Finite Element Analysis of Cutting Deformation of Stacked Polycarbonate Sheets Subjected to Two-Line Wedge Indentation*

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Abstract
This paper describes a pushing-cut process of stacked polycarbonate (PC) sheets. In this work, the cutting line force of a 30°/90° facet blade on the upper PC work sheet and the deformation of its PC work sheet were simulated by an FEM code (MSC.MARC-2005R3) in order to reveal the effect of stacked structure on deformation flow of the PC work sheet mounted on a PC underlay. The deformation profile of the work sheet was observed with respect to indentation of the blade by varying the friction coefficients of the work sheet with the blade and that with the underlay, and also the anaphase work-hardening effect of the PC sheet was discussed for estimation of the final cutting stage.

Key words: Laminated Construction, High Polymer Materials, Friction, Cutting, Plastic Working

1. Introduction

A wedge indentation processing is widely used for cutting off a complicated-formed pattern from a sheet material such as carton boxes, labels, insulation films and similar metal thin sheets\(^1\)(\(^2\)). In the converting industry on printing and cutting off resin or paper work sheet, a single-line and a two-line bevel blade (facet wedge) are empirically used.

Traditional theories were insufficient to explain the cutting load response and the deformation behavior of stacked resin work sheets during the two-line bevel blade indentation process, especially for the stability of interface sticking/sliding phenomena. Pelletie et al.\(^3\) has studied determining the mechanical properties of thin films on substrate with a rigid indenter loaded for a nano-indentation test. This report is useful for understanding the influence of the Young’s modulus ratio and the yielding stress ratio of a worksheet with a substrate. However, since this model was based on a half infinite thickness of the substrate, it is insufficient for explaining the cutting deformation of a thin worksheet mounted on a finite thickness underlay. There are not almost any studies about the cutting mechanism of a worksheet on a flexible underlay during wedge shearing process.

Nagasawa et al.\(^4\)(\(^5\)) experimentally studied about the statistical stability of stacked polycarbonate (PC) sheets, the thickness of which was 0.5 mm for each, regarding the load response and the sheared profile modes. In order to cut off a thick (0.5 mm or more) resin sheet in stable conditions, the rigidity of underlay, the contact friction of blade and work sheet, and blade profile such as the apex angle seem to be the primary factors. Comparison of two-line with single-line blade\(^6\), and effect of tip height on the two-line blade\(^7\) were investigated with respect to the stacked PC sheets.
From the experimental results \(^{(5)}\), the friction coefficient of PC work sheet with a blade and/or that of PC work sheet with the underlay of PC were remarkably varied with the contact pressure and also by a certain kind of surface washing condition. The load response had statistically branched into two modes, named as the upper-bound state and the lower-bound state. This branching was obviously controlled by sliding condition between the work sheet and the underlay, because the branching of load response was relatively decreased when an adhesive tape was inserted between them. However, a small branching of load response was yet observed when the adhesive tape was inserted into the interface of them. Hence, it seems that the friction condition between the blade and the work sheet primarily causes the branching of load response. The sliding friction among the three bodies (the blade, the work sheet and the underlay) seems to fundamentally control the branching phenomena of load response and eventually decide the sheared profile of the work sheet. In order to confirm the effect of sliding friction of those three bodies on the load response and the sheared profile, since the friction forces are not observed in direct during the cutting process, any other methods such as an finite element method (FEM) analysis are required.

In this paper, therefore, an indentation of 30°/90° facet wedge into PC work sheet of \( t = 0.5 \) mm thickness, which is mounted on the underlay of PC, was numerically carried out by varying the friction coefficients of the PC work sheet with the blade and/or that of the PC work sheet with the underlay, in order to reveal the effect of the frictional restriction between the blade surface and the PC work sheet surface, and the underlay effect of the lower work sheet itself.

