Crushing Behavior of Combined Honeycomb Structure∗

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Abstract

In this work, the crushing behavior of a combined honeycomb structure is studied based on the numerical results of finite element method analysis. It is found that the compressive process of the combined honeycomb, which consists of two honeycombs H-1 and H-2 arranged in different directions, can be divided into two steps corresponding to compression of the constituent honeycombs separately. As a result, the compressive load of the combined honeycomb can be predicted from the theoretical results for each honeycomb H-1 and H-2. Also, the scale of the impactor affects the amount of compressive load, and if the radius of the impactor is 7 times larger than the length l of the cell wall, the compressive load reaches the value for the case of impact by a rigid wall.

Key words: Crushing Behavior, Compressive Load, Finite Element Method, Honeycomb, Absorbed Energy

1. Introduction

The honeycomb structure consisting of hollow hexagons is used in many materials, including the core of automobile bumpers,(1) the test barrier for evaluating automobile collision safety,(2) and the inner material of impact buffers at the crossroads of highways. For the honeycomb structure to absorb collision energy efficiently, various structural techniques are adopted such as a multilayer panel in which honeycombs with differently sized hexagons are laminated,(3) and a panel in which the honeycomb wall thickness is continuously varied.(4) As an example, Fig.1 shows the honeycomb structure found in a highway impact buffer. It can be found from Fig.1 that the inside of the device consists of a sandwich structure where the honeycomb elements are stacked in the horizontal and vertical directions. That is, the vertical honeycomb (H-1) easily deforms under a compressive load in the direction shown in Fig.1 with comparatively low shock energy. Also, the horizontal honeycomb (H-2) can absorb compressive load with comparatively high shock energy through out-of-plane buckling deformation.

In this paper, numerical analysis is carried out using the finite element method (FEM) on the crushing deformation of a honeycomb structure arranged in horizontal and vertical directions as described above, and the effects on the crushing deformation of factors such as the geometry and the material properties of the honeycombs are examined. Our aim is to provide useful guidelines for designing shock buffers by clarifying the crushing characteristics of the combined honeycomb structure.

2. Numerical analysis methods

In this study, the commercial FE analysis code MSC.Dytran is used for the collision analysis of the honeycomb structure. The model shown in Fig.2 is used in our analysis as the target honeycomb structure (hereinafter, called the combined honeycomb structure for simplicity),
Fig. 1 Schematic of combined honeycomb structure

(a) Struck by a rigid wall (b) Struck by a rigid ball

Fig. 2 Geometry of the combined honeycomb structure, with loading conditions for FE analysis

Fig. 3 Global and microstructural parameters of the combined honeycomb structure
in which two honeycomb structures are arranged in horizontal and vertical directions. As a loading condition, a rigid plate or a ball is dropped to the combined honeycomb structure with a constant speed \( V_0 \) from above in the \( z \)-direction. Here, the mass \( M \) of the impactor is assumed to be sufficiently large that the speed of the impactor decreases by less than 1\% during the process of crushing the combined honeycomb structure.

In the combined honeycomb structure, the cells are arranged as follows. The lower honeycomb is made of the cells arranged in 3 vertical layers and 5 horizontal layers, totaling 14 regular hexagonal cells. The upper honeycomb consists of vertical \( n \) layers (\( n \) is an odd number, and \( n = 3 \) is shown in Fig.3 as an example) and horizontal 5 layers, totaling \((9n+1)/2\) regular hexagonal cells \((\theta_0 = 30^{\circ})\), as shown in Fig.3. The wall length of the regular hexagonal cells is denoted as \( t \), and the thickness of the diagonal cell walls is expressed as \( t_v \) and \( t_h \). The thickness of the vertical cell wall is twice that of the diagonal cell wall because of the production method of the honeycomb. Panels (width \( B = 5l \), length \( L = 8.67l \)) of thickness \( t_p \) are placed at both the top and bottom ends of the structure, and between the two types of honeycomb structures.

