Numerical Simulation of Dynamic Deformation in Solid-Fluid System by Monolithic Approach of FEM using Three-Element Solid Model

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Abstract

The interaction problem of solid-fluid systems can be simulated by using unified constitutive models. One such model is the three-element solid model and it is useful for analyzing the dynamics concerning the deformation of the interaction problem. The model is also applied to analyze vibration absorption problems in systems. In this study, the dynamics of a solid-fluid system is numerically analyzed to investigate the absorption problem by using the three-element solid model. The monolithic approach of FEM using the model is applied to simulate the vibration phenomena of an elastic vessel filled with a viscous fluid. The FEM using the three-element solid model is formulated by the dynamic explicit method that is digitized by the central difference method in its time scale. Here, the behavior of the fluid is represented by neglecting the Young’s modulus in the constitutive model. Some material properties are applied to represent the behavior of the vessel and the fluid, and the simulated results are evaluated based on the response variation of the power spectral density for design sensitivity. Then it is known that the dominant factor of the absorber can be analyzed by the monolithic approach of FEM using the three-element solid model. An absorber system based on the analysis of the interaction problem can be optimized by tuning its eigenfrequency.

Key words: Numerical Analysis, Finite Element Method, Damping, Forced Vibration, Three-Element Solid Model, Solid-Fluid System

1. Introduction

Solid-fluid interaction problems can be simulated by the hybrid method or the monolithic method(1)–(4). Two or more dominant equations and constitutive relationships are used in the hybrid method, and total equilibrium is considered in the entire system. Meanwhile, one governing equation and one constitutive relationship is used in the monolithic method. Unified constitutive models can be used in this method, the three-element solid model being one such model. This model can be applied to represent solid systems, fluid systems and solid-fluid systems. By using the three-element solid model, Ogasawara et.al.(6) have already represented nonlinear deformations of biological soft tissues such as muscles with their strain dependency and numerically simulated the anterior bending of the human body. On the other hand, good damping effects of the human body have been studied experimentally for a railway vehicle carbody(7)–(12). These studies have indicated that a solid-fluid system such as the human body is capable of absorbing vibrations. In this study, the dynamics of a solid-fluid system is numerically simulated in order to analyze the absorption problem. A monolithic approach of FEM using the three-element solid model is applied to simulate the vibration phenomena of a
vessel filled with a viscous fluid. FEM using the three-element solid model is formulated by
the dynamic explicit method with a central difference. The behavior of a fluid is represented by
neglecting the Young’s modulus in the constitutive model. The simulated frequency responses
of the system are evaluated by the design sensitivity of the absorber.

2. FEM using Three-Element Solid Model

2.1. Governing Equation

The following energy conservation law is considered in order to formulate FEM for the
simulation of solid-fluid interaction problems:

\[ \dot{U} + \dot{K} = \dot{W} + \dot{Q} \] (1)

Here, \( U, K, W \) and \( Q \) denote the internal energy, kinetic energy, mechanical work, and ther-
mal energy respectively. By using the strain shape function \( B \) and mass matrix \( M \), the
discretized equation of motion is derived in terms of the displacement \( \{u\} \), equivalent nodal
force \( \{f\} \) and symmetricity condition of stress \( \{\sigma\} \) as follows:

\[ [B]^T\{\sigma\} + [M]\{\ddot{u}\} = \{f\} \] (2)

2.2. Three-Element Solid Model for Solid-Fluid System

The schematic of a three-element solid model is shown in Fig.1 with the Young’s modulus
\( E \) and viscous compliance \( C \) as parameters. Here, the subscripts e, v, and ve indicate the
elastic, viscous, and viscoelastic parts respectively.

2.3. Formulation of FEM (13)

In a constitutive relationship, the vector form of the total strain rate \( \dot{\varepsilon} \) is defined as the
sum of the elastic strain rate \( \dot{\varepsilon}^e \) and viscoelastic strain rate \( \dot{\varepsilon}^v \) as follows:

\[ \dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^v \] (3)

Here, the rates of elastic strain \( \dot{\varepsilon}^e \) and viscoelastic strain \( \dot{\varepsilon}^v \) can be written as follows:

\[ \dot{\varepsilon}^e = [E^e]^{-1}[\dot{\sigma}] \] (4)

\[ \dot{\varepsilon}^v = [C][\sigma^v] = [C](\{\sigma\} - \{\sigma^{ve}\}) \] (5)

where \( \{\sigma^v\} \) and \( \{\sigma^{ve}\} \) are the viscous and elastic stresses of the viscoelastic part, respectively,
and \( [E^e] \) is an elastic matrix and \( [C] \) is a viscous compliance matrix.

