Prediction of Residual Tensile Strength after Fatigue in Unidirectional Brittle Fiber-Reinforced Ceramic Composites*

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Abstract

This paper presents models for predicting residual tensile strength after fatigue in unidirectional brittle fiber-reinforced ceramic composites. First, the subcritical crack growth law was employed to predict residual strength of fiber. Next, two models were proposed for residual strength of a composite. The first model, Model I, is established based on Curtin's probabilistic model that describes the relationship between strengths of the fiber and the composite. In the second model, Model II, the composite strength is directly derived from the survival probability of fiber. Thirdly, the relationship between lives of the fiber and the composite was investigated. Finally, a case study was conducted for a ceramic matrix composite (CMC) to obtain the residual strength of the CMC after static fatigue loading. It was proven that the life and the initial strength of the fiber and the composite depend on the shape parameter of fiber strength and the fiber/matrix interfacial shear stress. In contrast, the residual strength ratio of the composite was found to be almost independent of these two parameters.

Key words: Fiber-Reinforced Ceramic Composite, Fatigue, Residual Strength, Life

1. Introduction

As fiber-reinforced composites have been increasingly applied to structures in many industrial fields, long-term structural reliability has become a critical issue. Especially, it is essential to predict the degree of strength degradation after fatigue loading, as well as to predict a fatigue life, in a composite structure. Residual strength of the composite after fatigue loading is predicted by combination of a model for residual strength of fiber, with a model for relationship between strengths of the fiber and the composite. However, as far as the authors know, such modeling has not been performed thus far.

Since stress in the fiber is considerably higher than that in the matrix for a unidirectional fiber-reinforced composite, composite strength is dominated by fiber strength. A lot of work (1-11) has been conducted on the relationship between fiber strength and composite strength. The theories of the strength of fibrous composites to date are based on two conflicting concepts. One is local load sharing (LLS) and another is global load sharing (GLS). Curtin and coworkers (5-11) proposed theoretical models based on the above two concepts, considering the probabilistic characteristics of fiber strength and the fiber/matrix interfacial shear stress. The composite strength predicted using their models is in good agreement with experiment results. In contrast, residual strength of brittle fiber after fatigue can be estimated based on the subcritical crack growth (SCG) model, where the residual strength is expressed as a function of the number of cycles and applied stress.

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This paper presents probabilistic models for predicting residual tensile strength after fatigue in unidirectional brittle fiber-reinforced ceramic composites. First, residual strength as well as a fatigue life of fiber was obtained using the SCG law. Next, two models were proposed to predict residual strength of a composite. The first model, Model I, is established based on the Curtin's probabilistic model that describes the relationship between the fiber strength and the composite strength. In the second model, Model II, the composite strength is directly derived from the survival probability of fiber. In addition, the relationship between lives of the fiber and the composite was investigated for Model I. Finally, a case study was performed for a ceramic matrix composite (CMC) to obtain the residual strength of the CMC after static fatigue loading. Predictions were compared with experiment results only for initial composite strength and time to failure (or stress rupture time) of the fiber because experimental data for residual strength of the fiber and the composite were not found. Instead, effects of the shape parameter of fiber strength and the interfacial shear stress on the fatigue life and residual strength of the CMC were analyzed.

2. Modeling

2.1 Residual strength of fiber

The SCG behavior of a single crack in a fiber subjected to cyclic stress \( \sigma^f(t) = \sigma_{\text{max}}^f f(t) \) (maximum stress \( \sigma_{\text{max}}^f \), period \( \tau \)) is presented here. The stress intensity factor \( K_1 \) of a crack (length \( a \)) is expressed as

\[
K_1 = Y \sigma^f(t) \sqrt{a} = Y \sigma_{\text{max}}^f f(t) \sqrt{a}
\]

where \( Y \) is a constant depending on the size and geometry of the crack. The SCG law gives the relationship between the crack propagation rate \( da/dt \) and \( K_1 \) as

\[
\frac{da}{dt} = A \left( \frac{K_1}{K_{IC}} \right)^n
\]

where \( A \) and \( n \) are material constants and \( K_{IC} \) denotes fracture toughness of the fiber. Integrating Eq. (2) using Eq. (1) from \( a_i \) to \( a_j \) \((i = 1, 2, \ldots, N)\) and summing the \( N \) (the number of cycles) integrals, we obtain

\[
a_{i} - a_{i} = A \nu \left( \frac{Y \sigma_{\text{max}}^f}{K_{IC}} \right)^n \nu \tau_e
\]

