Modeling of Ultrasonic Guided-Wave Reflection from a Discontinuity in a Plate Structure*

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Abstract

In this paper, a combined model was proposed for reflection from a discontinuity and for a wave field of a fundamental shear horizontal guided wave in a plate structure. The reflection coefficient of a guided wave from a rectangular discontinuity can be modeled in the same way as the total reflection coefficient of a bulk wave. For the discontinuity having a slightly changed cross-sectional area, the reflection coefficient from the whole discontinuity can be calculated by spatial integration of the waves reflected from the divided regions. Some experiments and analyses were performed for 4mm-thick stainless steel plate specimens with EDM notches using a 300kHz frequency shear horizontal angle beam sensor, and for a cylindrical shell specimen (4.2m inner diameter, 9mm wall thickness) with discontinuities using a 40kHz frequency shear horizontal inter-digital sensor. The experimental reflection amplitudes were in good agreement with the analytical reflection amplitudes derived by the proposed model.

Key words : Ultrasonic Guided Wave, Reflection Coefficient, Plate

1. Introduction

The numbers of aged plants have been increasing in the power generation, oil, gas, chemical, and petro-chemical industries. A technique to efficiently evaluate integrity, especially for thickness, of many plate structures, for example pipes or tanks used in the plants, is becoming more important as present conventional non-destructive testing methods, i.e. radiographic testing, ultrasonic testing and eddy current testing, cannot meet these new inspection needs. This is because the methods naturally cover a narrow area, almost equal to their sensor size, and it is very time-consuming and cost-consuming to cover a long range or a wide area of the plate structures. Furthermore, if inspection points are placed in overhead locations, installation of scaffolds is needed before the inspection and then they must be removed afterwards; and except for radiographic testing, heat insulators must also be removed and then replaced for all inspection points.

One technique to overcome these issues is a long range inspection technique using an ultrasonic guided wave that was theoretically formulized by Gazis⁽¹⁾⁽²⁾. In recent years, several approaches have been proposed and some techniques have been adopted commercially⁽³⁾⁽⁴⁾⁽⁵⁾⁽⁶⁾. In these inspection techniques, a ring sensor is used, and the discontinuity size is measured as a cross-sectional area change of the pipe by using

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amplitude of a reflected guided wave. However, the amplitude of a reflected guided wave is known to depend on the length of the discontinuity; therefore the authors previously proposed a reflection model for the guided wave in a pipe⁽⁷⁾⁽⁸⁾. On the other hand, for tanks, the reflection amplitude also depends on the relative position between the sensor and the discontinuity.

Based on this background, the authors propose an analysis method to obtain the receiving gain of the guided wave. The model is based on combining the reflection from a discontinuity and a guided-wave field of a fundamental shear horizontal wave. In this paper, first, the details of the combined model are described and then the model is verified by some experiments.

2. Modeling of the guided-wave field and reflection from a discontinuity

2.1 Modeling of reflection from a discontinuity

In general, the reflection coefficient R_0 and transmission coefficient T_0 of an ultrasonic bulk wave from medium 1 to medium 2 are given by $R_0 = (z_{02} - z_{01}) / (z_{02} + z_{01})$ and $T_0 = 2z_{02} / (z_{02} + z_{01})$, respectively, where $z_{01} = \rho_1 c_1$ is the specific acoustic impedance of medium 1 (ρ_1 and c_1 are density and velocity of medium 1) and $z_{02} = \rho_2 c_2$ is the specific acoustic impedance of medium 2 (ρ_2 and c_2 are density and velocity of medium 2).

The reflection coefficient R and transmission coefficient T of a fundamental shear horizontal guided wave propagating in a medium of finite thickness are given by $R = (z_2 - z_1) / (z_2 + z_1)$ and $T = 2z_2 / (z_2 + z_1)$, respectively, where $z_1 = \rho_1 c_1 A_1$ and $z_2 = \rho_2 c_2 A_2$ are respective characteristic acoustic impedances of medium 1 and medium 2, and A_1 and A_2 are respective cross-sectional areas of medium 1 and medium 2 as shown in Fig. 1(a)⁽⁹⁾. The reason why the reflection and transmission coefficients are represented by the simple equations is because the fundamental shear horizontal guided wave has same displacement through the medium and the propagation direction is parallel to the medium interface. Therefore, for guided waves, reflection occurs at a discontinuity even if the medium density or velocity is not changed between medium 1 and medium 2, i.e. $\rho_1 = \rho_2$ and $c_1 = c_2$. If the discontinuity is a rectangular shape, the amplitude reflection coefficient is given by Eq.(1), like the total amplitude reflection coefficient of a bulk wave:⁽¹⁰⁾

$$R = \left(\frac{z_2}{z_1} - \frac{z_1}{z_2} \right) / \sqrt{4 \cot^2 \alpha + \left(\frac{z_2}{z_1} + \frac{z_1}{z_2} \right)^2} \quad (1)$$

where $\alpha = 2\pi a / \lambda$, a is axial length of the discontinuity (medium 2) and λ is the wavelength of the guided wave.