As for the material characteristics, both the honeycomb and the panel materials are assumed to be isotropic, homogeneous and ideally elastic-plastic with the elastic modulus \( E = 71 \) GPa, Poisson’s ratio \( \nu = 0.3 \), and yield stress \( \sigma_y = 190 \) MPa. Furthermore, the von Mises yield criterion is obeyed in our analysis. The length of the honeycomb cell wall and the panel thickness are set as \( l = 5 \) mm and \( t_p = 1 \) mm, respectively.

In general, the thin-walled structures exhibit inertia and strain rate effects, and they are both velocity and impact mass sensitive. On the other hand, it is often assumed that the influence of strain rate on the dynamically loaded aluminum structure is negligible.\((5),(6)\) In the present study, the collision speed is always assumed to be \( V_0 = 40 \) km/h, and strain-rate insensitive material as a first step, and the effects of impact velocity on the crushing behaviour of combined honeycomb structures will be discussed in the next work.

Moreover, for the actual honeycomb structure, there are vast amount of cells in the structure, so that bigger model or another numerical model with cyclic symmetry condition should be used for making the general conclusion. Instead, we have studied two kinds of problems with different loading conditions as shown in Fig.2(a) and (b). If a striker from the upper side is sufficiently larger than the cross-section of the honeycomb structure, the obtained result is independent of the number of cells, and close to the numerical result by Fig.2(a). On the contrary, if the striker becomes small, the deformation behaviour depends on not only the number of cells, but also the ratio the striker’s width and the cell size. Such a characteristic can be found in the numerical result by Fig.2(b).

In this paper, we discuss the effect of the loading condition on the crushing behaviour as a first step, and such a size effect would be discussed in the next paper.

In the FE analysis, finite element discretization is carried out using four-node quadrilateral thick-shell elements of \( 1 \times 1 \) mm\(^2 \) so that wrinkles appearing on the inside of the structure are smoothed out in the crushing process. In addition, the boundary condition is assumed such that the top and bottom ends of a honeycomb structure are perfectly fixed to the panels. Also, the dynamic and static friction coefficient for the friction between honeycomb and panel are set as 0.2 and 0.3, respectively.

Furthermore, to investigate the deformation mechanism of the combined honeycomb structure in greater detail, the crushing behavior is analyzed in this study for the horizontally and vertically oriented honeycomb structures as shown in Fig.4 (a)(b). These analytical results are then used to evaluate the crushing characteristics of the combined honeycomb structure. Moreover, the honeycombs shown in Fig.4(a)(b) are referred as H-1 and H-2, respectively, and in the following discussion, the parameters with \( h \) as a subscript represent the H-1 honeycomb, and the ones with \( v \) represent the H-2 honeycomb.
3. Results of numerical analysis and discussions

3.1. Compressive deformation process of the combined honeycomb structure

The solid line in Fig.5 shows the variation of compressive load applied to the combined honeycomb structure \((n = 3)\) with \(t_v/t_h = t_h/l = 0.06\) when a rigid plate is made to move quasi-statically in the \(z\)-direction to crush the honeycomb structure. Here, Fig.6 shows the deformation behavior of the combined honeycomb structure at the points A, B, C, and D in Fig.5. As Figs.5 and 6 show, when the crushing load is applied to the combined honeycomb structure, the crushing deformation occurs first in the H-1 honeycomb because the crushing load of the H-1 honeycomb is low and approximately constant (up to \(U_z = 0.01\) m in Fig.5 in the compressive deformation stage). As deformation develops further, the compressive load increases as the H-1 honeycomb becomes denser (in the compressive deformation stage from \(U_z = 0.01\) m to \(U_z = 0.02\) m in Fig.5), and then compressive deformation begins to develop also in the H-2 honeycomb.

Figure 7 shows the variation of compressive load in honeycomb structures H-1 and H-2, both of which have the same structural geometry as the combined honeycomb structure used in the analysis in Fig. 5. The compressive load curve for the H-1 honeycomb shown in Fig. 7 is re-plotted in Fig. 5 (dotted line) to be compared with the earlier result on the combined honeycomb structure. It is found that the two compressive load curves agree with each other in the initial deformation stage. Furthermore, based on the compressive load curves for the H-1 and H-2 honeycomb structures shown in Fig. 7, the sum displacement curve given by

\[
U_z(P_z) = U_{zh}(P_z) + U_{vh}(P_z)
\]  

