Nonlinear Bending Stiffness of Plates Clamped by Bolted Joints under Bending Moment*

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Abstract

Equivalent stiffness of plates clamped by bolted joints for designing should be evaluated according to not only the strength of bolted joints but also the deformation and vibration characteristics of the structures. When the applied external axial load or the bending moment is sufficiently small, the contact surfaces of the bolted joint are stuck together, and thus both the bolt and the clamped plates deform linearly. Although the sophisticated VDI 2230 code gives the appropriate stiffness of clamped plates for the infinitesimal deformation, the stiffness may vary nonlinearly with increasing the loading because of changing the contact state. Therefore, the present paper focuses on the nonlinear behaviour of the bending stiffness of clamped plates by using Finite Element (FE) analyses, taking the contact condition on bearing surfaces and between the plates into account. The FE models of the plates with thicknesses of 3.2, 4.5, 6.0 and 9.0 mm tightened with M8, 10, 12 and 16 bolts were constructed. The relation between bending moment and bending compliance of clamped plates is found to be categorized into three regions, namely, (i) constant compliance with fully stuck contact surfaces, (ii) transition showing the nonlinear compliance, and (iii) constant compliance with one-side contact surfaces. The mechanical models for these three regions are proposed and compared with FEM solutions. The prediction on the bounds of three regions is in a fairly good agreement except the case with smaller bolts and thicker plates.

Key words: Nonlinear Bending Stiffness, Clamped Plates by Bolted Joints, Finite Element Method, Bending Moment, Contact State, Stress Analysis

1. Introduction

For bolted joints, it is well known that the external forces and moments are linearly shared between the bolt and clamped plates in proportion to the stiffness ratio of them, when the external forces and moments are so small that the bolt and clamped plates are deformed into one body. Present strength evaluations have been carried out on bolted joints^{(1)~(4)} by using the stiffnesses of the clamped plates under the external force and the moment in this linear region. In particular, a bolted joint subjected to offset axial force was estimated by

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using the superposition of axial force on the bolt axis and the bending moment ⁽⁴⁾.

Moreover, a bolted joint was modelled with a beam element for use in some finite element (FE) analyses that had an equivalent stiffness calculated from that of the clamped plates ^{(5), (6)}. For example, the bending stiffness of a bolted joint affected the natural frequency of the structure jointed with bolts ⁽⁵⁾. Therefore, the FE model and the strength evaluation of the bolted joints both require the stiffnesses of the clamped plates under the axial load and the bending moment are precisely estimated.

When the contact surfaces between two clamped plates are opened up due to an increase in external axial force and bending moment, the bolted joint nonlinearly deforms. When the contact surfaces between two clamped plates separate under axial force, the nonlinear relationship between the deformation and the force can be estimated under the assumption that the bolt is subjected to the entire axial load ⁽¹⁾⁻⁽⁴⁾. On the other hand, German engineering society code, VDI 2230 (2003) provides an analytical model for the nonlinear deformation under a bending moment such as the offset axial force, only when the contact surfaces between the clamped plates are small enough that the contact pressure of the bolt preload spreads over the entire area of the contact surfaces ^{(1), (3), (4)}. The code warns that the analytical model accounts for a large error when the outer diameter of the clamped plates is larger than the sum of the bearing radius and the thickness of the clamped plates. A small number of studies have been carried out for the nonlinear deformation of the clamped plates under the bending moment such as the offset axial force for large and thin plate structures. The limit state of the nonlinear deformation under the bending moments should be clarified when designing highly reliable structures.

The contact conditions between two clamped plates and between the bearing surfaces should be taken into consideration when estimating the nonlinear deformation under the external force and moment. Previously, a number of studies on bolted joints were conducted using FE analysis while taking the contact condition into consideration. Most of them focused on the thread surface contact conditions such as the root stress ^{(7), (8)} and nut loosening behaviour according to the thread surface friction ⁽⁹⁾. However, little consideration has been given to the stiffness of the clamped plates, which is most important when evaluating the bolted-joint characteristics. We previously calculated the stiffnesses of clamped plates under an axial load and a bending moment by using FE models while taking the contact condition between clamped plates into consideration ⁽¹⁰⁾. The equivalent stiffness model of the clamped plates in VDI 2230 (2003) was also improved to make it possible to precisely estimate the stiffness for clamped plates with different thicknesses ⁽¹¹⁾.