If the cross-sectional area of the discontinuity is slightly changed due to corrosion, for example, the reflection coefficient from the whole discontinuity is calculated by spatial integration of the waves reflected from the divided regions as shown in Fig. 1(b) and (c). Assuming that the guided wave is continuous and multiple reflection waves having over two reflection times can be ignored because of their low amplitude, the reflection wave from the whole region is given by Eq.(2):

$$f(x, t) = \sum_{i=1}^N \left(\prod_{m=1}^{i-1} \left(\frac{2z_{m+1}}{z_{m+1} + z_m} \frac{2z_m}{z_{m+1} + z_m} \right) \frac{z_{i+1} - z_i}{z_{i+1} + z_i} \exp j(\omega t - k(x - 2x_{i-1})) \right) \quad (2)$$

where x is axial position, t is time, z_i is characteristic acoustic impedance of the i th small region, ω is angle frequency, k is wave number and j is imaginary unit.

Example analysis results of two types of typical discontinuities (rectangular and semi-elliptical) are shown in Fig. 2. For the simplest rectangular discontinuity, the reflection coefficient has its maximum value when the axial length of the discontinuity is $(2n - 1)/4$

times the wavelength ($n=1, 2, \dots$). On the other hand, the reflection coefficient has its minimum value at $n/2$ times the wavelength ($n=1, 2, \dots$). This periodicity is caused by interference between the anti-phase reflection wave from the position where the thickness is decreasing and the in-phase reflection wave from the position where the thickness is increasing. The reflection waves from the semi-elliptical discontinuity are also periodically changed for the same reason. But, its reflection coefficient decreases slightly for a long axial length. Therefore, the axial discontinuity length or shape is estimated by measuring the relationship between the frequency and reflection coefficient.

Fig. 1 Analysis models of guided-wave reflection

Fig. 2 Reflection coefficients calculated by the proposed model

2.2 Modeling of the wave field

For a bulk wave, the amplitude of the reflected wave changes in a complicated way if a discontinuity is located in the near-field region. Conventionally, the size of a discontinuity can be evaluated as equivalent-defect-diameter and the diameter of a circular defect shape is assumed to be a circle using the DGS (distance-gain-size) diagram⁽¹¹⁾. This concept can be used for a guided wave also.

A guided wave has two characteristics. First, the amplitude of the guided wave is decreased in proportion to the square root of the distance from the source because the guided wave is a cylindrical wave. Second, the sound source is homogeneous through the wall and it can be treated as a line source on a two-dimensional plane as shown in Fig. 3. Therefore, the reflected wave received by the wave source (a transducer) is calculated by the Kirchhoff method using the following procedure. First, the vibration of element q on the discontinuity is expressed by Eq.(3)

$$y_q(t) = \sum_p \frac{A_p}{\sqrt{r}} \exp j(\omega t - kr) \quad (3)$$

where $r = |r_1 - r_0|$, r_0 is the position vector of element p on the wave source, r_1 is the position vector of element q on the reflection source, ω is angular frequency, t is time, j is imaginary unit, and k is wave number. Next, the vibration $x_p(t)$ of element p on the wave source (transducer) is expressed by Eq.(4)

$$x_p(t) = \sum_q \frac{1}{\sqrt{r'}} y_q(t) \exp(-jkr') \quad (4)$$

where $r' = |r_2 - r_1|$, and r_2 is the position vector of element p . Finally, the amplitude C of the reflected wave received by the wave source is expressed by Eq.(5).

$$C = \sqrt{\sum_p x_p^2} \quad (5)$$

When the reflection source is in front of the wave source, the DGS diagram of the guided wave that has $ka = 4.49$ (k is wave number and a is the half length of the sound source) is shown in Fig. 4(a). In the figure, the horizontal axis is normalized distance (distance L is normalized by wavelength λ), and the vertical axis is relative amplitude normalized by the amplitude of the reflected wave from the reflection source $w/W = \infty$ (w is width of the reflection source and W is width of the wave source) at the normalized distance = 1. The amplitude from the reflection source $w/W = \infty$ decreases 10dB for each power of ten change in distance, but the amplitude from the finite width of the reflection source decreases 20dB for each power of ten change in distance at the far-field, and the relative amplitude is proportional to the width of the reflection source. The relationship between the normalized lateral distance D/λ and relative amplitude at $L/\lambda = 100$ is shown in Fig. 4(b).