(1)

is also plotted in Fig. 5 (dashed line). Here, \(U_{zh}(P_z)\) and \(U_{vh}(P_z)\) express compressive displacements in the H-1 and H-2 honeycomb structures, respectively, under the same compressive load of \(P_z\). Comparing the load curve of the combined honeycomb structure with this
dashed line, both lines agree well, except that the peak load of the combined honeycomb structure is lower than that of the H-2 honeycomb structure. The peak loads probably differ because, as shown in Fig. 6, in the combined honeycomb structure, deformation of the H-1 honeycomb occurs earlier than deformation of the H-2 honeycomb, and the panel between them is bent and bowed, thus the compressive load is concentrated on a part of the H-2 honeycomb structure.

Therefore, it can be concluded that the compressive load of the combined honeycomb structure, which is made by combining H-1 and H-2 honeycomb structures, can be approximately predicted from the results on the compressive load in the two separate honeycomb structures. Because longer calculation times are required to calculate the performance of a combined honeycomb structure than individual honeycomb components, and because the economic cost of carrying out crushing experiment is also greater for a combined structure, a method for predicting the behavior of a combined honeycomb structure from the results of analyzing its individual components would be useful.

3.2. Crushing characteristics of H-1 honeycomb

To evaluate the crushing characteristics of the H-1 honeycomb, FE analysis was carried out for three H-1 honeycomb structures with different numbers of layers \((n = 3, 5, 9)\). The solid lines in Fig. 8 show the relation between compressive load \(P_z\) and displacement \(U_z\).

In discussing the crushing characteristics of H-1 honeycomb under a uniaxial crushing
Fig. 8 Compressive load-displacement behavior of the combined honeycomb with different number of layers \( n = 3, 5 \) and 9

Fig. 9 Schematic diagram of H-1 honeycomb

load, Klintworth et al.\(^{(7)}\) proposed a formula for predicting compressive stress \( \sigma_z \) under the symmetrical compressive deformation of the H-1 honeycomb by equating the work done by the external force with the work done by the plastic hinge. According to their theory, with respect to the angle \( \theta \) of the diagonal cell wall in the basic unit (the area ABCD surrounded by dotted lines) in Fig. 9, the compressive stress \( \sigma_z \) and the compressive displacement \( U_z|_1 \) are given by

\[
\sigma_z = \frac{\sigma_s}{2 \cos \theta_0 \cos \theta} \left( \frac{l_0}{l} \right)^2, \tag{2}
\]

\[
U_z|_1 = l(\sin \theta_0 - \sin \theta). \tag{3}
\]

Equations (2) and (3) indicate that the compressive load \( P_z \) on the H-1 honeycomb as discussed in this study is given by

\[
P_z = \sigma_z BL = \frac{\sigma_s}{2 \cos \theta_0 \cos \theta} \left( \frac{l_0}{l_e} \right)^2 BL \tag{4}
\]

and the compressive displacement of the H-1 honeycomb with \( n \) layers is given by

\[
U_z = (n - 1)U_z|_1 = (n - 1)l_e(\sin \theta_0 - \sin \theta). \tag{5}
\]

Here, considering the decrease in the effective length of a diagonal cell wall affected by the cell wall thickness, \( l \) in Eqs.(2) and (3) is replaced by the effective length \( l_e \), as shown in Fig. 9. The effective length \( l_e \) is given analytically by

\[
l_e = l - t_h \tan \theta_0. \tag{6}
\]

The dashed lines in Fig. 8 show the relation between the compressive load and displacement of H-1 honeycomb as given by Eqs.(4) and (5). These curves agree very well with the result of numerical FE analysis (solid lines in Fig. 8).
Equation (5) gives the compressive displacement $\delta_{ahl}$ ($\delta_{ahl}$ is shown in Fig. 8 for the H-1 honeycomb with $n = 9$ layers). Also, in the present analysis, the honeycomb structure consists of regular hexagonal cells ($\theta_0 = 30^\circ$), and when the angle $\theta = -\theta_0$, the upper and lower inclined cell walls are contacted from each other. That is, Eq. (5) corresponds to the change in the slope of the diagonal cell wall from $\theta = \theta_0$ to $\theta = -\theta_0$, which is given by