The nonlinear deformation of the clamped plates under the external pure moment was estimated for this paper, aiming to expand the strength evaluation method of VDI 2230 (2003) for bolted joints in thin plate structures. We also developed a mechanical model for the nonlinear deformation of clamped plates under the bending moment.

2. Analysis Method

2.1 Analysis model

The analysis model from previous studies ^{(10), (11)} was used in this study as well. We estimated the stiffness of the clamped plates under a clamping force and a bending moment by analysing a 3D FE model that took the contact conditions between the clamped plates into consideration. Figure 1 shows the analytical model and its material properties. Figure 2 shows the 3D FE model. The analysis model consisted of two steel plates clamped together by a steel bolt. The clamped plates and bolt had the elastic material properties of steel. The upper and lower bearing surfaces and the surface between the plates were covered with contact elements, and the friction coefficient of these contact surfaces was $\mu = 0.15$. The plate thicknesses t_1 and t_2 were 3.2, 4.5, 6.0 or 9.0 mm (a thickness of 3.2 mm is expressed

as $t3.2$), and t_1 and t_2 were varied for all combinations except $t_1=t_2=9.0$. The bolt section on the surface between the clamped plates was fixed in the axial direction (z direction). The cross point of the bolt axis and the surface between the plates was constrained in the x direction as well. We used a half model because of the symmetry of the bending moment, and the symmetric sections of the bolt and the clamped plates were fixed in the y direction. The diameter of the clamped plates was equal to $5D_o$ so as not to affect the contact pressure distribution between the clamped plates. The analyses were carried out with ANSYS® ver.11.

Table 1 lists some of the dimensions in Fig. 1, the clamping force (preload) F_f , the thread stress under the preload, σ_{zs} , and the average pressure on the bearing surface under the preload, p_w . The nominal bolt diameter was varied from M8 to M16, and the material strength of the bolts was 4.8 in ISO grade. The bolts were assumed to be full thread bolts, and the bolt was modelled as a cylinder with a minor diameter of the bolt thread. The diameter of the hole was defined as a Class 2 value in JIS B1001. The outer diameter of the bearing surface, D_o , was defined as being equal to that of a hexagon head with the washer stipulated in JIS B1180. The nut and bolt height was equal to that of the hexagon head bolt specified in JIS B1180, although the nut height is higher than that of the bolt head in the actual bolt. The preloads were given so that they were 70% of the yield pretension defined in JIS B1083. The pretension elements of ANSYS® were applied to the bolt section to provide the preload.

Fig. 1 Dimensions and material properties of analysis model [mm]

Fig. 2 3D finite element model of bolted joint

Table 1 Dimensions of analysis model and preload

	M8	M10	M12	M16
d_3 [mm]	6.647	8.376	10.106	13.835
D_i [mm]	9	11	13.5	17.5
D_o [mm]	11.63	14.63	16.63	22
k [mm]	5.3	6.4	7.5	10
F_f [N]	7,670	12,130	17,610	32,620
σ_{zs} [MPa]	221	220	220	217
p_w [MPa]	180	166	238	234

2.2 Load condition

The bending moments were supplied to the analysis models, as shown in Figs. 1 and 2, and the bending stiffness value of the clamped plates were estimated^{(10), (11)}. We aimed to estimate the ideal stiffness of the clamped plates such as when the entire external bending moment contributes to the deformation on the bearing surface of the clamped plates. The bending moment, therefore, was applied within the bearing surface.

Figure 3 shows the load condition of the bending moment. When a bending moment that is a non-axisymmetric load was applied to an axisymmetric model (such as a cylinder) with axisymmetric elements, it is expanded into the first order Fourier series. Consequently, the result obtained is a first order Fourier integral of the axisymmetric results⁽¹²⁾. In this

study, 3D solid models were used because nonlinear analyses including the contact conditions were carried out. The bending moment was modelled as the pressure p of a first order Fourier function, i.e., a cosine function of the angle θ , that is $p = 1000 \cos\theta$ [MPa], as shown in Fig. 3. The minimum pressure of -1000 MPa was larger than the bearing pressure under the preload p_w , even though the average pressure on the bearing surface varied from 170-240 MPa in accordance with the nominal diameter of the bolt. The bending moment, therefore, may be large enough to open the contact surface between the clamped plates on the tension side. On the other hand, the contact surfaces on the bearing surfaces did not open under this bending moment in the analyses. The loading process was divided into a large number of short time steps to estimate the nonlinear behaviour of the contact surfaces.