2. Experimental and simulation condition

2.1 Method of pushing cut and specimens

Nagasawa et al. \(^{(7)}\) have reported the experimental method for the pushing cut of the PC sheet. The in-plane MD (machine direction) tensile properties of PC sheet were as follows: the Young’s modulus \( E = 2.65 \) GPa, the proof stress \( \sigma_Y = 52.9 \) MPa, the ultimate strength \( \sigma_B = 66.0 \) (153.7) MPa, and the true strain of breaking point \( e_B = 0.57 \) under the strain rate of \( 0.1 \) s\(^{-1}\). All the rectangle formed PC sheets had the thickness of \( t = 0.5 \) mm, the width of \( B = 20 \) mm and the length of \( L = 40 \) mm. Figure 1 shows a representative stress-strain diagram on the same tensile testing. The stress-strain diagram of cross machine direction of the PC sheet was almost same as that of the machine direction. Regarding the thickness direction, the compressive test was inspected by using stacked 10 pieces of squared-form PC sheets of 15 mm x 15 mm x 0.5 mm with the compressive strain rate of 0.1s\(^{-1}\). From this compressive test, Young’s modulus of 2.40 GPa was derived.

Figure 2 illustrates the experimental setup for a blade pushing into a stacked specimen of two pieces of PC sheet. On the experimental apparatus, the upper crosshead moved downward with a feed velocity \( V \), which was chosen as \( 0.05 \) mm\(^{-1}\). This velocity was chosen as to make the cutting strain rate \( V/t = 0.1 \) equal to the tensile test strain rate, because the material properties of PC sheet generally depended on the strain rate. The position of the cutting blade was vertical to the specimen, while the angle \( \phi \) of the cutting line direction with respect to MD of the PC sheet was chosen as \( \phi = 90^\circ \).

The facet (wedge) cutting blade, which was made with Cemented Carbide (FM10K), had the length of \( L = 30 \) mm, the height of \( H = 9 \) mm, the thickness of \( S = 0.90 \) mm, and the initial tip thickness of \( w = 0.8 \) \( \mu \)m in average. The apex angle \( \alpha \) of the first facet was chosen as 90°. The height of the first facet \( h \) was 0.15 mm (\( h/t = 0.3 \)), while the secondary bevel angle \( \alpha' \) was chosen as 30°.

As the PC sheet was initially coated with a water soluble glue layer which was used for attaching a masking film, all the specimens were prepared with the following two conditions: (i) they were sufficiently washed and dried before cutting, (ii) the others were used without washing. The three kinds of friction coefficient \( \mu_C \), \( \mu_U \) and \( \mu_P \) were
experimentally measured by the horizontal method based on JIS-P8147, and shown in Table 1. Here, they were measured with the applied contact pressure $p = 2$–$6$ kPa.

![Fig.1 Stress strain diagram on in-plane tensile testing of PC sheet.](image)

**Fig.1** Stress strain diagram on in-plane tensile testing of PC sheet.

![Fig.2 Schematic diagram of pushing test apparatus.](image)

**Fig.2** Schematic diagram of pushing test apparatus.

Notation of primary variables:

- $w$: tip thickness $5 \mu m$
- $f$: line force $kN \cdot m^{-1}$
- $t$: thickness of upper/lower work sheet $0.5 \text{ mm}$
- $h$: height of first line of wedge $0.15 \text{ mm}$
- $\alpha$: apex angle $90^\circ$, $\alpha'$: angle of second line $30^\circ$
- $c$: indentation displacement $\text{ mm}$
- $d$: indentation depth $\text{ mm}$

**Table 1** Friction coefficients measured by the horizontal method ($2$–$6$ kPa)

<table>
<thead>
<tr>
<th>Symbol (contact surface)</th>
<th>$\mu_C$ (Blade-PC)</th>
<th>$\mu_U$ (PC-PC)</th>
<th>$\mu_P$ (PC-C.Plate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Washed</td>
<td>0.06 (0.05–0.08)</td>
<td>0.04 (0.03–0.06)</td>
<td>0.06 (0.05–0.11)</td>
</tr>
<tr>
<td>(ii) Unwashed</td>
<td>0.40 (0.38–0.42)</td>
<td>0.49 (0.44–0.60)</td>
<td>0.30 (0.20–0.40)</td>
</tr>
</tbody>
</table>