The model is applied to theoretically represent both solid and fluid regions. The former
can represent the behavior of a solid directly by considering the parameter \( C \) to be 0. The total
strain rate can be redefined from equation (3) and equation (5) as follows:

\[ \lim_{C \to 0} \dot{\varepsilon} = \dot{\varepsilon}^e \] (6)
The latter can represent the behavior of a fluid by neglecting the elastic deformation with the minimized Young’s modulus $E^{ve}$ and maximized Young’s modulus $E^e$. The total strain rate is derived from equation (3) and equation (4):

$$\lim_{E^{ve} \to \infty} \{\dot{e}\} = \{\dot{e}^v\}$$  (7)

Then, the time scale is discretized by the central difference method between $t$ and $t + \Delta t$, and the following FE equation can be derived from the abovementioned equation (2) of motion and the constitutive equation (3).

$$\left( \frac{1}{\Delta t} [M] + \frac{1}{\Delta t} [B]^T [D] [B] \right) \{u'(t + \Delta t)\} = \{f\} + \frac{1}{\Delta t} \{\dot{u} - \dot{u}^{\Delta t}\}$$

$$+ \frac{1}{\Delta t} [B]^T [D] [B] \{u'(t)\} - [B]^T [D] \left( \frac{2}{\Delta t} [E^e]^{-1} \{\sigma'(t)\} + [V] \{\sigma^{ve,t}\} \right)$$  (8)

Here, $[V]$ and $[D]$ are defined as follows with the elastic matrix of the viscoelastic part $[E^{ve}]$.

$$[V] = \left( [I] + \frac{1}{2} [E^{ve}] [C] \Delta t \right)^{-1} [C]$$  (9)

$$[D] = \left( [V] + \frac{2}{\Delta t} [E^e]^{-1} \right)^{-1}$$  (10)

It is difficult to obtain the eigenfrequency of equations (8) - (10). This is because these equation cannot be transformed to solve the eigenvalue problem easily. Then, the transient response is computed using these equations to analyze the problem.

3. Dynamics of Elastic Vessel Filled with Fluid

3.1. Condition of Simulation

3.1.1. FE Model  By using the FEM described in the previous section, the vibration behavior of a vessel filled with fluid is simulated in order to inspect the dynamics of a solid-fluid system and the design problem concerning vibration absorption.

The analysis model shown in Fig.2(a) is a quarter model of a cubic-shape vessel with a thin wall, and the FE mesh shown in Fig.2(b) is used in the simulation; this mesh is divided by using tetra meshes with 4-nodes. This model is comprises 837 nodes and 3498 elements. The fundamental material properties adopted in the simulations are listed in Table 1. The model is perfectly constrained at the lower $x - y$ plane perfectly, and an enforced sinusoidal vibration is added along the $z$ - direction for the fundamental study of the mechanics of the solid-fluid system. The oscillation of the wave is 1mm, and the frequencies and corresponding objectives are listed in Table 2. Here, the time increment $\Delta t$ is $1.0 \times 10^{-6}$s in the computation of FEM.
3.1.2. Problems on Dynamics

In this study, problems concerning the dynamics of the solid-fluid system are analyzed by using power spectral density (PSD) in this paper.

In the first part of the analysis, some frequencies of the wave are inputted to the system for the fundamental observation of the dynamic response of the system, where the response of a commuter vehicle carbody is considered as an example of the dynamics. The application of FEM to the system is validated by inspecting the variety caused by the difference in modulus in the fluid region.

Then, some of the other material properties are changed to inspect their sensitivity for the solid-fluid system for the design of the absorber. The properties changed in the simulation are listed in Table 3(a)-(d). Here, the responses are evaluated by the z-direction mode of the observation point; this is the center of the top surface of the model shown in Fig.2(a) in order to evaluate the fundamental behavior of the vessel.

Table 1 Properties of vessel and liquid.