Fig. 3 Bending moment condition of analysis model

3. Analysis Results

The analysis model in Figs. 1 and 2 was first subjected to the preload of the bolt, and the bending moment, as shown in Fig. 3, came second. Figure 4 shows the contact pressure distribution on the surfaces between two plates with $t4.5$ and $t6.0$ plates fastened together by an M10 bolt under several time steps. Figure 5 shows the contact pressure distribution along the x axis. The horizontal axis was normalized by $2/D_o$, and therefore, the place at ± 1.0 is the outer diameter of the bearing surface, D_o . In this bolted joint, the maximum diameter of the equivalent cone, $D_{A,Gr}$, which will be described in the next chapter, was $1.37 D_o$. The bending moment M was the sum of the moment on the bolt, M_b , and that on the clamped plates, M_c . The moment on the plates, M_c , can be calculated by integrating the product of the contact pressure shown in Fig. 4 and the x coordinate with the contact area. The moment on the plates, M_c , is also described in Figs. 4 and 5. The contact pressure decreases on the tension side as the moment is increased. In Fig. 4 (b) - (d), it becomes zero, which means the contact surfaces open up between two plates. On the compression side, the outer diameter of the pressured area does not change and is almost equal to the maximum diameter of the equivalent cone, $D_{A,Gr}$, although the contact pressure value increases as the moment is increased.

Figure 6 shows the z -direction (i.e. axial direction) stress contour of the plates with $t4.5$ and $t6.0$ plates fastened together by an M10 bolt. The stress distribution on the hole surface is described on the surface among the hole diameter D_h , as shown in Fig. 6. The z -direction stress on the compression side decreases and that on the tension side increases as the moment is increased. Thus, the contact pressure on the compression side is positive and the contact surfaces on the tension side open up, as shown in Fig. 6 (b) - (d). When the moment is M_{c3} or M_{c4} , the contact pressure and the compression stress on the compression side increase as the moment is increased. However, the contact area and the shape of clamp solid only slightly change.

Figure 7 shows the displacements of the upper and lower bearing surfaces on $y=0$ when $t4.5$ and $t6.0$ plates are fastened together by an M10 bolt. When the preload that is when $M_c=0$ is applied, the upper and lower bearing surfaces are parallel. As the bending moment increases, the upper and lower bearing surfaces incline. The bending gradient ξ was defined

at the position of the average diameter of the bearing surface, $(D_i + d_w)/2$. Thus, the bending gradient ξ was calculated from the differences in deformation on the left and right sides at the bearing diameters, $\Delta u_{l, D_o}$ and $\Delta u_{r, D_o}$, and those at the hole diameters, $\Delta u_{l, D_i}$ and $\Delta u_{r, D_i}$, by using the following equation:

$$\xi = \frac{(\Delta u_{l, D_i} + \Delta u_{l, D_o}) - (\Delta u_{r, D_i} + \Delta u_{r, D_o})}{D_i + D_o} \quad (1)$$

Figure 8 shows the relationship between the moment M_c and the bending gradient ξ when $t4.5$ and $t6.0$ plates are fastened together by an M10 bolt. The bending compliance of the clamped plates can be estimated as the slope of the line in Fig. 8. Figure 8 shows that the clamped plates have two constant compliances. Figure 9 shows the bending compliance of the clamped plates, β_c , which was calculated by dividing the difference in bending gradient ξ between the two incremental time steps by the difference in the bending moment, M_c , between the time steps. Thus, the bending compliance between i and $i+1$ steps, $\beta_{c(i, i+1)}$, is defined by the bending gradient of i step, $\xi_{c(i)}$, and the bending moment of i step, $M_{c(i)}$, as following equation:

$$\beta_{c(i, i+1)} = \frac{\xi_{c(i+1)} - \xi_{c(i)}}{M_{c(i+1)} - M_{c(i)}} \quad (2)$$

Fig. 4 Contact pressure distribution on surface between two plates (M10, $t4.5$, $t6.0$)