where \( a_{i} \) and \( a_{j} \) denote the initial crack length and the crack length at \( N \) cycles, respectively, \( \nu \) equals \( n/2 - 1 \) and

\[
\tau_e = \int_0^\tau f(t) \nu dt
\]

is the equivalent period. These crack length are related with the fracture toughness, initial strength \( \sigma_{\text{ic}}^f \), and residual strength \( \sigma_{\text{R}}^f \) of the fiber as

\[
K_{IC} = Y\sigma_{\text{ic}}^f \sqrt{a_0} = Y\sigma_{\text{R}}^f \sqrt{a_{\nu}}.
\]

Substitution of Eq. (5) into Eq. (3) yields

\[
\sigma_{\text{R}}^f = \left[ 1 - g \frac{\nu N(\sigma_{\text{max}}^f)}{(\sigma_0^f)^\nu} \right]^{\frac{1}{2\nu}} \sigma_0^f
\]

with

\[
g = \frac{\tau_e}{\tau} = \sigma_0 \cdot \sigma_0 = A \nu \left( \frac{Y \sigma_{\text{max}}^f}{K_{IC}} \right)^2.
\]

Fatigue fracture occurs when applied stress equals residual strength. Thus, putting \( \sigma_{\text{max}}^f = \sigma_{\text{R}}^f \) into Eq. (6) gives the life of fiber for a given maximum stress \( \sigma_{\text{max}}^f \) as
Equation (6) is rewritten by using Eq. (8) as

\[ f = \left[ 1 - \left( 1 - \frac{\sigma_{\text{F}}}{\sigma_{\text{t}}} \right)^{N} \right]^{\frac{1}{N}} \sigma_{\text{t}}. \]  

It should be noted that this equation overestimates the residual strength because \( \sigma_{\text{F}} \) becomes \( \sigma_{\text{F}} = \sigma_{\text{max}} \) for \( N = N_{\text{F}} \).

### 2.2 Tensile strength of a composite

When the strength of fiber (length \( L \)) obeys Weibull distribution, cumulative failure probability is given by

\[ P = 1 - \exp \left[ - \left( \frac{L}{L_{\text{s}}} \right)^{m} \right] \]  

where \( m \) denotes a shape parameter, \( L_{\text{s}} \) the reference length, and \( \sigma_{\text{s}} \) the scale parameter for \( L_{\text{s}} \). Curtin and coworkers (8, 9) derived a relationship among stress in the composite, \( \sigma_{\text{s}} \), the number of fragment per fiber length, \( \rho \), and applied stress in the fiber, \( \tau \), assuming GLS (see Appendix). Maximizing \( \sigma_{\text{s}} \), the strength is given by

\[ \sigma_{\text{s}} = V_{\text{f}} \sigma_{\text{t}} \left( \frac{m}{2} \right) \left[ 1 - \exp \left( - \frac{2}{m} \right) \right] \]  

where \( V_{\text{f}} \) denotes fiber volume fraction and \( \sigma_{\text{t}} \) denotes the critical strength of fiber defined by (A2). Thus the initial composite strength is

\[ \sigma_{\text{c}} = V_{\text{f}} \sigma_{\text{t}} \left( \frac{m}{2} \right) \left[ 1 - \exp \left( - \frac{2}{m} \right) \right] \]  

Average strength of fiber is expressed using the scale parameter \( \sigma_{\text{s}} \) for length \( L_{\text{s}} \) from Eq. (10) as

\[ \bar{\sigma}_{\text{s}} = \sigma_{\text{s}} \Gamma \left( 1 + \frac{1}{m} \right). \]  

Using \( \bar{\sigma}_{\text{s}} \) in place of \( \sigma_{\text{s}} \), Eq. (12) yields

\[ \sigma_{\text{c}} = k \left( \bar{\sigma}_{\text{s}} \right)^{m} \]  

where

\[ k = V_{\text{f}} \left( \frac{L_{\text{s}}}{r} \right) \left[ 1 - \exp \left( - \frac{2}{m} \right) \right] \Gamma \left( 1 + \frac{1}{m} \right) \]  

Hence, composite strength is expressed using the average strength of fiber with the reference length. When \( m \) is infinite, Eq. (15) becomes \( k = V_{\text{f}} \) and \( \alpha = 1 \). Therefore, the deterministic strength is \( \sigma_{\text{c}} = V_{\text{f}} \sigma_{\text{t}} \).

