Changes in ISO 15653-Based CTOD for Specimens of $a_0/W=0.45^*$

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Abstract

ISO 15653 is a Crack Tip Opening Displacement (CTOD) testing standard for welds, and has issued since 2010. This new testing standard consists of two different CTOD calculations, displacement-conversion CTOD for deep-notched specimens and J-conversion CTOD for shallow-notched specimens, and ISO 15653-based CTOD changes at the boundary of crack length between deep-notched and shallow-notched specimens. In this study, changes in ISO 15653-based CTOD were investigated by using experimental and analytical data for the wide range of steel mechanical properties. J-conversion CTOD did not correspond to displacement-conversion CTOD, and there were significant mismatches in ISO 15653-based CTOD at the crack length boundary particularly for the steels of the low strain hardening exponent of the Ramberg-Osgood relation. Displacement-conversion CTOD calculation by using the modified plastic rotational factor or using the CTOD transformation equation proposed in this study is helpful in moderating the underestimated CTOD for low strain hardening exponent shallow-notched specimens.

Key words: CTOD, J-Integral, Elastic-Plastic Fracture, Fracture Toughness, Shallow-Notched Specimen, Testing Standard, ISO

1. Introduction

CTOD has been widely used as an elastic-plastic fracture mechanics parameter for structural steel. CTOD fracture toughness testing was initially standardized by the British Standards Institution in BS5762⁽¹⁾, which was replaced by BS7448⁽²⁾, and was extended to other CTOD testing standards such as ASTM E1290⁽³⁾ and WES1108⁽⁴⁾. These standards conventionally assume the plastic hinge model, and a few constant plastic rotational factors are used for converting Crack Mouth Opening Displacement (CMOD) into the plastic component of CTOD. On the other hand, the normalized crack length and the strain hardening exponent affect the plastic rotational factor⁽⁵⁾⁽⁶⁾, and this is disadvantageous to the accurate calculation of CTOD. ASTM International revised E1290 in 2002⁽⁷⁾, and its latest edition was issued in 2008⁽⁸⁾, where CTOD was not calculated by using the plastic hinge model, but was converted from J-integral. Because structural steel qualifications such as API RP2Z⁽⁹⁾ and DNV-OS-C401⁽¹⁰⁾ have essentially required fracture toughness in accordance with CTOD tests using the plastic hinge model, a discrepancy between two different kinds of CTOD, conventional displacement-conversion CTOD and new J-conversion CTOD, should be avoided.

*Received 4 Oct., 2011 (No. 11-0597)

[DOI: 10.1299/jmmp.6.645]

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Table 1 CTOD testing standard comparison

Standard		Year	CTOD	Initial normalized crack length
BS7448: Part 1		1991	Displacement-conversion	$0.45 \leq a_0/W \leq 0.55$
WES1108		1995	Displacement-conversion	$0.45 \leq a_0/W \leq 0.55$
ASTM E1290		2008	J-conversion	$0.45 \leq a_0/W \leq 0.70$
ISO 12135		2002	Displacement-conversion	$0.45 \leq a_0/W \leq 0.70$
ISO 15653	Normative	2010	Displacement-conversion	$0.45 \leq a_0/W \leq 0.70$
	Annex E (informative)	2010	J-conversion	$0.10 \leq a_0/W \leq 0.45$

The International Organization for Standardization (ISO), which promulgates worldwide industrial and commercial standards, has set a CTOD testing standard of ISO 12135⁽¹¹⁾ for deep-notched bend specimens since 2002, where CTOD is displacement-conversion and determined by using the plastic hinge model. In addition, ISO has issued another CTOD testing standard of ISO 15653⁽¹²⁾ specially for welds since 2010. This new CTOD testing standard consists of a normative test method for a deep-notched bend specimen of $0.45 \leq a_0/W \leq 0.70$ and an informative test method in Annex E for a shallow-notched bend specimen of $0.10 \leq a_0/W \leq 0.45$, as shown in Table 1, where a_0 is the initial crack length and W is the specimen width. Although CTOD in ISO 15653 is displacement-conversion for deep-notched specimens, it is J-conversion for shallow-notched specimens; both CTODs are available for a specimen of $a_0/W=0.45$. However, it has been unclear whether two different CTOD calculations result in the identical value of CTOD for a specimen of $a_0/W=0.45$. The authors previously investigated the relationship between displacement-conversion CTOD and J-conversion CTOD for deep-notched⁽¹³⁾⁻⁽¹⁵⁾ and shallow-notched⁽¹⁶⁾ specimens, and demonstrated that J-conversion CTOD was occasionally much lower than displacement-conversion CTOD for brittle fracture and the CTOD relationship was particularly affected by the strain hardening exponent. This suggests that there is a significant mismatch between two CTODs at $a_0/W=0.45$ in ISO 15653.