Fig. 6 z -direction stress distribution of plates (M10, $t4.5$, $t6.0$)

Fig. 5 Contact pressure distribution on surface between two plates along x axis (M10, $t4.5$, $t6.0$)

Fig. 7 Displacement on bearing surfaces (M10, $t4.5$, $t6.0$)

The relationship between the bending compliance and the moment looks to be divided into three regions with two moment thresholds M_{cth1} and M_{cth2} . These regions are (i) $0 < M_c < M_{cth1}$, (ii) $M_{cth1} < M_c < M_{cth2}$ and (iii) $M_{cth2} < M_c$. In region (i), the bending compliance of the clamped plates has a constant value, which is defined as the bending compliance in the linear relation as shown in previous papers^{(10), (11)}. In region (iii), the contact surfaces between the plates on the tension side perfectly open and the clamp solid on the compression side supports the bending moment. When the preload is large enough to enlarge the clamp solid to the maximum diameter of the equivalent cone, $D_{A,Gr}$, the clamp solid does not further increase, even though a larger preload is applied. When the bolted joint is subjected to the bending moment, the clamp solid on the compression side does not change as previously mentioned. The bending compliance in region (iii), therefore, is constant. Region (ii) is a transition region from regions (i) to (iii), in which the centre of the bending moment moves from the bolt axis to the clamp solid on the compression side. The constant bending compliances in regions (i) and (iii) are described in two straight lines, and the transition region (ii) is expressed as a curved plot between two lines in Fig. 8.

Fig. 8 Relation between moment and bending gradient of plate surfaces (M10, $t_{4.5}$, $t_{6.0}$)

Fig. 9 Relation between moment and bending compliance (M10, $t_{4.5}$, $t_{6.0}$)

4. Mechanical model for nonlinear behaviour of bending stiffness

From the previous chapter, the relationship between the bending compliance and the moment is divided into three regions, namely (i) $0 < M_c < M_{cth1}$, (ii) $M_{cth1} < M_c < M_{cth2}$ and (iii) $M_{cth2} < M_c$. Region (ii) is a transition region from regions (i) to (iii), and the bending compliance can be estimated along a straight line of the relationship between the compliance and the moment as shown in Fig. 9. What is needed to estimate the bending stiffness of the clamped plates is the mechanical models for regions (i) and (iii).

4.1 Mechanical model for (i) $0 < M_c < M_{cth1}$

Figure 10 schematically presents a bolted joint. When two plates are joined with sufficient clamping force and the contact conditions between the two plates are maintained under an external force, the subjected external load is transmitted from one plate to the other through the clamp solid shown in the figure. The stiffness of the clamped plates, therefore, is expressed as the stiffness of the clamp solid.

According to VDI 2230 (2003)⁽⁴⁾, the clamp solid is assumed to be an equivalent cone, as shown with the dashed lines in Fig. 10. We had improved the equation for the helix angle of the equivalent cone, φ , in VDI 2230 (2003) to be able to take the difference in thickness of the clamped plates into consideration in our previous paper⁽¹¹⁾. The improved $\tan \varphi$ equation is as follows.

$$\tan \varphi = 0.323 + 0.032 \ln \left(\frac{l_f}{2D_o} \right) + 0.153 \ln \left(\frac{D_c}{D_o} \right) + 0.0717 \ln \left(\frac{t_1}{t_2} \right) \quad (\text{when } t_1 \leq t_2) \quad (3)$$

In this equation, D_o is the outer diameter of the head-bearing surface, l_f is the clamping length calculated by adding the thicknesses of the two clamped plates, and D_c is the outer diameter of the clamped plates. The maximum diameter of equivalent cone $D_{A,Gr}$ is calculated as follows:

$$D_{A,Gr} = D_o + w l_f \tan \varphi \quad (4)$$

where w is the joint coefficient, and $w = 1$ for a through bolted joint. The bending stiffness is calculated from the second moment of the equivalent cone area as follows:

$$I_{Bers} = \frac{3\pi (D_c - D_o) D_o^3 D_c^3}{64 (D_c^3 - D_o^3)} - \frac{\pi D_i^4}{64} \quad (5)$$

