Bingham fluid behavior of plagioclase-bearing basaltic magma: Reanalyses of laboratory viscosity measurements for Fuji 1707 basalt

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Datasets obtained by viscometry of Fuji 1707 basalt at a pressure of one atmosphere (Ishibashi, 2009) were analyzed by using the Bingham fluid model, and both the yield stress (τ_y) and Bingham viscosity (η_B) were determined. The relation between η_B and the crystallinity (Φ) was compared with the Krieger-Dougherty equation, and both the maximum packing fraction of crystals (Φ_m) and intrinsic viscosity (ν) for Bingham viscosity were determined as Φ_m = ~0.45 and ν = ~5.25, respectively; thus, it was found that Φ_m decreased and ν increased concomitantly with an increase in the shape anisotropy of crystals. However, the obtained value of νΦ_m (~2.36) was similar to that in the case of uniform, isotropic particles (2.5). This indicates that the effect of crystal shape anisotropy on η_B might be predicted only on the basis of a change in Φ_m. For Φ > 0.133, τ_y was found to be finite; it increased with Φ, which suggests that the critical crystallinity for the onset of yield stress, Φ_c, is at least lower than 0.133. The upper limit of Φ_c is close to the value calculated numerically for randomly oriented uniform particles by Saar et al. (2001) (their value is 0.10-0.15 for a width/length (W/L) ratio of 0.1 to 0.2, which is similar to the ratios in the case of basalt).

Keywords: Magma, Viscosity, Bingham fluid parameters, Yield stress, Plagioclase, Fuji

INTRODUCTION

Most natural magmas are suspensions of crystals and/or bubbles in a silicate melt. The suspended particles are known to affect the viscosity of magma (e.g., Harris et al., 2008). The focus of this study is the effects of crystals on magma. Crystals are known to cause two major effects on magma; they increase the absolute value of the apparent viscosity and induce non-Newtonian behavior, or, in other words, shear strain rate-dependence of viscosity. The former effect is well-described in a quantitative manner by the Einstein-Roscoe (ER) equation (Roscoe, 1952) only for certain suspensions that have uniform isotropic particles with low crystallinity and that behave as a Newtonian fluid (e.g., Lejeune and Richet, 1995). The equation, however, fails to represent the apparent viscosity when suspended crystals are anisotropic, when the crystallinity is high, or when non-Newtonian behaviors are observed (e.g., Sato, 2005). The effects of shape anisotropy and high concentration of crystals are one of the most important issues in the rheology of crystal-bearing magma (e.g., Carrich et al., 2007). However, only a few studies on magma with shape anisotropy of crystals have been carried out (e.g., Ishibashi and Sato, 2007). This study was undertaken in order to examine the effects of tabular crystals on the flow properties of moderately concentrated magmas with crystallinities of up to ~25%.

In the investigation of magma with anisotropic crystals, the treatment of non-Newtonian behavior is important because the behavior of magmas readily departs from Newtonian behavior. The two simplest non-Newtonian fluid models are the power law fluid model and the Bingham fluid model. In an earlier study (Ishibashi, 2009), the power law fluid model was adopted to explain the non-Newtonian behavior of tabular plagioclase-bearing basaltic magma, and its relative viscosity (the ratio of the viscosity of magma to the melt viscosity) was formulated as a function of the crystallinity and the shear strain rate. However, the power law fluid model is, at present,
suitable for use in volcanology because yield stress data are required in order to explain the steep margin of lava flows. In addition, there have been few studies that involve numerical investigations of volcanic phenomena by adopting a power law fluid model to describe the rheology of magmas (Fillippucci et al., 2010). On the other hand, the Bingham fluid model has commonly been adopted to approximate the magma rheology in many geological, numerical, and analog investigations (e.g., Fink and Griffiths, 1998; Miyamoto and Sasaki, 1998), although the quantitative relationship between textural characteristics and the Bingham fluid behavior of magma is yet to be examined experimentally. In this study, we tried to examine the relationship between Bingham flow properties and textural parameters by carrying out analyses of datasets that include both viscosity measurements and textural analyses of tabular plagioclase-bearing basaltic magma.