### 2.3 Residual strength of a composite

Here we propose two models for predicting residual strength of a composite after fatigue. In Model I, the concept in the previous section is extended to residual strength. In Model II, residual strength is obtained from the survival probability of fiber.

#### 2.3.1 Model I

It is assumed that the relationship between residual strength of the composite and average residual strength of the fiber after fatigue is expressed by the same form as Eq. (14). When the average strength of the fiber is reduced to \( \bar{\sigma}_{\text{t}} \) after fatigue loading, the
residual strength of the composite is expressed as
\[ \sigma^c_\infty = k \left( \sigma^c_0 \right)^m. \] (16)
Putting \( \sigma_0^c = \bar{\sigma}_0^c \) into Eq. (6), \( \bar{\sigma}_0^c \) is approximately obtained. Thus, Eq. (16) yields
\[ \sigma^c_\infty = k \left[ 1 - \frac{g \tau N (\sigma_{\max}^t)^{2/\nu}}{(\sigma^c_0)^{2/\nu}} \right]^{\frac{m}{\nu}} (\sigma^c_0)^m. \] (17)
Since the maximum stress applied to the composite, \( \sigma_{\max}^c \), is approximated by \( V_f \sigma_{\max}^t \), Eq. (17) is rewritten using Eq. (14) as
\[ \sigma^c_\infty = \left[ 1 - \frac{g \tau N k^{2/\nu} / (V_f)^{2/\nu}}{(\sigma^c_0)^{2/\nu}} \right]^{\frac{m}{\nu}} (\sigma^c_0)^m. \] (18)
Equation (18) denotes the residual strength of the composite for a given maximum stress in the composite, \( \sigma_{\max}^c \), and the number of cycles, \( N \).

2.3.2 Model II
The number of cycles to failure of the fiber, subjected to fatigue loading with maximum stress \( \sigma_{\max}^t \), is given by Eq. (8). Rewriting Eq. (8) after replacing \( \sigma_0^t \) with \( * \sigma_{\max}^t \), we obtain
\[ \sigma^t = \left[ 1 + \frac{g \tau N (\sigma_{\max}^c)^{2/\nu}}{(\sigma^c_0)^{2/\nu}} \right]^{\frac{1}{\nu}} (\sigma^c_0)^m. \] (19)
The existing probability of initial cracks with the number of cycles to failure less than Eq. (8) is equal to that with static strength smaller than applied stress given by Eq. (19). This stress is called equivalent static stress in fatigue loading \( (12, 13) \). Fracture and survival probabilities are obtained by substituting the equivalent static stress into applied stress.

Now the survival probability of fiber (length \( L_{ns} \)) after fatigue (the number of cycles \( N \)) is given by
\[ R = \exp \left[ - \frac{\sigma^t}{\sigma_S} \right]^m. \] (20)
The residual strength equals \( R \sigma^c_\infty \), assuming that survived fibers keep initial strength. Using \( \sigma_{\max}^t = \sigma_{\max}^c / V_f \), we obtain
\[ \sigma^c_\infty = \exp \left[ - \left[ 1 + \frac{g \tau N (\sigma_{\max}^c)^{2/\nu}}{(\sigma^c_0)^{2/\nu}} \right] \frac{m}{\nu} \left( \frac{\sigma_{\max}^c / V_f}{\sigma^c_0} \right)^m \right]. \] (21)
Note that this model gives \( \sigma^c_\infty < \sigma^c_0 \) for \( N = 0 \).

2.4 Life of a composite
Substituting \( \sigma_{\max}^c = \sigma^c_0 \) into Eq. (18) and rearranging the equation, we obtain the life of the composite as
\[ N^c_F = \frac{\left( \frac{\sigma^c_0}{\sigma_S} \right)^{2/\nu} - \left( \frac{\sigma^c_\infty}{\sigma_S} \right)^{2/\nu}}{g \tau k^{2/\nu} / (V_f)^{2/\nu}}. \] (22)
The average fiber life \( N^c_F \) is given by substitution of \( \sigma_0^t = \bar{\sigma}_0^t \) into Eq. (8). Substituting \( N^c_F \) into Eq. (22), we obtain
\[ N^c_F = \frac{\left( \frac{\sigma^c_0}{\sigma_S} \right)^{2/\nu} - \left( \frac{\sigma^c_\infty}{\sigma_S} \right)^{2/\nu}}{g \tau k^{2/\nu} / (V_f)^{2/\nu}} \frac{1}{N^c_F}. \] (23)
When the maximum stress \( \sigma_{\max}^c \) is smaller than a critical value \( \left( k / V_f \right)^{(1-\nu)} = \sigma^c_\infty \), the composite life \( N^c_F \) becomes larger than the average fiber life \( N^c_F \). From Eq. (15), the critical applied stress is given by
Thus, the critical applied stress is proportional to interfacial shear stress. For \( m = \infty \), \( N_f^{*} \) equals \( N_f^{t} \) because of \( k = V_f \) and \( \alpha = 1 \). Moreover, from Eqs. (18) and (22), we obtain