where D_i is the hole diameter. When the clamped plates are large enough such as $D_c > D_{A,Gr}$, $D_{A,Gr}$ is substituted for D_c in Eq. (5). Thus the bending compliance of the clamped plates, β_c , is defined in the following equation with the Young's modulus of the clamped plates, E_c :

$$\beta_c = \frac{l_f}{E_c I_{Bers}} \quad (6)$$

In region (i), the contact pressure between the plates redistributes with the elastic deformation of the equivalent cone, and the centre of the contact pressure leans to the compression side. The limit value for this region, namely the moment threshold M_{cth1} , can be calculated as the condition that the contact pressure at the maximum diameter of the equivalent cone, $D_{A,Gr}$, on the tension side becomes zero. The contact pressure between the plates, p , is the sum of the pressure under the preload F_f and that under the bending moment of the plates, M_c , as follows:

$$p = \frac{F_f}{\frac{\pi}{4} (D_{A,Gr}^2 - D_i^2)} + \frac{M_c}{\frac{\pi}{64} (D_{A,Gr}^4 - D_i^4)} x \quad (7)$$

The moment threshold M_{cth1} can be obtained in the condition when the contact pressure becomes zero at the maximum diameter of the equivalent cone, $x = -D_{A,Gr}/2$, in Eq. (7) as follows:

$$M_{cth1} = \frac{F_f (D_{A,Gr}^2 + D_i^2)}{8 D_{A,Gr}} \quad (8)$$

Fig. 10 Equivalent cone

4.2 Mechanical model for (iii) $M_{cth2} < M_c$

In this region, the contact surfaces between the plates on the tension side open perfectly and the clamp solid on the compression side supports the bending moment. From Figs. 4, 5, and 6, the shape of the clamp solid was assumed to be an equivalent solid, which is obtained by subtracting the bolt hole from the truncated cone with an upper diameter of $D_o/2$ and a lower diameter of $D_{A,Gr}/2$, as shown in Fig. 11. The section of the equivalent solid on the x - y plane in Fig. 11 is described in Fig. 12. The bending centre of the bending stiffness of the equivalent solid was placed on the bolt axis, which is the centre axis of the external bending moment.

The deformation model of the bolt and the equivalent solid is shown in Fig. 13. When

the tension side of the clamped plates is perfectly opened, the bolt axial force F increases to $F=F_f+\Delta F$ from the preload F_f by adding incremental force ΔF , which is caused by the bending moment. The axial compression force of the equivalent solid is equal to the bolt axial force F . The bending moment of the clamped plates, M_c , is obtained by multiplying the axial compression force of the equivalent solid, F , and the offset length from the bolt axis, x_{c0} .

$$M_c = Fx_{c0}, \quad (9)$$

where x_{c0} is thought to be the gravity centre of the contact surface of the clamped plates, namely the gravity centre of the equivalent-solid section at $z=0$.

Under the equilibrium of forces of the axial tension force of the bolt, F , and the axial compression force of the equivalent solid, F , the bolt extends to the amount of Δ_b and the equivalent solid shrinks by the amount of Δ_c . The bending gradient ξ is calculated as

$$\xi = \frac{\Delta_b + \Delta_c}{x_{cw}} = \frac{F(\delta_b + \delta_{c2})}{x_{cw}}, \quad (10)$$

where x_{cw} is the gravity centre of the bearing surface of the equivalent solid, δ_b is the axial compliance of the bolt, and δ_{c2} is the axial compliance of the equivalent solid, respectively. The bending compliance of the equivalent solid, β_{c2} , can be calculated from Eqs. (9) and (10) as follows:

$$\beta_{c2} = \frac{\xi}{M_c} = \frac{\delta_b + \delta_{c2}}{x_{cw}x_{c0}}. \quad (11)$$

Although there are some equations for the axial compliance of the bolt, δ_b , we used the following equation from VDI 2230 (2003):

$$\delta_b = \frac{0.5d}{E_b \frac{\pi}{4} d^2} + \frac{l_f}{E_b \frac{\pi}{4} d_3^2} + \frac{0.5d}{E_b \frac{\pi}{4} d_3^2} + \frac{0.4d}{E_b \frac{\pi}{4} d^2}, \quad (12)$$

where d is the nominal diameter of the bolt, d_3 is the thread diameter of the bolt, and E_b is the Young's modulus of the bolts. When we calculated the compliance of the bolt from the results under an axial force for the FE models in Figs. 1 and 2, the analysis results agreed well within a 10% error of the values predicted by Eq. (12).