Regarding the blade position, the indentation displacement $c$, which was measured by a displacement meter $^{(5)}$, was calibrated using Eq. (1) due to the elastic deformation of blade and counter plate system. The indentation depth $d$ was experimentally estimated from Eq. (1) and compared with the simulation results, which was mentioned later. Here, the line force $f\text{ kN} \cdot \text{ m}^{-1}$ was considered.

$$d/t = c/t - 0.004498 f$$

As there was ripple of friction force when a loader moved on the specimen, the dispersion of measured friction force was apt to be large. When the applied contact pressure was increased with the horizontal method, those friction coefficients were empirically increased, although the washed surface was apt to be slippery. The contact pressure on the
blade can be theoretically estimated from the proof stress of the work sheet and the apex angle of the blade \(^8\). Assuming that the compressive stress in the thickness direction is \(1.15\sigma_{0.2}\) beneath the blade tip and the stress in the lateral (in-plane) direction is nearly zero, the contact pressure on a 90° single-line bevel blade is roughly estimated as \(1.15 \times 52.9 \times 0.5\) = 30.4 MPa. This is calculated from the Mohr’s stress circle diagram concerning the normal direction of the blade surface which is located at the angle of 45° with the thickness direction. Since it is normally difficult to measure the friction force under such a high pressure condition in the horizontal method, if the friction coefficients are really required, the friction coefficients should be experimentally and numerically identified from several cutting load responses. From the authors’ prior experiment, the cutting load response was fairly scattered when the width of rectangle-formed specimen was shorter than 6 mm, while the cutting load response was stably repeated when the width of specimen was larger than 12 mm. Furthermore, the sheared profile was almost uniform along the width direction of specimen. So far, the cutting deformation seemed to be sufficiently in the plane-strain condition at the middle position of specimen in case of the width of 20 mm.

The applied pushing force \(F\) was measured by the load cell. The indentation displacement of the cutting blade into the PC sheet \(c\) was measured as the upper crosshead displacement, and \(f = F/b\) kN \(\cdot\) m\(^{-1}\) is the line force applied in vertical to the cutting blade. During this cutting process, the bent-up angle of the PC work sheet was remarkably observed in case of the upper-bound load response. The bent-up angle at the peak load point was roughly 10–40° for unwashed condition, while it was roughly 10–20° for washed condition. In this study, those parameters were compared with that of simulation in case of washed condition.

2.2 Condition of FEM simulation

An elasto-plastic finite element analysis with non-linear contact problem was carried out to simulate the center bevel blade indentation on the stacked PC sheet. The PC work sheet and the PC underlay were considered as deformable bodies, while the cutter and counter plate were modeled as rigid contact bodies as shown in Fig. 3.
The apex angle of blade $\alpha$ was $90^\circ$, and the second angle $\alpha'$ was $30^\circ$, the tip thickness $w$ was $1\,\mu m$, and the height of the first facet $h$ was $0.15\,mm$. The PC work sheet and the PC underlay were assumed to be a half symmetric model of a rectangle with the side length of $1.0\,mm$ and the sheet thickness $t$ of $0.5\,mm$. The quadrilateral first order plane strain element type was considered. The work sheet was initially divided into 5000 elements for the half model, while the underlay was divided into 1250 elements. In process of auto-remeshed calculation (using ADVANCING FRONT QUAD of MSC.MARC 2005R3), the maximum side (edge) length was restricted to $l_s = 4$–$6\,\mu m$ for the work sheet and $l_u = 50\,\mu m$ for the underlay, respectively. The ratio of side length $l_u/l_s$ was roughly kept to be less than 10 in order to avoid the invasion among multiple bodies. The boundary constraint among multiple bodies was explicitly controlled by CONTACT of MSC.MARC.