Fig. 3 Analysis model of guided-wave field

(a) Distance-gain-size diagram: $ka=4.49$ where k means wave number and a means half length of wave source

(b) Relationship between lateral distance and amplitude: $ka=4.49$, $L/\lambda=100$.

Fig. 4 Analysis results of relative amplitude from several size discontinuities by the wave field model

3. Experiments

3.1 Experiment with plate specimens

In order to measure amplitude of the reflected wave from rectangular discontinuities, 2 types of stainless steel plate specimens (thickness = 4mm) with artificial discontinuities were fabricated. One type specimen had 4 discontinuities with width of 100mm and height of $h=0.5\text{mm}$ to $h=2.0\text{mm}$ in steps of 0.5mm (Fig. 5(b)). The other type specimen had 2 discontinuities with width of 100mm and each height of $h=2.5\text{mm}$ or $h=3.0\text{mm}$.

A block diagram of the experimental system is shown in Fig. 5(a). An arbitrary function generator, power amplifier, diplexer receiver amplifier and A/D converter (maximum sampling rate = 1GS/s, vertical resolution = 8 bits) configured the system. The transducer was a shear-vibration type with a resonance frequency of 300kHz. It was bonded on an acrylic wedge with a refraction angle of 90deg which matched the shear horizontal wave velocity.

The transducer was placed on a plate specimen using viscosity medium coupling, opposite the fabricated discontinuities, and a normal load was given until the amplitude

became stable. Then, the transducer was placed 700mm from the discontinuities, and the reflected wave was recorded. The amplitude of the reflected wave from the plate end was also measured. In the experiments, the transducer was driven by a 300kHz tone-burst wave.

Fig. 5 Experimental setup and shape of plate specimens (unit: mm)

The amplitude reflection coefficients of rectangular discontinuities obtained from the experiment with the plate specimens are shown in Fig. 6. The filled circles are experimental results normalized by the reflection amplitude of the through-thickness discontinuity with width of 100mm. Solid lines are theoretical results obtained by Eq.(1). In this experiment, the frequency of the guided wave was 300kHz and the length of the rectangular discontinuity was 1mm, then the length of the discontinuity normalized by the wavelength ($\sim 10.5\text{mm}$) was about 0.1. In this figure, experimental results are plotted between analytical values of 1.0mm length discontinuity and 1.5mm length discontinuity. Nominal length of the discontinuity was 1mm, and the results obtained for the proposed model were approximately in agreement with the experimental results.

Fig. 6 Amplitude reflection coefficients of rectangular discontinuities on plate specimens (frequency = 300kHz): experimental values are normalized by signal amplitude of through-thickness discontinuity

3.2 Experiment with the cylindrical shell specimen

The guided-wave system and sensor are shown in Fig. 7(a). The system included arbitrary function generators, power amplifiers, an A/D converter and a computer for control. The sensor had 8 inter-digital transducer groups (4 x 2 rows). The inter-digital transducers had a shear mode element and the fundamental shear horizontal mode guided wave with no dispersion was excited.

In order to control the transmission direction, 4 pairs of transducers were set on the cylindrical shell specimen with a separation of 20mm (Fig. 7(b)), which is about one-quarter of the wavelength of the 40kHz shear horizontal mode, and a one-quarter period delayed waveform was applied to each pair. Backward transmission was actively suppressed by the inter-digital transducers according to the following principle. A synthetic traveling wave in the plus (right hand) direction $A^+(x, t)$ and a synthetic traveling wave in the minus (left hand) direction $A^-(x, t)$ are given by Eqs.(6) and (7), respectively:

$$A^+(x, t) = A_1 e^{j(\omega t - kx)} + A_2 e^{j\left(\omega\left(t - \frac{T}{4}\right) - k\left(x - \frac{\lambda}{4}\right)\right)} \quad (6)$$

$$A^-(x, t) = A_1 e^{j(\omega t + kx)} + A_2 e^{j\left(\omega\left(t - \frac{T}{4}\right) + k\left(x - \frac{\lambda}{4}\right)\right)} \quad (7)$$

where x is the axial coordinate of the pipe, A_1 and A_2 are amplitudes of the transmitting wave of positions 1 and 2, respectively, ω is angle frequency, t is time, k is wave number, $T(=2\pi/\omega)$ is period, $\lambda(=2\pi/k)$ is wavelength, and j is imaginary unit. For $A_1 = A_2$ in Eqs.(6) and (7), a synthetic wave traveling in the plus direction is doubly super-positioned, and a synthetic wave traveling in the minus direction is suppressed to zero. This is shown in Fig. 8.