\[
\sigma^e_r = \left[ 1 - \left( \frac{\sigma^{\max}_c}{\sigma^e_0} \right)^{2v/a} \right] \frac{N}{N_f^{t}} \sigma^e_0.
\]

It should be noted that the composite residual strength becomes the applied stress \( \sigma^{\max}_c \) when the number of cycles reaches the life corresponding to \( \sigma^{\max}_c \), denoted by \( N_f^{*} \), as the residual strength of the fiber becomes \( \sigma^{f}_{\max} \) for \( N = N_f^{*} \).

### 2.5 Static fatigue

In this section, we consider a composite under static fatigue loading with constant stress \( \sigma^c_0 \). Substituting \( \sigma^{\max}_c = \sigma^c_0 \), \( rN = t \), and \( g = g_0 \) in Eqs. (18) and (21), the residual strength for Models I and II are respectively given by

\[
\sigma^e_r = \left\{ - g_0 t k^{2a/v} \left( \frac{\sigma^c_0 / V_f}{\sigma^e_0} \right)^{f} \right\}^{2v/a} \sigma^e_0,
\]

and

\[
\sigma^e_r = \exp \left\{ - \left[ 1 + g_0 t \left( \frac{\sigma^c_0}{V_f} \right) \right]^{m} \left( \frac{\sigma^c_0}{V_f \sigma^c} \right)^{m} \sigma^e_0 \right\}.
\]

Time to failure is obtained from Eq. (22) as

\[
t^* = \frac{\left( \sigma^e_0 \right)^{2v/a} - \left( \sigma^c_0 \right)^{2v/a} \left( \sigma^c_0 / V_f \right)^{f}}{g_0 k^{2v/a} \left( \sigma^c_0 / V_f \right)^{f}} = \frac{\left( \sigma^e_0 \right)^{2v/a} - \left( \sigma^c_0 \right)^{2v/a} \left( \sigma^c_0 / V_f \right)^{f}}{g_0 \left( \frac{\sigma^f_0 / V_f}{\sigma^c} \right)^{v}} \tau^*.
\]

where \( \tau^* \) denotes the average time to failure of the fiber expressed as

\[
\tau^* = \frac{\left( \sigma^e_0 \right)^{2v/a} - \left( \sigma^c_0 \right)^{2v/a} \left( \sigma^c_0 / V_f \right)^{f}}{g_0 \left( \frac{\sigma^f_0 / V_f}{\sigma^c} \right)^{v}}.
\]

From Eqs. (26) and (28), we obtain

\[
\sigma^e_r = \left\{ - g_0 t k^{2a/v} \right\}^{2v/a} \sigma^e_0
\]

for Model I. This gives \( \sigma^e_r = \sigma^c_0 \) for \( t = \tau^* \).

### 3. Case study and discussion

Nicalon/LASII is used for a case study. The material properties summarized from literature are presented in Table 1. Strength properties of the fiber and the composite are given in Ref. (6) while the parameters associated with SCG behavior of the fiber are given in Ref. (14).

Figure 1 presents the initial strength of the composite using Eq. (14) for two values of the interfacial shear stress \( \tau_i \). The initial strength \( \sigma^e_0 \) increases with increasing \( \tau_i \) although the effect of \( \tau_i \) becomes smaller as \( m \) becomes larger. This is because \( \sigma^e_0 \) becomes \( V_f \sigma^c_0 \), regardless of the value of \( \tau_i \), when \( m \) approaches infinity. The calculated values agree well with the experiment result as demonstrated in Ref. (6).