**Table 2** Assumed friction coefficients on simulation

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>BL-PC $\mu_C$</td>
<td>0.05</td>
<td>0.25</td>
<td>0.5</td>
<td>1.0</td>
<td>0.05</td>
<td>0.05</td>
<td>1.0</td>
</tr>
<tr>
<td>PC-UL $\mu_C$</td>
<td>0.05</td>
<td>0.25</td>
<td>0.5</td>
<td>1.0</td>
<td>0.05</td>
<td>1.0</td>
<td>0.05</td>
</tr>
<tr>
<td>UL-CP $\mu_P$</td>
<td>0.05</td>
<td>0.25</td>
<td>0.5</td>
<td>1.0</td>
<td>1.0</td>
<td>0.05</td>
<td>0.05</td>
</tr>
</tbody>
</table>

The arctangent-coulomb friction coefficient was assumed to be chosen from Table 2. Here, the relative velocity threshold of arctangent model was empirically chosen as 0.01. Effect of average magnitude of friction coefficients $\mu_C$, $\mu_U$, $\mu_P$ was discussed with respect to the cases 1, 2, 3 and 4. The cases 5, 6 and 7 were compared with each other in order to confirm the effect of each sliding position.

The material properties were assumed to be isotropically elasto-plastic. The material properties of simulation model were considered as two cases A, B.

![Fig.4 Relationship between stress and strain on FEM model of PC sheet](image)

The simplified model A was as follows: Young’s modulus $E = 2.65$ GPa, Poisson’s ratio $\nu = 0.3$, the initial yield stress $\sigma_Y = 61.8$ MPa, the work-hardening up to the ultimate point of $\sigma_B = 70.6$ MPa, $\varepsilon_B = 1.28$. Where, this data set was based on the nominal specification (catalogued data without any tensile testing diagrams). Adding this, the full model B, where $E = 2.65$ GPa, $\nu = 0.3$, $\sigma_Y = 61.8$ MPa, $\sigma_B = 153.7$ MPa, $\varepsilon_B = 0.57$ as shown in Fig.4, was considered in order to discuss with the effect of anaphase work-hardening property. The early stage of B was assumed to be same as that of A up to the initial yielding point. Regarding the anaphase stage $(0.1 < \varepsilon < 0.57)$, the experimental data of Fig.1 was approximated with a polygonal line at the strain chosen as 0.1, 0.2, 0.3, 0.4, 0.5 and 0.57.

The fracture model such as Cockcroft and Latham criterion (9) was not considered, because any large cracks and force drop were not detected in the experiment of $h/t = 0.3$,.
\( \alpha/\alpha' = 90^\circ/30^\circ \) (5). If the fracture (half-breaking) is considered in the material model, the deformation resistance of cutting would be a little reduced, although any cracks would not be opened due to the high pressure state in the thickness direction.

3. Results and discussion

3.1 Experiment

In order to know the fundamental cutting behavior of two pieces of stacked PC sheet without any adhesive tape, the two-line (facet wedge) blade was indented to the upper PC sheet with the feed velocity of \( V = 0.05 \text{ mm} \cdot \text{s}^{-1} \).

Figure 5 shows the representative examples (as the quasi-upper bound and the quasi-lower bound) of load response. The surface of specimen was considered as (i) washed and (ii) unwashed condition. Here, the indentation depth \( d/t \) was estimated from the displacement of blade \( c/t \) (5) and the elastic deformation of the machine device using Eq. (1).

![Fig.5 Relationship between cutting line force and indentation displacement of blade (t=0.5mm).](image)

![Fig.6 Relationship between bent-up angle and indentation displacement of blade (t=0.5mm).](image)

![Fig.7 CCD side views of PC sheets sheared by 30°/90° facets blades (V= 0.05mm · s⁻¹, t=0.5mm, in case of sticky (unwashed) surface w/o adhesive tape) [Ref. (5): Fig.2 (a)].](image)
When the upper work sheet was completely cut off in the case of washed condition, $d/t$ tended to exceed 1.0 (roughly reached 1.14) due to the plastic deformation of the underlay.