<table>
<thead>
<tr>
<th>Object</th>
<th>(E', \text{ Pa})</th>
<th>(E'', \text{ Pa})</th>
<th>(C, (\text{Pa} \cdot \text{sec})^{-1})</th>
<th>Poisson ratio</th>
<th>Density, kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vessel</td>
<td>(4.0 \times 10^7)</td>
<td>0</td>
<td>0</td>
<td>0.4</td>
<td>(0.5 \times 10^3)</td>
</tr>
<tr>
<td>Fluid</td>
<td>(2.0 \times 10^7)</td>
<td>0</td>
<td>(1.0 \times 10^3)</td>
<td>0.499</td>
<td>(1.0 \times 10^3)</td>
</tr>
</tbody>
</table>

Table 2 Frequencies and objectives of input wave.

<table>
<thead>
<tr>
<th>Input Frequency, Hz</th>
<th>Objective</th>
</tr>
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<tbody>
<tr>
<td>7.9</td>
<td>Fundamental observation</td>
</tr>
<tr>
<td>13.12</td>
<td>Typical frequencies of commuter vehicle carbody</td>
</tr>
</tbody>
</table>

Table 3 Properties used the evaluation.

(a) Evaluation of the validity of numerical simulation.

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(b) Evaluation of the relationship between the Young’s modulus of the vessel and the eigenfrequency.

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<thead>
<tr>
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(c) Evaluation of the relationship between the density of the system and the eigenfrequency.

<table>
<thead>
<tr>
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(d) Evaluation of the effect of the fluid viscosity.

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</table>

|          | \(4.0 \times 10^7\) | 0                 | \(1.0 \times 10^3\) | 0.499        | \(1.0 \times 10^3\) |
|          | \(2.0 \times 10^7\) | 0                 | \(1.0 \times 10^3\) | 0.499        | \(1.0 \times 10^3\) |

3.2. Eigenfrequency Evaluation

The variation of the PSD caused by the difference in the input frequency is shown in Fig.3. In this result, the mild peaks at lower frequencies are related to the input. The sharp peaks at around 42Hz are independent of the input frequency, and can be considered as the...
eigenfrequency of this system. This implies that the analysis of eigenfrequency of the system can be analyzed by monolithic FEM using the three-element solid model.

3.3. Difference in Response with Material Properties

3.3.1. Validation on Young’s Modulus of Fluid

The fundamental material properties described in section 3.1 are changed to investigate the effect of material properties such as density, Young’s modulus, and viscosity. Here, the frequency of the input wave is fixed at 1Hz in the following simulations.

In the first investigation, the Young’s modulus in the fluid region is changed to validate the FEM. The simulated results shown in Fig.4 indicate that many peaks that are excited in the high-frequency region when the Young’s modulus is low, vanish when the Young’s modulus is high. Then, the value of the modulus indicated here is adopted as the Young’s modulus on the fluid region in the vessel to be filled up, as given by equation (7).

3.3.2. Sensitivity of Young’s Modulus of Vessel

The difference in response caused by the variation of the Young’s modulus of the vessel is shown in Fig.5. In this result, the excited peak shifts a high frequency with an increase in the Young’s modulus of the vessel. This implies that the eigenfrequency of this system depends on the stiffness of the vessel, as in the case of general eigenfrequency problems.

3.3.3. Sensitivity of Density

The difference in response caused by the variation of the densities of the vessel and fluid is shown in Fig.6. Here, the variation of the densities is indicated in the figure. In this result, the excited peak shifts to a low frequency with an increase in the fluid density even if the increase in the vessel density causes a reduced shift of the excited peak. This implies that the sensitivity of the fluid density is greater than that of the vessel density in this system.
3.3.4. Sensitivity of Viscosity

The difference in response caused by the variation of the fluid viscosity is shown in Fig.7. In this result, the excited peaks shift to a lesser extent as compared to the results in the case of the Young’s modulus and the density described above. This implies that the sensitivity of the fluid viscosity is lesser than that of the Young’s modulus and density in this system.

4. Conclusion

The dynamics of a structure comprising an outer vessel and an inner fluid is simulated by FEM using the three-element solid model, and the frequency response of the structure is investigated to study the availabilities of FEM. The followings conclusions are drawn from this study:
The transient response analysis using the three-element solid model can be applied to the eigen value problem of solid-fluid interaction systems.

The increase in the Young’s modulus of the vessel causes a shift of the eigenfrequency of the solid-fluid system to a higher region.

The densities of the vessel and the viscosity of the fluid have low sensitivity in the dynamic response of the system.

In the future, we intend to practically analyze of the eigenvalue problem for solid-fluid systems by the FEM using the three-element solid model.

References