**ANALYTICAL METHOD**

**Summary of Ishibashi (2009) experiment**

In this study, we reanalyzed the results of viscosity measurements for Fuji 1707 basalt, details of which were reported by Ishibashi (2009). Here, we briefly summarize the experimental results.

The basalt used in this experiment was a high-alumina basalt. Ishibashi (2009) conducted viscosity measurements at eight temperatures from 1297 °C to 1157 °C using 20 °C intervals under a near Ni-NiO oxygen buffered condition using a concentric cylinder rotational viscometer. At each temperature, measured samples attached to the rotational rods were collected and quenched. The quenched samples were then processed into polished thin sections for EPMA analysis.

In the basalt, plagioclase first crystallized at 1247 ± 10 °C, followed by olivine at 1167 ± 10 °C; crystallinity increased monotonously with cooling up to 0.248 at 1157 °C (Table 1). Major element compositions of silicate melt varied only slightly with cooling except for Al₂O₃ and FeO⁺ (= FeO + 0.9 Fe₂O₃) contents: Al₂O₃ content decreased from 17.0 wt% to 13.6 wt% and FeO⁺ content increased from 9.9 wt% to 12.5 wt%. In all subliquidus run samples, plagioclase crystals were tabular. Both apparent sizes and width/length ($W/L$) ratios varied significantly, even in one sample. The apparent length of plagioclase varied from 5 µm to 3000 µm and crystal size distributions were similar to those of natural basaltic lavas. The $W/L$ ratios of plagioclase crystals varied from 1 down to 0.02 in every sample. Their distributions were approximated with log-normal frequency distribution. The means and standard deviations of log ($W/L$) are shown in Table 1. Tabular plagioclase crystals tended to align their long axes parallel to the tangential direction of rotation in all subliquidus run samples.

Ishibashi (2009) measured shear stress at the surface of the rotational cylinder ($τ_i$), at each rotational angular velocity ($Ω$). The measured $τ_i/Ω$ dataset for the basalt are presented in Figure 1. During all measurements, $τ_i$ first decreased with increasing total strain applied (thixotropy); then, it achieved a steady state. The value of $τ_i$ used for analyses are those at a steady state (measurement errors are below ~ 6 relative percent in 1σ). At $Ω=1$ s⁻¹, the $τ_i/Ω$ ratio increased from $10^{1.9}$ Pa s to $10^{3.2}$ Pa s with cooling from 1297 °C to 1157 °C. At temperatures higher than 1217 °C, the $τ_i/Ω$ ratio was constant irrespective of $Ω$, although the ratio decreased concomitantly with increasing $Ω$ at temperatures less than 1197 °C, indicating that the basalt behaved as a non-Newtonian fluid at temperatures less than 1197 °C. In this study, we adopted the Bingham fluid model to approximate the behavior of the basalt.

**Theory for determining $τ_i$ and $η_B$**

A concentric cylinder rotational viscometer allows measurement of a suspension’s viscosity and Spera et al.
(1988) and Ishibashi (2009) have summarized its principles and advantages in detail. For the viscometer, shear stress, \( \tau_i \) and shear strain rate, \( \dot{\varepsilon}_i \), are generally related to \( \Omega \) by the following equation (Krieger and Elrod, 1953).

\[
\Omega = 0.5 \int_{\tau_i}^{\infty} \left( \frac{\dot{\varepsilon}}{\tau} \right) \, d\tau
\]

(1)

Here \( \tau_i \) signifies shear stress at the surface of the outer cylinder, \( \tau_i \) and \( \tau_c \) are related as \( \tau_i = k \tau_c \), in which \( k = (R_i/R_c)^2 \), and \( R_i \) and \( R_c \) represent radii of the inner and the outer cylinders respectively. By substituting the arbitrary model describing the \( \dot{\varepsilon} - \tau \) relation into Eq. (1), and solving it, the relation between \( \tau_i \) and \( \Omega \) is obtainable.