Figure 2 depicts comparison of time to failure or static fatigue life of the fiber between
Table 1 Material properties and geometrical parameters of Nicalon fiber and Nicalon/LASII.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$m$</td>
<td>3.8*, 5.5**</td>
</tr>
<tr>
<td>$V_f$</td>
<td>0.46*</td>
</tr>
<tr>
<td>$\tau_f$</td>
<td>2*, 3*</td>
</tr>
<tr>
<td>$L_{G}$</td>
<td>25*</td>
</tr>
<tr>
<td>$r$</td>
<td>8*</td>
</tr>
<tr>
<td>$\sigma^c_0$ (MPa)</td>
<td>717-795*</td>
</tr>
<tr>
<td>$\sigma_s$ (MPa)</td>
<td>1740*</td>
</tr>
<tr>
<td>$K_{IC}$ (MPa m$^{1/2}$)</td>
<td>1.25**</td>
</tr>
<tr>
<td>$n$</td>
<td>2.8**</td>
</tr>
<tr>
<td>$g_0$ (MPa$^{-2}$ s$^{-1}$) at 600 °C</td>
<td>$2.25 \times 10^{-10}$ **</td>
</tr>
</tbody>
</table>

* Ref. (6), ** Ref. (14)

Fig. 1 Comparison of composite strength vs. shape parameter between experiment results and calculations.

Fig. 2 Comparison of time to failure of the fiber vs. applied stress in the fiber between experiment results (Ref. (14)) and calculations.
experiment results and predictions. The predictions are in reasonably good agreement with the experiment results, although the abrupt decrease in life is not expressed by the predictions. Figure 3 plots static fatigue life of the fiber and the composite vs. applied stress of the composite, calculated using Eqs. (28) and (29). Here we employed the values in Table 2 as initial composite strength. The fatigue life dramatically decreases as the applied stress $\sigma_{con}^c$ approaches the initial strength $\sigma_0^c$. The ratio of the fiber life to the composite life (life ratio) depends on the applied stress and the interfacial shear stress $\tau_i$. The relative time to failure calculated using Eq. (28) is illustrated in Fig. 4. The life ratio is larger for higher $\tau_i$. First, we consider the case of $\tau_i = 2$ MPa (dotted curves). The life ratio for $m = 5$ increases slightly for the stress ranging from 100 to 300 MPa, and then decreases for the stress larger than 300 MPa. The life ratio is smaller than unity for the stress higher than 552 MPa. In contrast, the life ratio for $m = 20$ decreases monotonically with stress and becomes smaller than unity for the stress higher than 174 MPa. Next, the case of $\tau_i = 3$ MPa (solid curves) is considered. The life ratio for $m = 5$ increases monotonically with stress and is kept to be larger than unity. The life ratio for $m = 20$ decreases monotonically with stress and becomes smaller than unity for the stress higher than 260 MPa. In order to
Table 2 Calculated values of parameters associated with strength of Nicalon and Nicalon/LASII.

<table>
<thead>
<tr>
<th></th>
<th>$m = 5$</th>
<th>$m = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$ (MPa$^{(1/m)}$)</td>
<td>1.4907</td>
<td>1.6046</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.833</td>
<td>0.952</td>
</tr>
<tr>
<td>$\sigma_0^c$ (MPa)</td>
<td>1579.6</td>
<td>1693.9</td>
</tr>
<tr>
<td>$\sigma_{\alpha}^c$ (MPa)</td>
<td>700.76</td>
<td>749.76</td>
</tr>
<tr>
<td>$\sigma_{cr}^c$ (MPa)</td>
<td>552.43</td>
<td>828.65</td>
</tr>
</tbody>
</table>

Fig. 5 Critical applied stress vs. shape parameter.

Fig. 6 Relative residual strength vs. static fatigue loading time for three values of applied stress and two values of $m$, calculated based on (a) Model I and (b) Model II ($\tau = 2$ MPa).

Explain the above behavior of life ratio, the critical applied stress $\sigma_{cr}^c$ (Eq. (24)) is plotted against $m$ in Fig. 5 and is also presented in Table 2. The critical applied stress decrease with increasing $m$. For the three cases except for the case of $m = 5$ and $\tau_i = 3$ MPa, $\sigma_{cr}^c$ is smaller than 700 MPa. Thus, it depends on applied stress whether the life ratio is smaller than unity. In contrast, for the case of $m = 5$ and $\tau_i = 3$ MPa, $\sigma_{cr}^c$ is 829 MPa. Then, the fatigue life is always larger than unity. In this case, the fiber life is short because the average
fiber strength is low due to small $m$. In addition, the composite life is long since the composite strength is high because of high $\tau_i$. Consequently, $t_F^i$ is always longer than $t_F$. 