In this study, eighteen force vs. CMOD curves of deep-notched and shallow-notched specimens were referred to, and their critical CTOD values were calculated by using ISO 15653. Changes in ISO 15653-based CTOD were investigated at $a_0/W=0.45$ for the wide range of steel mechanical properties.

Nomenclature

- A : plastic area under the force versus crack mouth opening displacement curve
- a_0 : initial crack length
- B : specimen thickness
- E : Young's modulus
- F : force
- F_0 : reference force as the limit load in the EPRI scheme
- f_1, f_2 : CTOD transformation factors
- h_1, h_2 : factors in the EPRI scheme
- J_0 : J-integral
- $J_{0,el}$: elastic component of J_0
- $J_{0,pl}$: plastic component of J_0
- K_0 : stress intensity factor
- m : constraint factor

n : strain hardening exponent of the n-power hardening law
 N : strain hardening exponent of the Ramberg-Osgood relation
 R_{mb} : tensile strength
 $R_{p0.2b}$: 0.2% offset yield strength
 R_Y : effective yield strength
 r_{pl} : plastic rotational factor
 S : span in SE(B) specimens
 V_{pl} : plastic component of crack mouth opening displacement
 W : specimen width
 z : distance of the knife edge measurement point from the front face
 α : material factor in the plastic term of the Ramberg-Osgood relation
 δ_{disp} : displacement-conversion CTOD
 $\delta_{disp,el}$: elastic component of δ_{disp}
 $\delta_{disp,pl}$: plastic component of δ_{disp}
 $\delta_{ISO15653}$: ISO 15653-based CTOD
 δ_J : J-conversion CTOD
 $\delta_{J,el}$: elastic component of δ_J
 $\delta_{J,pl}$: plastic component of δ_J
 ε : strain
 ε_{ys} : yield strain
 η_c : plastic J-calibration eta factor
 ν : Poisson's ratio
 σ : stress

2. Background of CTOD calculation

2.1 Displacement-conversion CTOD

One kind of CTOD in ISO 15653 is the displacement-conversion CTOD, δ_{disp} , for $0.45 \leq a_0/W \leq 0.70$, and is given by the following equation⁽¹²⁾, where $\delta_{disp,el}$ is the elastic part of δ_{disp} , $\delta_{disp,pl}$ is the plastic part of δ_{disp} , K_0 is the stress intensity factor, ν is Poisson's ratio, E is Young's modulus, $R_{p0.2b}$ is the 0.2% offset yield strength, r_{pl} is the plastic rotational factor, V_{pl} is the plastic part of CMOD, a_0 is the initial crack length and z is the distance of the knife edge measurement point from the front face on Single Edge-notch Bend (SE(B)) specimens. The plastic hinge model is used for calculating $\delta_{disp,pl}$, and r_{pl} is set at 0.4 for SE(B) specimens.

$$\begin{aligned}
 \delta_{disp} &= \delta_{disp,el} + \delta_{disp,pl} \\
 &= \frac{K_0^2(1-\nu^2)}{2R_{p0.2b}E} + \frac{r_{pl}(W-a_0)V_{pl}}{r_{pl}(W-a_0) + a_0 + z}
 \end{aligned} \quad (1)$$

2.2 J-conversion CTOD

The other kind of CTOD in ISO 15653 is the J-conversion CTOD, δ_J , for $0.10 \leq a_0/W \leq 0.45$, and is given by the following equation⁽¹²⁾, where J_0 is the J-integral, R_Y is the effective yield strength given by the average of $R_{p0.2b}$ and the tensile strength, R_{mb} , $\delta_{J,el}$ is the elastic part of δ_J , $\delta_{J,pl}$ is the plastic part of δ_J , $J_{0,el}$ is the elastic part of J_0 , $J_{0,pl}$ is the plastic part of J_0 , A is the plastic area under the plot of force, F , versus CMOD and B is the specimen thickness.