For the Bingham fluid model, \( \tau \) is related to \( \dot{\varepsilon} \) as

\[
\tau = \tau_y + \eta_B \dot{\varepsilon},
\]

(2)

where \( \tau \) denotes the yield stress and \( \eta_B \) is the Bingham viscosity. Additionally, \( \dot{\varepsilon} = 0 \) when \( \tau \leq \tau_y \). By solving Eq. (1) with substitution of Eq. (2), a set of equations relating \( \tau \) and \( \Omega \) was obtained as follows (e.g., Spera et al., 1988).

\[
(1-k) + (\tau_i/\tau_c) \ln k - 2\eta_B (\tau_i/\Omega)^{-1} = 0 \quad \text{for } \tau_i/k \leq \tau_y \tag{3a}
\]

\[
(1-\tau_i/\tau_c) + (\tau_i/\tau_c) \ln(\tau_c/\tau_y) - 2\eta_B (\tau_i/\Omega)^{-1} = 0 \quad \text{for } \tau_y < \tau_i < \tau_i/k \tag{3b}
\]

For \( \tau_y = 0 \), Eq. (3a) is rewritten as.

\[
(1-k) - 2\eta_B (\tau_i/\Omega)^{-1} = 0 \tag{4}
\]

Eq. (4) shows the \( \tau_i/\Omega \) ratio is constant irrespective of \( \Omega \) for a Newtonian fluid. Both \( \tau_i \) and \( \eta_B \) can be determined simultaneously from the measured \( \tau_i/\Omega \) dataset using Eqs. (3a) and (3b).

**RESULTS AND DISCUSSION**

In this study, least-squares fitting of Eqs. (3a) and (3b) was applied for the \( \tau_i/\Omega \) datasets measured by Ishibashi (2009) to determine both \( \tau_i \) and \( \eta_B \). The determined \( \tau_i \) and \( \eta_B \), were listed with calculated melt viscosities, \( \eta_m \), crystallinity, \( \Phi \), and mean \( W/L \) ratios for each temperature in Table 1. The value of \( \eta_B \) increased monotonously from 38.8 Pa s to 800 Pa s with cooling. Finite \( \tau_i \) was detected at temperatures less than 1197 \(^\circ\)C. It increased with cooling up to 19.5 Pa at 1157 \(^\circ\)C. The value of \( \eta_B \) was calculated from measured glass compositions and run temperatures for each run sample using the method of Shaw (1972), and melt viscosities increased from 37.4 Pa s to 127 Pa s with cooling. Preference for the Shaw (1972) method over that of Giordano et al. (2008) is discussed in the following section.

Substituting the obtained \( \eta_B \) and \( \tau_i \) into Eqs. (3a) and (3b), \( (\tau_i/\Omega) - \Omega \) relations were calculated for each temperature. They are shown in Figure 1. The calculated \( (\tau_i/\Omega) \) represents measured datasets within \( 1\sigma < \sim 3.7 \) relative %, demonstrating the validity of the obtained values of both \( \eta_B \) and \( \tau_i \). We also showed the fitting lines by using the power law fluid model (Ishibashi, 2009) for comparison (dotted lines in Fig. 1). The model represents the measured \( (\tau_i/\Omega) \) within \( 1\sigma = \sim 8 \) relative %. Therefore, we inferred that the Bingham fluid model is slightly better than the power fluid model to describe the non-Newtonian behavior of Fuji 1707 basalt.

**The effect of crystals on Bingham viscosity**

In Figure 2a, \( \eta_B \) and \( \eta_m \) are shown against reciprocal absolute temperatures. Melt viscosities with the composition of starting material are also shown for comparison, as calculated using the method of Shaw (1972), \( \eta_{0G} \), and Giordano et al. (2008), \( \eta_{0G} \) respectively. These methods are based on the Arrhenian and the Tamman–Vogel–Fulcher (TVF) equation, respectively.