Figure 6 demonstrates the residual strength ratio of the composite after static fatigue, calculated using Models I and II. In Model I, the residual strength ratio abruptly decreases when loading time approaches the life $t_F^i$ corresponding to applied stress $\sigma_{con}^c$. However, the residual strength does not reach zero even when the loading time exceeds $t_F^i$. Therefore, it is concluded that Model I is inappropriate for predicting residual strength after long loading time. Besides, it is found that effect of $m$ on residual strength ratio is limited for $m < 20$. In Model II, the residual strength becomes almost zero when loading time approaches $t_F^i$, in contrast to in Model I. However, the residual strength is maintained to be approximately constant for the loading time lower than $t_F^i$. Furthermore, the initial residual strength is lower than the initial strength, especially for the case of $m = 5$. Accordingly, it is proven that Model II is unsuitable for predicting the residual strength after short loading time. In addition, the effect of $m$ appears more clearly in Model II than in Model I, and the residual strength is maintained for longer loading time. However, Model II does not

![Fig. 7](image_url) 
**Fig. 7** Comparison of relative residual strength vs. static fatigue loading time for three values of applied stress between Models I and II for $\sigma_{con}^c = 300$ MPa, $\sigma_{con}^c = 400$ MPa, and $\sigma_{con}^c = 500$ MPa.

![Fig. 8](image_url) 
**Fig. 8** Effect of shape parameter and interfacial shear stress on residual strength vs. static fatigue loading time for $\sigma_{con}^c = 300$ MPa, calculated based on Model I.
address the effect of interfacial shear stress.

Figure 7 compares residual strength of the composite after static fatigue between Models I and II. As mentioned above, Model I is appropriate for shorter side of loading time, while Model II is suitable for longer side. Figure 8 presents the effect of shape parameter $m$ and interfacial shear stress $\tau_i$ on residual strength of the composite, based on Model I. As shown in Fig. 1, initial strength is higher for higher $\tau_i$, and the effect of $m$ on the initial strength depends on $\tau_i$. In contrast, the decrease behavior of residual strength is similar among the four cases. Hence, the residual strength ratio does not exhibit strong dependency of $\tau_i$ and $m$.

4. Conclusions

Probabilistic models for residual strength after fatigue in unidirectional fiber-reinforced composites were proposed. Two models (Models I and II) were established to predict residual strength of a composite while the SCG law was employed to predict residual strength of fiber. Additionally, the relationship between lives of the fiber and the composite was investigated. A case study was performed for a CMC to obtain the residual strength after static fatigue loading. The effects of shape parameter of fiber strength and interfacial shear stress on residual strength are analyzed. It was proven that the life and the initial strength of the fiber and the composite depend on the shape parameter and the fiber/matrix interfacial shear stress. In contrast, the residual strength ratio of the composite was found to be almost independent of these two parameters.

Appendix. Probabilistic strength model

We consider a single fiber with length $L$ broken into $N$ fragments due to applied stress $T$. Curtin and Zhou (8) assumed that the distribution of fragment length is random, and that fiber strength obeys Weibull distribution expressed by Eq. (10). When the yield strength in matrix is negligibly small compared with $T$, the composite stress is expressed as

$$\sigma_c = \frac{V_i}{\rho} \frac{\sigma_m}{\bar{\rho}} \left[1 - \exp \left(-\frac{\bar{T}}{\bar{T}}\right)\right]$$  \hspace{1cm} (A1)

where $\bar{\rho} = \rho \delta_c$ and $\bar{T} = T/\sigma_c$ are dimensionless variables of $\rho$ and $T$. $\sigma_{cr}$ and $\delta_{cr}$ denote the critical strength and length given by

$$\sigma_{cr} = \left(\frac{\sigma_m}{\tau_i} L_i \right)^{1/m}$$ \hspace{1cm} \text{and} \hspace{1cm} \delta_{cr} = \left(\frac{\sigma_m L_1 \tau_i^{1/m}}{\tau_i} \right)^{1/m}.$$ \hspace{1cm} (A2)

where $r$ denotes radius of fiber. Maximizing Eq. (A1) noting $\bar{\rho} = \bar{T}^m$, Eq. (11) is obtained.

Iyenger and Curtin (9) derived a constitutive relation for fiber-reinforced composites subjected to variable loading depending on time, considering SCG of fiber. This relation has the same form as Eq. (A1), from which time to failure is derived.

References


