Model	l_w (mm)	a_w or a_{wb} (mm)	a_{wt} (mm)	Buckling mode (5 wrinkles)	$e_V (\times 10^3 \text{ kJ/m}^3)$
Straight tube [†]	—	—	—	Triangular (all)	134.4
Model1	9	0.6	—	Triangular (all)	58.5
Model2	9	0.8	—	Triangular (all)	46.4
Model3 [‡]	9	1	—	Axisym. (all)	62.6
Model4	9	1.2	—	Axisym. (all)	56.7
Model5	9	1.4	—	Axisym. (all)	45.6
Model6	7.69	1	—	Axisym.→elliptic (4th, 5th)	48.7
Model7	8.33	1	—	Axisym. (all)	60.4
Model8	10	1	—	Elliptic (all)	51.7
Model9	11	1	—	Elliptic (all)	58.0
Model10	9	1	0.5	Axisym.→elliptic (5th)	57.2
Model11	9	1	0.6	Axisym. (all)	66.5
Model12	9	1	0.7	Axisym. (all)	63.7
Model13	9	1	0.8	Axisym. (all)	63.7
Model14	9	1	0.9	Axisym. (all)	63.5

Fig. 12 Results of corrugated circular tubes ($L_W \approx 10.2\text{mm}$, $\Delta L_e = 40\text{mm}$)

Fig. 13 Results of inclined corrugated circular tubes ($L_W \approx 10.2\text{mm}$, $\Delta L_e = 40\text{mm}$)

On the other hand, P - ΔL relations in Fig.16(a) show the tendency of the overall amplitude of P increasing with n_a , as reported in the previous numerical investigation⁽²⁷⁾. Fig.16(b) also shows that of e_V increasing with n_a . To obtain the buckling modes for $n_a = 5$ or 6, S25C-A and S20C-A tubes were made with high accuracy by the electric discharge machining ('-A' means annealed material). In these cases, the absolute error in a is less than 0.05mm as well as that of the SPCC square tube is. Figs.15(e) and (f) show the buckling modes obtained for $n_a = 5$ and 6 respectively. The mode of the pentagonal tube of $h = 1.2\text{mm}$ is a little broken due to the tendency of getting harder to yield the polygonal wrinkle as h grows, while that of the hexagonal tube of $h = 0.5\text{mm}$ is very clear. Thus, the initial imperfection and h greatly affect the buckling mode, and the polygonal tube, of which n_a is larger than 4, also can yield polygonal wrinkles if the process accuracy is high enough, while only the extensional mode has been reported so far. The buckling mode that emerged from this investigation can be also confirmed as one eigenmode by the eigenvalue analysis, and the pattern of polygonal wrinkle changes whether n_a is odd or even as shown in Fig17. The side deforms inward or outward in turn along the periphery, but if n_a is a odd number, two adjacent sides deform inward or outward in succession somewhere, as shown in the gray scale region in Fig.17. In the stacking direction of wrinkles, each side always deforms inward or outward in turn.

Fig. 14 Example cross sections of regular polygonal tubes (unit: mm, $L = 100\text{mm}$)

Fig. 15 Buckling modes of regular polygonal tubes ($L = 100\text{mm}$)

Next, the axial crushing behavior of the regular polygonal tube was studied by the FEM simulation. Because the model without the initial imperfection causes the first wrinkle of the extensional mode only, the initial imperfection is introduced by adding the pseudo-random numbers Δd_i ($i = 1 \sim 3n_n$), which are obtained by the linear congruential generator within $|\Delta d_i| \leq 0.04$ (unit: mm), to the coordinates of n_n nodes in the tube model. The disturbance magnitude is determined so as to be large enough to reproduce the observed buckling mode, which occurs at the least load as compared with any other possible mode, in the error range measured for the actual specimen. Figure 18(a) shows the deformed tube and $P-\Delta L$ relation calculated for the square tube, and Fig.18(b) shows the same results calculated for the hexagonal tube. Those numerical models correspond to the specimens shown in Figs.15(d) and (f) respectively. Both the calculated deformations reproduce the buckling modes obtained for the respective experiments, and the calculated $P-\Delta L$ relations also agree well with the respective experimental ones.