The analytical equation for the axial compliance of the equivalent solid shown in Fig. 12 is not easily obtained. We simplified the section of the equivalent solid to a section that is obtained by subtracting an ellipse with a short diameter of $D_i/2$ and a long diameter of $D_i\sqrt{1-D_i^2/D^2}$ from a circle with a diameter of $D(z)/2$, as shown in Fig. 14. The diameter $D(z)$ is calculated as follows:

$$\begin{aligned} D(z) &= D_{A,Gr} - \frac{D_{A,Gr} - D_o}{t_1} z & (0 \leq z \leq t_1) \\ &= D_{A,Gr} + \frac{D_{A,Gr} - D_o}{t_2} z & (-t_2 \leq z < 0). \end{aligned} \quad (13)$$

The gravity centres on the equivalent-solid sections, x_c , x_{cw} and x_{c0} are given as follows:

$$\begin{aligned} x_c(D) &= \frac{1}{4} \frac{D^3 - 2D_i^3 \sqrt{1 - \frac{D_i^2}{D^2}}}{D^2 - 2D_i^2 \sqrt{1 - \frac{D_i^2}{D^2}}}. \\ x_{cw} &= x_c(D_o), \quad x_{c0} = x_c(D_{A,Gr}). \end{aligned} \quad (14)$$

The stiffness of the truncated cone, K_{crc} , can be estimated from the circle area, $A_{crc}(D)$, with a diameter of $D(z)/2$ as

$$K_{crc} = \frac{E_c}{\int_{-t_2}^{t_1} \frac{dz}{A_{crc}(D)}} = \frac{\pi}{16} \frac{E_c D_{A,Gr} D_o}{l_f}. \quad (15)$$

The stiffness of the elliptic cylinder, K_{elp} , can be calculated from the ellipse area, $A_{elp}(D)$, as

$$K_{elp} = \frac{E_c}{\int_{-l_2}^{l_1} \frac{dz}{A_{elp}(D)}} = \frac{\pi E_c}{8l_f} \frac{D_i^2 (D_{A,Gr} - D_o)}{\sqrt{D_{A,Gr}^2 - D_i^2} - \sqrt{D_o^2 - D_i^2}}. \quad (16)$$

The stiffness of the equivalent solid is thought to be calculated by subtracting the stiffness of the elliptic cylinder, K_{elp} , from the stiffness of the truncated cone, K_{crc} . The axial compliance of the equivalent solid, δ_{c2} , therefore, is estimated as follows:

$$\frac{1}{\delta_{c2}} = K_{crc} - K_{elp}. \quad (17)$$

The bending compliance of the clamped plates, β_{c2} , can be obtained by substituting the axial compliance of the bolt, δ_b , shown in Eq. (12), the axial compliance of the equivalent solid, δ_{c2} , calculated from Eqs. (13)~(17), and the gravity centres of the equivalent-solid section, x_{cw} and x_{c0} .

Moreover, the moment threshold M_{cth2} can be obtained by using Eq. (9) in the condition when the axial force begins to increase from the preload F_f as follows:

$$M_{cth2} = F_f x_{c0}. \quad (18)$$

Fig. 11 Equivalent solid model when $M_{cth2} < M_c$

Fig. 12 Section of equivalent solid model when $M_{cth2} < M_c$

Fig. 13 Deformation model when $M_{cth2} < M_c$

Fig. 14 Approximated section of equivalent solid model when $M_{cth2} < M_c$

5. Comparison of mechanical model and analysis result

We compared the bending compliances calculated from FEM solutions by using Eqn. (2) for analysis results and the results of the mechanical model. Figure 15 shows a comparison of the mechanical model as shown in the previous chapter and the analysis result as shown in Fig. 9, when $t4.5$ and $t6.0$ plates are fastened together by an M10 bolt. The two bending compliances β_c and β_{c2} calculated from the mechanical models agreed well with the analysis results. The same with both of two moment thresholds M_{cth1} and M_{cth2} . Figures 16 and 17 show some comparisons when the thickness combinations were $t3.2$ and $t4.5$ plates and were $t4.5$ and $t4.5$ plates. The developed mechanical model can help to describe the nonlinear behaviour of the bending compliance of the clamped plates, when the thicknesses of the plates are varied. Figures 18-20 show some comparisons when the bolt was varied to M8, 12, and 16 for the $t4.5$ and $t6.0$ plates. The developed mechanical

model can be used to describe the nonlinear behaviour of the bending compliance of the clamped plates, when the bolt was also varied.