$$\delta_{ahl} = 2(n - 1)l_1 \sin \theta_0. \quad (7)$$

As Fig. 8 shows, the compressive load obtained from FE analysis is roughly constant in the deformation stage with compressive displacement up to $\delta_{ahl}$. This can be understood from Eq.(4). That is, Eq.(4) shows that the compressive load is the same for the slopes of the diagonal cell wall $\theta = \theta_0$ and $\theta = -\theta_0$. Furthermore, in the compressive deformation from $\theta = \theta_0$ to $\theta = -\theta_0$, the compressive load first decreases and takes a minimum value at $\theta = 0$. The minimum value is $\cos \theta_0 (\approx 0.87)$ times the value of the compressive load $P_z$ at $\theta = \theta_0$. Therefore, the compressive load is roughly constant in this deformation stage.

Furthermore, in this stage, the average compressive load $P_{ave|h}$ is found using Eqs.(4) and (5) to be

$$P_{ave|h} = \frac{\theta_0 \sigma_s}{2 \cos \theta_0 \sin \theta_0} \left(\frac{t_1}{t_e}\right)^2 BL. \quad (8)$$

Figure 10 shows a comparison of the average compressive load $P_{ave|h}$, as given by Eq.(8), and the average compressive load up to the compressive displacement $U_z = \delta_{ahl}$, as calculated from the compressive load obtained by FE analysis.

The compressive displacement before complete collapse (hereinafter, called the effective compression distance) of the H-1 honeycomb is denoted as $\delta_{ahl}$ and is shown in Fig.8 ($\delta_{ahl}$ is shown in Fig.8 for each type of H-1 honeycomb). Fig.11 shows the deformed shape at the displacement $U_z = \delta_{ahl}$ for the honeycomb with $n=5$ layers. As seen in Fig.11, the thickness of the H-1 honeycomb (distance AB in the Fig.11) remaining after the final crushing is approximately equal to the length $l$ of the cell wall before the deformation starts. This indicates
that the effective compression distance $\delta_{ulh}$ for the complete collapse is approximately given by

$$\delta_{ulh} = \left[(n-1) + (n+1) \sin \theta_0 \right] l - n t. \quad (9)$$

Figure 12 compares the estimated effective compression distance $\delta_{ulh}$ given by Eq.(9) and the effective compression distance $\delta_{u lh}$ obtained by FE analysis. As Fig.12 shows, both results approximately agree, irrespective of the number of layers $n$. This clearly indicates the effectiveness of Eq.(9).

### 3.3. Crushing characteristics of H-2 honeycomb

Figure 13 shows the compressive load for the H-2 honeycomb. As shown in Fig.13, in this paper, the compressive displacement up to the point where densification begins (where the compressive load begins to increase sharply) is called the effective compression distance $\delta_{ulv}$, and the average compressive load in this crushing process is $P_{avev}$. Figure 14 shows the relation between average compressive load $P_{avev}$ and thickness $t_v$. The average load (solid line) calculated according to Wierzbicki’s theory(8) is also shown in Fig.14. In his study, Wierzbicki(8) suggested that the average load $P_{avev}$ of honeycomb H-2 as studied here can be evaluated by using

$$P_{avev} = 6.63\sigma_s \left(\frac{t_v}{l}\right)^{5/3} \times BL. \quad (10)$$

It can be found from Fig.14 that the value obtained in our analysis is somewhat larger than the value predicted by Wierzbicki’s theory(8). This is probably due to the constraints at the upper and lower ends of the honeycomb, because the number of units used in our analysis is small as compared with the standard honeycomb H-2.