Figures 19(a)~(g) show typical results which were calculated by varying h/R and n_a . In Figs.19(a)~(d), typical buckling modes of $n_a = 7 \sim 9, 12$ are shown in the view through the axial direction. The modes of $n_a = 7$ and 8 clearly obey the patterns in Figs.17(a) and (b) respectively, while the crushing appearance of $n_a = 9$ deviates a little from the symmetry in Fig.17(a). Furthermore, the model of $n_a = 12$ initially shows the extensional (axisymmetric) mode and subsequently shows the triangular mode with ignoring the constraint at ridgelines.

Fig. 16 Examples of experimental results (SPCC, $h = 1.2$ mm, $A \approx 135$ mm²)

Fig. 17 Schematic of polygonal wrinkles

Fig. 18 Examples of calculated results ($h = 0.5$ mm, $A \approx 55$ mm²)

The circular tube model also yields the axisymmetric mode firstly and then yields the triangular mode as shown in Fig.19(e). In Fig.19(f), the models for basic circular tubes of (R, h) = (14.25mm, 0.5mm), (17.5mm, 0.5mm), (21.75mm, 0.5mm), (17.7mm, 0.9mm), and (17.85mm, 1.2mm) were calculated to evaluate the effects of the relative thickness h/R and n_a on e_V . The results for the basic circular tubes are also shown in the figure. e_V increases monotonically with h/R . It also increases with n_a but seems to saturate more than $n_a = 7$. e_V of polygonal tube is larger than that of circular tube if $n_a \geq 7$ or 8. Figure 19(g) shows the e_V - n_a relation for $h/R \approx 0.029$. In this case, e_V has a peak at $n_a = 9$ and then approaches the value of circular tube. n_a for this peak is smaller than that reported in the previous study ($n_a = 12$)⁽²⁷⁾. This difference seems to be caused by the introduce of the initial imperfection, and the obtained realistic buckling mode reduces e_V on the whole.

7. Conclusions

In order to examine the effects of the circular ribs, the corrugation of tube and the regular polygonal tube (the ridgeline) on the efficiency of the energy absorbing component, a series of numerical simulations and experiments for thin-walled circular and regular polygonal tubes of some dimensions was carried out. The circular ribs, which are placed at regular intervals in the axial direction, improves the absorbed energy per unit volume e_V and the overshoot of the load η_O , while the coefficient of variation of the load η_V shows the complex behavior and the geometric efficiency η_G always disimproves. If the interval of ribs is less than the length of one wrinkle yielded without the effect of ribs, all the indexes except η_G will improve. η_G

Fig. 19 Typical buckling modes, e_V - h/R and e_V - n_a relations (S20C-A, $\Delta L_e = 40\text{mm}$)

can be improved to some extent by reducing the dimensions of rib.

Corrugating tube can decrease the load fluctuation but also decreases the overall energy absorption. Moreover, the increase of the corrugation amplitude decreases e_V , while the increase of the corrugation period increases e_V a little in the narrow range. The inclined corrugation is another improving method for e_V , however, the effect is not so large, too. The inclined corrugation enables to design the load history and the order of wrinkle generation.

The regular polygonal tube was also investigated to understand the axial crushing behavior of the general polygonal-tube-type shock absorber. The experiments using the specimen of high process accuracy reveals that the regular polygonal tube also crushes in the specific buckling mode. Both the overall load amplitude and e_V increase with the number of apexes n_a . A series of numerical simulations with varying h/R shows that e_V also increases with h/R (R is the mean radius of the base circular tube). e_V of polygonal tube saturates more than $n_a = 7$ and is larger than that of circular tube if $n_a \geq 7$ or 8. In the e_V - n_a relation of $h/R \approx 0.029$, e_V has a peak at $n_a = 9$ and then approaches the value of circular tube. n_a for this peak is smaller than that reported in the previous study ($n_a = 12$)⁽²⁷⁾. This difference seems to be caused by the introduce of the initial imperfection, and the obtained realistic buckling mode reduces e_V on the whole.

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