Fig. 15 Comparison of analysis result and prediction of bending compliance (M10, $t_{4.5}$, $t_{6.0}$)

Fig. 16 Comparison of analysis result and prediction of bending compliance (M10, $t_{3.2}$, $t_{4.5}$)

Fig. 17 Comparison of analysis result and prediction of bending compliance (M10, $t_{4.5}$, $t_{4.5}$)

Fig. 18 Comparison of analysis result and prediction of bending compliance (M8, $t_{4.5}$, $t_{6.0}$)

Fig. 19 Comparison of analysis result and prediction of bending compliance (M12, $t_{4.5}$, $t_{6.0}$)

Fig. 20 Comparison of analysis result and prediction of bending compliance (M16, $t_{4.5}$, $t_{6.0}$)

The bending compliances when (iii) $M_{ch2} < M_c$ were calculated by using the mechanical model with the M8, 10, 12 and 16 bolts varying plate thicknesses, t_1 and t_2 among $t_{3.2}$, $t_{4.5}$, $t_{6.0}$ and $t_{9.0}$ except $t_1 = t_2 = 9.0$. The comparisons of the mechanical model and the analysis results are shown in Figs 21-24. The calculated values from the mechanical model were smaller than the analysis results when the clamping length l_f was long for the small M8 and M10 bolts. The reason for this is believed to be that the stiffness of the clamped plates decreases when the clamping length is increased because of the influence of the local

deformation on the bearing surface, even though our mechanical model assumes that the equivalent solid uniformly deforms. The calculated compliances from the mechanical model were agreed well with the analysis results for the M12 and M16 bolts. Our mechanical model presented a good prediction of the nonlinear behaviour of the bending stiffness of the clamped plates, except when the ratio of the clamping length l_f and the bearing diameter D_o was large, i.e. $l_f/D_o > 1.0$, namely when smaller bolts and thicker plates are used.

Fig. 21 Comparison of bending compliance of clamped plates with M8

Fig. 22 Comparison of bending compliance of clamped plates with M10

Fig. 23 Comparison of bending compliance of clamped plates with M12

Fig. 24 Comparison of bending compliance of clamped plates with M16

6. Conclusion

We evaluated the nonlinear behaviour of the bending compliance of clamped plates by using FE models of the bolted joints while taking the contact conditions into consideration. We also developed the mechanical models for estimating the nonlinear behaviour of the bending compliance. The following results were obtained in our work.

- (1) The relationship between the bending compliance and the moment were divided into three regions by the moment thresholds M_{cth1} and M_{cth2} , namely (i) $0 < M_c < M_{cth1}$, (ii) $M_{cth1} < M_c < M_{cth2}$ and (iii) $M_{cth2} < M_c$. In region (i), the bending stiffness of the plates could be estimated by using the equivalent cone of VDI 2230 (2003).
- (2) Mechanical models were developed for estimating the moment thresholds M_{cth1} and M_{cth2} . The mechanical models presented good predictions even when the bolt diameters and the plate thicknesses were varied.
- (3) In region (iii) $M_{cth2} < M_c$, the contact surface on the tension side of the plates opened

perfectly. The compression area did not change, although the contact pressure increased. We developed the mechanical model using the equivalent solid, which consisted of the truncated cone of the compression side, and the equations for the bending stiffness of the plates were defined.

- (4) Our mechanical model presented a good prediction for the nonlinear behaviour of the bending stiffness of the clamped plates, although the model estimated a lower value when smaller bolts and larger clamping lengths l_f were used.

This study shows that the bending stiffness of the plates defined by VDI 2230 (2003) can be used when the bending moment is lower than the moment threshold M_{cth1} for plates with larger diameters than the maximum diameter of the equivalent cone, $D_{A,Gr}$. When the contact surface does not allow for openings such as in ordinary structures, the structural design would be limited to within this threshold.

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