Furthermore, Fig.15 shows the relation between the effective compression distance $\delta_{ulv}$ and the ratio $t_v/l$ under the same height $h_v$. Referring to Abramowicz’s study(9) on the effective compression distance of a thin rectangular tube in the axial direction, the ratio between effective compression distance $\delta_{ulv}$ and height $h_v$ is estimated to be

$$\frac{\delta_{ulv}}{h_v} \approx 0.73. \quad (11)$$

Figure 15 also shows the value obtained by the approximate formula (11). Good agreement is obtained between the result of our FE analysis and the approximate Equation (11), regardless of the thickness $t_v$. 
Fig. 13  Comparison of compressive load and displacement for the H-2 honeycomb for three wall thicknesses $t_v$

Fig. 14  Variation of average compressive load $P_{av1}$ with $t_v/l$ in the H-2 honeycomb

Fig. 15  Variation of crushing distance $\delta_{uc}$ with $t_v/l$ in the H-2 honeycomb
3.4. Designing the combined honeycomb structure based on the crushing characteristics of H-1 and H-2 honeycombs

The above discussions indicate that the crushing behavior of the combined honeycomb structure composed of H-1 and H-2 honeycombs can be predicted from the crushing behavior of the separate honeycomb structures. In this section, therefore, using the formulas obtained in the previous sections for the compressive load \( P_{ave | h} \) and \( P_{ave | v} \) of the H-1 and H-2 honeycombs and for the effective compression distance \( \delta_{ul | h} \) and \( \delta_{ul | v} \), we examine a method for predicting the absorbed impact energy \( W \) and the compressive load ratio \( P_{ave | h} / P_{ave | v} \) in the two-step deformation process of the combined honeycomb structure (first the H-1 honeycomb deforms, and then the H-2 honeycomb deforms). The absorbed impact energy and the compressive load ratio are needed for designing combined honeycomb structures.

3.4.1. Prediction of absorbed impact energy \( W \)

Figure 16 shows the method for predicting the amount of impact energy \( W \) absorbed by the combined honeycomb structure based on the compressive load and the effective compression distance of the H-1 and H-2 honeycombs. When the combined honeycomb structure receives a compressive load, the area of H-1 honeycomb first deforms up to displacement \( U_z = \delta_{ul | h} \) under the compressive load equivalent to \( P_{ave | h} \), and then the area of H-2 honeycomb deforms from \( U_z = \delta_{ul | h} \) up to \( U_z = \delta_{ul | h} + \delta_{ul | v} \) under the compressive load equivalent to \( P_{ave | v} \). Therefore, the amount of energy \( W \) absorbed by the combined honeycomb structure before the complete collapse is evaluated using the following equation:

\[
W = P_{ave | h} \times \delta_{ul | h} + P_{ave | v} \times \delta_{ul | v}.
\]  

(12)

Figure 17 shows a comparison of the prediction from Eq.(12) and the results of FE analysis of impact energy \( W \) absorbed by the combined honeycomb structure. As seen in Fig.17 that the approximation (Eq.(12)) can predict well the amount of impact energy \( W \) absorbed by the combined honeycomb structure.

3.4.2. Ratio of compressive load in the two-step deformation of combined honeycomb structure

As discussed above, in the crushing of a combined honeycomb structure, the deformation process is divided into two stages, where the average compressive load \( P_{ave | h} \) in the first step (H-1 honeycomb deformation), is smaller than the load \( P_{ave | h} \) in the second step (H-2 honeycomb deformation), and the difference between the two is significant. In order to increase the amount of energy absorbed by the combined honeycomb structure, it is necessary to appropriately control the difference between the two. Here, by expressing the ratio between the two as \( P_{ave | h} / P_{ave | v} \), the effect of the thickness \( t_h \) of H-1 honeycomb on \( P_{ave | h} / P_{ave | v} \) is discussed in this section.
Figure 18 shows the compressive load curve of the combined honeycomb structure for the five thicknesses $t_h$ of H-1 honeycomb. Furthermore, Fig. 19 shows the finally deformed shapes for $t_h/t_v=1.0$ and 3.3. The compressive load at the first stage increases with the thickness $t_h$ of H-1 honeycomb. While the deformation progresses from H-1 to H-2 when the thickness $t_h$ is small, the deformation proceeds simultaneously in both areas if the thickness $t_h$ is large. Also, it can be seen in Fig. 19(b) that the deformation of H-2 honeycomb begins at $t_h/t_v \approx 3.3$ before the deformation of H-1 honeycomb finishes. If the thickness $t_h$ of the cell wall of the H-1 honeycomb is as large as, for example, $t_h/t_v = 5$, the ratio of average load satisfies $P_{aveh} > P_{avew}$, and the compressive deformation of the combined honeycomb structure begins in the H-2 honeycomb, instead of the H-1 honeycomb. Note that, for a combined honeycomb structure like this, the method shown in Fig. 16 cannot be applied to predict the amount of energy absorbed by the combined honeycomb structure.