In practice, the transmitting signals used for inspection are not continuous waves but burst waves that have a limited duration. Then, the waveform applied to the front row of the transducer pairs was the inverse of the waveform applied to the rear row and it had a one-quarter period delay. In this method, the amplitude of the synthetic traveling wave in the plus direction could not be completely doubled, but the amplitude of the synthetic traveling wave in the minus direction could be completely decreased to zero in theory. For inspection, distinguishing between the reflection wave from one direction (signal) and the

reflection wave from another direction (noise or false signal) is important. Therefore, almost the same processing was performed for received signals.

(a) Guided-wave system and sensor on a cylindrical shell specimen

(b) System configuration

Fig. 7 Photo of the guided-wave system and sensor and a block diagram of the system

Fig. 8 Transmission direction control by the inter-digital transducers

A cylindrical shell specimen was used as shown in Fig. 9. The specimen was SUS304, with 4200mm inner diameter, 2800mm height, and 9mm nominal wall thickness. Two axial butt welds were positioned, one at 0deg and the other at 180deg. Three different shapes of artificial discontinuities (O, A, R) were fabricated on the inner diameter (ID) side of the specimen. The guided-wave sensor was positioned at 6 locations (P1 to P6), and the guided

wave was transmitted in the direction indicated by the arrows. The reflection signal amplitude from the discontinuity was measured at several frequencies.

(b) Discontinuity shapes

Fig. 9 Cylindrical shell specimen and artificial discontinuities

The relative reflection amplitudes of the discontinuities obtained from the experiment with the cylindrical shell specimen are shown in Fig. 10. Filled marks are experimental results. Solid lines are analytical amplitudes obtained by Eq.(1) or Eq.(2), and Eq.(5). For comparison of the experimental amplitudes and the analytical amplitudes, the experimental amplitudes were corrected by using the ratio between the analytical amplitude of $w/W = \infty$ and an experimental amplitude of the top end of the cylindrical shell.

In the reflection coefficient analysis, sizes of discontinuities were precisely measured by the replica method, and were modeled by the values of Table 1. For example, discontinuity O, 100mm square length and 1mm depth, was approximately modeled as a 1mm-step discontinuity, because reflection waves at the decreasing step and increasing step hardly interfere with each other since the difference of propagation distance was 200mm and the duration time of waveform was about 3 cycles as shown in Fig. 8. Discontinuities A and R were modeled as a semi-elliptical shape, with depths of 6.5mm and 1.27mm, respectively, and width of 10mm. Axial weld, i.e. excess weld metal, was also modeled as a semi-elliptical shape, with length of 14mm and depth of -0.8mm, where the minus value means thickness was increasing.

Widths of the discontinuities for the wave field analysis were modeled by the values of Table 1. The width w of discontinuity A, a pair of 10mm-diameter drilled holes, was modeled as 20mm. Width of the axial weld was defined as ∞ . The guided-wave sensor was modeled as $W=720\text{mm}$ width, 40kHz frequency. The solid lines in Fig. 10 were calculated by this wave field analysis, and the values were corrected by the reflection coefficients in Table 1.

In Fig. 10, the relative amplitudes of discontinuities were in good agreement with

analytical values, and the proposed model was verified. By using the model, detectable discontinuities were estimated by providing their shape and size, and in the future, it is expected that the shape and size can be estimated using the amplitude of received signals with multiple frequency transmissions.

Table 1 Reflection coefficients of discontinuities

Discontinuity	Reflection model				Wave field model
	Model	Length (mm)	Depth (mm)	Reflection coef. @40kHz (dB)	Width (mm)
		a	d		w
Discontinuity O	Step	-	1.0	-24.6	100
Discontinuity A	Semi-ellipse	10	6.5	-8.1	20
Discontinuity R		10	1.3	-21.3	40
Axial weld		14	-0.8	-24.0	∞

Fig. 10 Relative signal amplitude of discontinuities of the cylindrical shell specimen (frequency = 40kHz): experimental results and analytical results obtained by the proposed model

4. Conclusions

A combined model for reflection from a discontinuity and for the guided-wave field of the fundamental shear horizontal guided wave in a plate structure was proposed. Some experiments and analyses were performed for 4mm-thick stainless steel plate specimens and for a cylindrical shell specimen with an inner diameter of 4.2m and wall thickness of 9mm.

It was found that reflection coefficient of the guided wave from a rectangular discontinuity could be modeled in the same way as the total reflection coefficient of a bulk wave. For a discontinuity having a cross-sectional area that slightly changed, the reflection coefficient from the whole discontinuity could be calculated by spatial integration of the waves reflected from the divided regions. The experimental results were in good agreement with the analytical values obtained using the proposed combined model.

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