$$\begin{aligned}
 \delta_J &= \frac{J_0}{mR_Y} \\
 &= \frac{1}{mR_Y} (J_{0,el} + J_{0,pl})
 \end{aligned}$$

$$= \frac{1}{mR_Y} \left[\frac{K_0^2(1-\nu^2)}{E} + \frac{\eta_c A}{B(W-a_0)} \right] \quad (2)$$

m and η_c are the constraint factor and the plastic J-calibration eta factor, respectively, and given by the following equations for SE(B) specimens.

$$m = A_0 - A_1 \left(\frac{R_{p0.2b}}{R_{mb}} \right) + A_2 \left(\frac{R_{p0.2b}}{R_{mb}} \right)^2 - A_3 \left(\frac{R_{p0.2b}}{R_{mb}} \right)^3 \quad (3)$$

$$A_0 = 3.18 - 0.22 \left(\frac{a_0}{W} \right), \quad A_1 = 4.32 - 2.23 \left(\frac{a_0}{W} \right)$$

$$A_2 = 4.44 - 2.29 \left(\frac{a_0}{W} \right), \quad A_3 = 2.05 - 1.06 \left(\frac{a_0}{W} \right)$$

$$\eta_c = 3.667 - 2.199 \left(\frac{a_0}{W} \right) + 0.437 \left(\frac{a_0}{W} \right)^2 \quad (4)$$

3. CTOD fracture toughness testing

3.1 CTOD test data

Eighteen F vs. CMOD curves of four steels, YR70⁽¹⁷⁾, YR90⁽¹⁷⁾, A517⁽¹⁸⁾ and HY80⁽¹⁹⁾, were referred to, and their mechanical properties and CTOD test conditions are shown in Table 2, where S is the span of SE(B) specimens. N is the strain hardening exponent of the Ramberg-Osgood relation denoted by the following equation, where ε is the strain, ε_{ys} is the yield strain, σ is the stress and α is the material factor in the plastic term.

$$\frac{\varepsilon}{\varepsilon_{ys}} = \frac{\sigma}{R_{p0.2b}} + \alpha \left(\frac{\sigma}{R_{p0.2b}} \right)^N \quad (5)$$

N is equivalent to the inverse of the strain hardening exponent of the n -power hardening law, n , and it can be estimated empirically from the following equation in ASTM E1290-02⁽⁷⁾.

$$n = \frac{1}{N} \\ = 1.724 - 6.098 \left(\frac{R_{p0.2b}}{R_{mb}} \right) + 8.326 \left(\frac{R_{p0.2b}}{R_{mb}} \right)^2 - 3.965 \left(\frac{R_{p0.2b}}{R_{mb}} \right)^3 \quad (6)$$

All the CTOD tests were conducted by using SE(B) specimens, and their a_0/W ranged from 0.17 to 0.61. The critical values of δ_{disp} and δ_I were evaluated by Eqs. (1) and (2), respectively, where r_{pl} was modified for $a_0/W < 0.45$ according to Kirk et al.⁽⁵⁾.

Table 2 Mechanical properties and CTOD test conditions

Steel	Mechanical properties at CTOD test temp.			CTOD test conditions					Ref.
	$R_{p0.2m}$ (MPa)	R_{mb} (MPa)	N	B (mm)	W (mm)	S (mm)	a_0/W	Temp. (°C)	
YR70	612	794	6.47	15	30	120	0.3, 0.5	-120	(17)
YR90	776	825	18.1	15	30	120	0.3, 0.5	-120	(17)
A517	834	924	11.4	25.4	25.4	102	0.17, 0.5	-76	(18)
HY80	630	735	8.53	25.4	50.8	203	0.29~0.61	21	(19)

3.2 Results of ISO 15653-based CTOD calculation

The relationships between the critical CTOD and a_0/W are shown in Figs 1 ~ 4, where the critical CTOD values were calculated for brittle fracture in YR70, YR90 and A517, and for plastic instability in HY80. Although the critical values of δ_{disp} and δ_j were scattered in YR70 and YR90, they tend to increase with decreasing a_0/W . The number of CTOD test data was not numerous in A517 and HY80, but δ_{disp} and δ_j obviously exhibited a rise in their critical values of shallow-notched specimens. There was a significant difference between δ_{disp} and δ_j in each low N steel, YR70 or HY80, while the difference between δ_{disp} and δ_j was quite small in each high N steel, YR90 or A517.