Since the upper PC sheet was simply stacked on the lower PC sheet (underlay), when the blade was indented to the upper PC sheet, it was basically bent up on the underlay. The measured bent-up angle $\theta_{BU}$ was shown in Fig.6.

**Figure 7** shows a few of representative side views of PC sheets sheared by the facet blade.

When $\theta_{BU}$ was large (a sample number: $n_C=17$), the interface between the blade and the upper PC sheet appeared to be fixed, while the same interface appeared to be smoothly slid when $\theta_{BU}$ was small ($n_C=11$). It was found from Fig.6 that $\theta_{BU}$ was similar with the quasi-lower bound on washed, unwashed condition, and it was also similar with the quasi-upper bound on washed, unwashed condition for $d/t < 0.8$.

### 3.2 Consideration of anaphase work-hardening property

**Figure 8** shows the cutting line force response by using the material properties with Model A, B in case of slippery state (Case 1 in Table 2), while **Fig. 9** (a) and (b) describes the deformation process of stacked PC sheets for the material model A, B.

![Fig.8](image)

**Fig.8** Relationship between cutting line force and blade indentation depth on FEM simulation (comparison of Model A, B of material properties in case of slippery state, $\mu_C=\mu_U=\mu_P=0.05$).

It is found that the response of the material model B (Fig.8) was fairly similar to the experimental result with the washed condition (Fig.5). Namely, the simplified material model A was insufficient for precisely estimating the load response, and the anaphase work-hardening effect was important for precisely estimating.

![Fig.9](image)

**Fig.9** Deformation process of stacked PC sheets during blade indentation for Model A, B of material properties in case of slippery state ($\mu_C=\mu_U=\mu_P=0.05$).
Fig. 10 Zoomed-up views of contact zone at \( d/t = 0.8 \) in case of 1 (slippery state)

Figure 10 shows the details of wear profile of contact zone for the material model A, B at \( d/t = 0.8 \) in Fig.8. The wear profile with the material model B was closer to the experimental than that of the material model A. The inflection points were ambiguous for \( d/t = 0.2 \sim 0.4 \) in case of the material model B. This seems to be caused by the anaphase work-hardening effect and resulted by the large wear profile.

As there was experimental dispersion of load response, which was recognized as the quasi-upper bound and the quasi-lower bound load response (as shown in Fig.5, Fig.7), and the bent-up angle \( \theta_{BU} \) had the similar tendency to the load response with respect to the indentation depth \( d/t \), we need to discuss about the unevenness of friction coefficients \( \mu_C \), \( \mu_U \), and \( \mu_P \).

So far, in the following, the material model B was mainly considered for discussing the deformation process of the upper PC sheet, while the friction coefficients \( \mu_C \), \( \mu_U \), and \( \mu_P \) were varied with Table 2.

Fig. 11 Cutting load response on FEM simulation by varying the friction coefficients \( \mu_C \), \( \mu_U \), \( \mu_P \) with Table 2

3.3 Effect of friction coefficients on cutting deformation

In order to compare the numerical cutting behavior of stacked PC sheets with the experimental, a 30°/90° facet wedge indentation was simulated in the seven cases of Table 2. Figure 11 shows the relationship between cutting line force and indentation depth of blade, while Fig. 12 shows deformation of the stacked PC sheets sheared by the 30°/90° facet wedge, regarding Case 2, 3, 4 and 6, 7.

Figure 13 shows the bent-up angle \( \theta_{BU} \) of the PC work sheet. From Figs.11~13 and related data set, the following features were revealed.
Fig. 12 Deformation process of stacked PC sheets during blade indentation for each case 2, 3, 4 and 6, 7 (based on the material model: B)

Fig. 13 Relationship between bent-up angle and indentation depth by varying the friction coefficients $\mu_C$, $\mu_U$, $\mu_P$ with Table 2

1. Comparing Case 5 with Case 1, the load response and the bent-up angle were almost same with each other. Hence, variation of $\mu_C$ is almost negligible for cutting response of upper work sheet.