On the other hand, it can be concluded from Fig. 18 that by appropriately controlling $t_h$, we can reduce the difference of compressive load in the two-step deformation and increase the absorption $W$. Figure 20 shows the result of FE analysis and the theoretical prediction of the relation between thickness $t_h/t_v$ and compressive load ratio $P_{aveh}/P_{avew}$. We find that both results agree well at the thickness ratio of approximately $t_h/t_v \leq 3$. 

Fig. 17 Comparison of absorbed energy $W$ obtained by FE analysis and theoretical analysis

Fig. 18 Relation between compressive load and displacement with the ratio $t_h/t_v$ as a parameter
Collison of a ball

Up to the above discussions, the colliding body has been assumed to be a rigid plate. In the following, another FE analysis of the crushing behavior of a combined honeycomb structure hit by a rigid ball is conducted in order to study the effect of the shape of the colliding body.

Figure 21 shows a comparison of the H-1 honeycomb side (Case A in the figure) and the H-2 honeycomb side (Case B in the figure) being hit by the ball (r = ∞ denotes the collision of a flat plate). In both cases, the compressive load decreases as the radius r of the ball decreases, and, depending on the contact area at the time of collision, the trend is seen more clearly at the stage of the compressive deformation of the H-2 honeycomb. The final shape of deformation is shown for both cases in Fig.22 with the radius to cell wall length ratio of $r/l = 2$. If the collision is on the H-1 honeycomb side (Case A), the load decreases less because the load is transmitted to the H-2 honeycomb via the H-1 honeycomb, which is more easily deformed. Here, to quantitatively examine the effect of the shape of a colliding body on the amount of absorbed energy, the relation between the energy ratio $W_{\text{ball}}/W_{\text{wall}}$ and the radius of the ball $r$ is shown in Fig.23, where $W_{\text{wall}}$ is the absorbed energy for the collision of a rigid flat plate and $W_{\text{ball}}$ is the absorbed energy for the collision of a ball. It can be concluded from Fig.23 that if the collision is on the H-1 honeycomb side and if $r/l \geq 7$ holds, the amount of energy absorption reaches 90% of the collision of a flat plate, and so the approximation as proposed in the present study can be well applied.

4. Conclusions

In this study, the crushing characteristics of a combined honeycomb structure during collisions were investigated by FE analysis, in particular the relation between the compressive load and the geometry of the honeycomb structure. The following conclusions were obtained.

1) Except for the case where the cell wall in the H-1 honeycomb, which is easily deformed, is extremely thick, the deformation behavior of the combined honeycomb structure is explained by a two steps process of initial H-1 honeycomb deformation followed by H-2...
Fig. 21 Compressive load-displacement behavior of the combined honeycomb hit by a rigid ball from different directions

(a) Case A  (b) Case B

Fig. 22 Deformed shapes of the combined honeycomb ($t_e = t_h = 0.3\text{mm}, \; n = 3$) hit by a rigid ball ($r/l=2$) from different directions

Fig. 23 The ratio of absorbed energy $W_{\text{ball}}/W_{\text{wall}}$ as a function of $r/l$
honeycomb deformation. As a result, the load of the combined honeycomb structure can be approximately predicted from the data on the separate honeycombs.

(2) The compressive load that the H-1 and H-2 honeycombs receive can be predicted by using Eq.(10) and Eq.(8), respectively, and the effective compression distance can be calculated by using Eq.(11) and Eq.(9), respectively. Therefore, the amount of energy absorbed by the combined honeycomb structure is evaluated by using the model shown in Fig.16.

(3) If the colliding body is a rigid ball, the compressive load is small compared with the case of a flat plate, and the decrease rate grows as the size of the ball decreases. However, the decrease of load is smaller in the collision on the H-1 honeycomb side than in the collision on the H-2 honeycomb side.

References