ISO 15653-based CTOD, $\delta_{ISO15653}$, is δ_{disp} for $0.45 \leq a_0/W$, and is δ_j for $a_0/W \leq 0.45$. The relationships between the critical $\delta_{ISO15653}$ values and a_0/W are also drawn in Figs 1 ~ 4. The lines of CTOD discontinuity were recognized in $\delta_{ISO15653}$ at $a_0/W=0.45$, and the discontinuity was remarkable in YR70 and HY80; there was a significant mismatch of $\delta_{ISO15653}$ at $a_0/W=0.45$.

Fig. 1 ISO 15653-based critical CTOD variation with the normalized crack length in YR70

Fig. 2 ISO 15653-based critical CTOD variation with the normalized crack length in YR90

Fig. 3 ISO 15653-based critical CTOD variation with the normalized crack length in A517

Fig. 4 ISO 15653-based critical CTOD variation with the normalized crack length in HY80

Table 3 Analytical conditions

Mechanical properties		Specimen		
$R_{p0.2b}$ (MPa)	N	a_0/W	B (mm)	Configuration
300, 600, 900	5, 10, 16	0.45	25.4	SE(B)

4. CTOD analysis

4.1 Analytical methodology

The Electric Power Research Institute (EPRI) scheme⁽²⁰⁾ was used for the calculation of $J_{0,pl}$ and V_{pl} in the following equations, where F_0 is the limit load of SE(B) specimens.

$$J_{0,pl} = \alpha \varepsilon_{ys} R_{p0.2b} (W - a_0) h_1 \left(\frac{F}{F_0} \right)^{N+1} \quad (7)$$

$$V_{pl} = \alpha \varepsilon_{ys} a_0 h_2 \left(\frac{F}{F_0} \right)^N \quad (8)$$

$$F_0 = \frac{1.455 B (W - a_0)^2 R_{p0.2b}}{S} \quad (9)$$

h_1 and h_2 are factors, which relate to a_0/W , N , specimen configuration and constraint conditions, and are listed in the EPRI report⁽²⁰⁾. δ_{disp} and δ_j were analytically obtained by substituting V_{pl} of Eq. (8) into Eq. (1) and by substituting $J_{0,pl}$ of Eq. (7) into Eq. (2), respectively⁽¹³⁾. Table 3 shows the analytical conditions used in this analysis. $R_{p0.2b}$ and N were widely arranged from 300 MPa to 900 MPa and 5 to 16, respectively.

4.2 Results of ISO 15653-based CTOD analysis

Figure 5 shows the relationships between δ_j/δ_{disp} and F/F_0 , in which a_0/W and N were set at 0.45 and 5, respectively, and three different $R_{p0.2b}$ values were used for the analysis. Although low $R_{p0.2b}$ steel gave slightly lower δ_j/δ_{disp} , three curves were closely located in the part of $\delta_j/\delta_{disp} < 1$. δ_j/δ_{disp} was not constant, but changed according to F/F_0 . The minimum δ_j/δ_{disp} value was 0.668 at $F/F_0 = 0.90$ for the mechanical model of $R_{p0.2b} = 300$ MPa. Three curves of δ_j/δ_{disp} vs. F/F_0 are shown in Fig. 6, in which a_0/W and $R_{p0.2b}$ were set at 0.45 and 600 MPa, respectively, and three different N values were used for the analysis. δ_j/δ_{disp} was reduced with decreasing N , and δ_j/δ_{disp} was smaller than 1 for $N = 5$. However, the curves of δ_j/δ_{disp} vs. F/F_0 were located in the part of $\delta_j/\delta_{disp} > 1$ for $N = 10$ and 16.

Fig. 5 Relationships between the ratio of J-conversion CTOD to displacement-conversion CTOD and the normalized force in three different 0.2% offset yield strength models

Fig. 6 Relationships between the ratio of J-conversion CTOD to displacement-conversion CTOD and the normalized force in three different strain hardening exponent models

5. Discussion

5.1 Changes in ISO 15653-Based CTOD at $a_0/W=0.45$

As shown in Figs 1 ~ 4, both the critical δ_{disp} and the critical δ_J increased with decreasing a_0/W . This is fundamentally caused by the loss of constraint⁽²¹⁾. In addition, the critical δ_J did not correspond to the critical δ_{disp} , and the critical δ_J was particularly smaller than the critical δ_{disp} in low N steels such as YR70 and HY80. This is probably because δ_J/δ_{disp} is highly affected by N , as shown in Fig 6, and the critical $\delta_{ISO15653}$ was conspicuously discontinuous at $a_0/W=0.45$ in YR70 and HY80.