2. There are two inflection points at $d/t \approx 0.2, 0.35$. The latter is coincident that the height of apex angle of 90° has been indented to the PC work sheet, while the former is related to the finite thickness of the PC work sheet and also depends on its shear sliding with the underlay. In case of experiment, those two inflection points seem to smoothly transfer during $d/t=0.2 \sim 0.4$.

3. The camber of upper work sheet starts at $d/t=0.3 \sim 0.4$. Here, due to the result of FEM simulation, $\mu_C$ seemed to be close $\mu_U$ in this stage.

4. From Cases 3, 4 and 7 in Fig. 11, when $\mu_C$ is larger than a certain value, supposed to be larger than 0.5, the interface between the wedge and the PC work sheet tends to be fixed. Eventually, the PC work sheet behaves as a built-in edge to the underlay and the load
response increases for $d/t > 0.8$.

(5) When $\mu_C >> \mu_U$ (Case 7), the bent-up angle $\theta_{BU}$ tends to be increased. From Fig.6, this situation ($\mu_C >> \mu_U$) seemed to occur for $d/t > 0.7$ in case of the quasi-upper bound load response.

(6) Comparing with Cases 2, 3 and 4 in Fig.13, it is found that bent-up angle $\theta_{BU}$ is almost independent of the magnitude of the friction coefficients under $\mu_C=\mu_U=\mu_P$. Seeing Case 6 ($\mu_C << \mu_U$), the load response and the bent-up angle were far from the experimental. Namely, this situation ($\mu_C << \mu_U$) was supposed to be rare. However, it was confirmed that un-even assignment of friction coefficients was essentially a primary factor to decide the camber of upper work sheet.

(7) Case 7 was similar to the experimental behavior of quasi-upper bound load response with unwashed condition, while Case 2 was similar to the experimental behavior of quasi-lower bound load response with un-washed condition.

(8) From (4), (5) and (6) mentioned above, the occurrence of quasi-upper and quasi-lower bound responses is determined by the un-even assignment of friction coefficients and also their increasing with respect to the blade indentation.

Fig. 14 Contour lines of normal stress $\sigma_{22}$ for the thickness direction (Case 1: $\mu_C=\mu_U=\mu_P=0.05$, $\sigma_Y=52.9$ MPa)

Figure 14 shows the normal stress for the thickness direction $\sigma_{22}$ in the slippery state ($\mu_C=\mu_U=\mu_P=0.05$, Case 1). Comparing the magnitude of $\sigma_{22}$ with the contact surfaces (the wedge and the underlay), it was found that the pressure on the wedge was remarkably higher than that of the underlay during the blade indentation.

Since we empirically know that the friction coefficient of PC sheet unstably increases with contact pressure and sliding motion \cite{5}, it is understood that un-evenness of friction coefficients: $\mu_C > \mu_U$ easily occurs at the contact surfaces. So far, it is found that the experimental load response can be estimated by the FEM simulation when the friction coefficients were appropriately varied with the blade indentation to the material model B.

3.4 Discussion for the future work

If $\mu_C$ and $\mu_U$ are varied with the blade indentation, namely they are increased with a certain kind of external sliding work (a contact energy), the load response seems to be increased in Fig.11. In this case, the friction coefficients should be considered as a function of the external work of friction force on the sliding surface.

On the other hand, as there is a certain criterion of half breaking at the final stage of cutting, the actual load response seems to be balanced and dispersed by the frictional-contact stability and the half-breaking phenomena.

4. Conclusions

The indentation of a $30^\circ/90^\circ$ facet wedge blade to stacked polycarbonate (PC) sheets was numerically carried out to investigate the effect of contact friction and the material properties of PC sheet under stacked structure. The simulation results were discussed with the experimental results. Through this work, the following are revealed:
When a combination of the friction coefficients $\mu_c, \mu_u$ is randomly varied with respect to the indentation of the facet wedge, it is found that the cutting load response is possibly changed and its dispersion is generated.

When the anaphase work-hardening properties are considered in the material properties (the stress-strain relationship), it is found that the calculated cutting load response is fairly matched to some experimental results.

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**References**