ISO 15653 uses two different CTODs, δ_{disp} and δ_J , but one kind of CTOD should be consistently utilized for qualification; almost all the CTOD requirements are originally prescribed in the critical δ_{disp} values⁽⁹⁾⁽¹⁰⁾. In addition, changes in CTOD from one to the other at $a_0/W=0.45$ make it difficult to show the effect of a_0/W on the critical CTOD in low N steels; although the critical δ_{disp} increased with decreasing a_0/W on average, the critical $\delta_{ISO15653}$ of $a_0/W=0.39$ was lower than those of $a_0/W=0.55$ and $a_0/W=0.61$ in HY80, as shown in Fig. 4. In other words, $\delta_{ISO15653}$ for shallow-notched specimens underestimates the critical CTOD. ISO 15653 does not use δ_{disp} for shallow-notched specimens, but the nominal δ_{disp} can be calculated for shallow-notched specimens by using r_{pl} modified for $a_0/W < 0.45$, as conducted in this study. Another way of calculation δ_{disp} is the transformation of δ_J into δ_{disp} .

5.2 Transformation of δ_J into δ_{disp}

The authors⁽¹³⁾ constructed an equation of the transformation of δ_{disp} to δ_J , and it was demonstrated that the transformation equation is reasonably accurate⁽¹⁶⁾ for shallow-notched specimens whose a_0/W is within the valid range of in the EPRI scheme. Inversely, the following equation can transform δ_J into δ_{disp} for the shallow-notched specimens.

$$\delta_{disp} = \frac{1}{f_1} \left(\delta_{J,el} + \frac{1}{f_2} \frac{F_0}{F} \delta_{J,pl} \right) \quad (10)$$

$$f_1 = \frac{2R_{p0.2b}}{mR_Y}, \quad f_2 = \frac{h_1[r_{pl}(W - a_0) + a_0 + z]}{2r_{pl}a_0h_2}$$

The critical δ_J was transformed into the critical δ_{disp} by using Eq. (10), and shown in Figs 7 ~ 10. The transformed critical δ_{disp} values were close to the original critical δ_{disp}

Fig. 7 Transformed critical displacement-conversion CTOD and the ISO 15653-based critical CTOD variations with the normalized crack length in YR70

Fig. 8 Transformed critical displacement-conversion CTOD and the ISO 15653-based critical CTOD variations with the normalized crack length in YR90

Fig. 9 Transformed critical displacement-conversion CTOD and the ISO 15653-based critical CTOD variations with the normalized crack length in A517

Fig. 10 Transformed critical displacement-conversion CTOD and the ISO 15653-based critical CTOD variations with the normalized crack length in HY80

values in YR70 and HY80, and Eq. (10) is useful for moderating the underestimated CTOD for shallow-notched specimens. On the other hand, the transformed critical δ_{disp} was almost invariable in YR90 and A517 because δ_J is essentially close to δ_{disp} for high N steels.

In this study, CTOD test data used for the experimental investigation into ISO 15653-based CTOD change at $a_0/W=0.45$ were limited; four kinds of steel were referred to the investigation. The effects of mechanical properties, specimen sizes, test temperatures and applied forces on the CTOD change should be experimentally investigated in detail in the future.

6. Concluding remarks

In this study, ISO 15653-based CTOD was investigated by using experimental and analytical data. According to two different CTOD calculations in ISO 15653, there were significant mismatches in ISO 15653-based CTOD at $a_0/W=0.45$ particularly for the steels of the low strain hardening exponent of the Ramberg-Osgood relation. Displacement-conversion CTOD calculation by using the modified plastic rotational factor or using the CTOD transformation equation proposed in this study is helpful in moderating the underestimated CTOD for shallow-notched specimens.

Acknowledgement

The authors would like to acknowledge the members of the Fracture Toughness Evaluation Committee of the Japan Welding Engineering Society.

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