Firstly, \( \eta_B \) at temperatures higher than 1237 \(^\circ\)C, at which the effect of crystals on \( \eta_B \) is negligible, were compared with both \( \eta_{0G} \) and \( \eta_{0S} \) (Fig. 2a). The results show that \( \eta_{0S} \) was almost identical to the measured \( \eta_B \), but \( \eta_{0G} \) was similar to \( \eta_B \) only at 1297 \(^\circ\)C and 1277 \(^\circ\)C. It was higher than \( \eta_B \) at temperatures less than 1257 \(^\circ\)C. Furthermore, apparent activation energy \([d[d\log(\eta)]/d(1/T)]\) of \( \eta_{0G} \) was much greater than that of \( \eta_B \), indicating the meth-
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Although \( \eta_{\text{B}} \), \( \eta_{\text{m}} \), and \( \eta_{\text{B}} \) were identical at temperatures above 1237 °C, \( \eta_{\text{B}} \) and \( \eta_{\text{m}} \) deviate upward and downward respectively from \( \eta_{\text{0S}} \) below 1217 °C (Fig. 2a). The deviation of \( \eta_{\text{m}} \) from \( \eta_{\text{0S}} \) is caused by compositional changes of melt induced by crystallization of plagioclase (and minor olivine at 1157 °C). Crystallization of plagioclase consumes \( \text{Al}_2\text{O}_3 \) content and enriches \( \text{FeO}^+ \) content in melt, resulting in decreased viscosity relative to \( \eta_{\text{0S}} \) at the same temperature. The difference between \( \eta_{\text{B}} \) and \( \eta_{\text{m}} \) is caused by the mechanical effect of crystals. To evaluate the mechanical effect of crystals on the viscosity of a suspension, \( \eta_{\text{B}} \) is well described by the Krieger-Dougherty (KD) equation (Krieger and Dougherty, 1959) as shown below.

\[
\eta / \eta_{\text{m}} = (1- \phi/\Phi_{\text{m}})^{-\nu}\phi
\]  

Therein, \( \Phi_{\text{m}} \) stands for the maximum packing fraction of suspended particles and \( \nu \) is an intrinsic viscosity. The ER equation is a specific version of Eq. (5) with \( \nu = 4.17 \) and \( \Phi_{\text{m}} = 0.6 \) (Marsh, 1981). We fit Eq. (5) to the \( \eta_{\text{B}}/\eta_{\text{m}} \) - \( \Phi \) relation using least-squares method. Thereby, both \( \Phi_{\text{m}} \) and \( \nu \) were determined simultaneously as \( \Phi_{\text{m}} = 0.45 \) (1σ = 0.10) and \( \nu = 5.25 \) (1σ = 0.64). The \( \eta_{\text{B}}/\eta_{\text{m}} \) - \( \Phi \) relation calculated from Eq. (5) with these values represents the experimental data with 1σ ~ 17 relative percent (Fig. 2b).

The obtained value of \( \Phi_{\text{m}} \sim 0.45 \) is lower than the random packing of uniform spheres, which is ~ 0.64 (e.g., Scott and Kilgour, 1969). This is because of the shape anisotropy of suspended crystals. Numerical studies show that \( \Phi_{\text{m}} \) varies with aspect ratios for randomly oriented uniform particles (e.g., Wouterse et al., 2007). For oblate particles, \( \Phi_{\text{m}} \) increases to ~ 0.71 at \( W/L \) ratio ~ 0.4 and then decreases concomitantly with decreasing \( W/L \) ratio. On the other hand, \( \Phi_{\text{m}} \) generally increases with the variance of particle size and/or the increase in the degree of parallel arrangement. Wouterse et al. (2007) calculated the \( \Phi_{\text{m}} \) of randomly oriented uniform oblate ellipsoids to be ~ 0.47-0.57 for a \( W/L \) ratio ~ 0.1-0.2. The \( \Phi_{\text{m}} \) value obtained in this study almost coincides with their results at the same mean \( W/L \) ratio within 1σ error, whereas plagioclase crystals were arranged moderately parallel and their size varied significantly. The coincidence might be explained as follows: The effects of size variation and parallel arrangement on \( \Phi_{\text{m}} \) are offset by the dispersion in shape anisotropy of crystals. Because the excluded volume of a particle increases as its aspect ratio deviates...
from unity, more anisotropically shaped crystals contribute more effectively to $\Phi_{\text{in}}$. Therefore, $\Phi_{\text{in}}$ at the same mean $W/L$ ratio should be lower for crystals with varied $W/L$ ratios than for uniform-shaped crystals. This effect is opposite in the sense of increasing size variation and/or the degree of parallel arrangement. Consequently, these effects offset each other.

It is also important that the obtained value of $v\Phi_{\text{in}}$ is 2.36 ($1\sigma = 0.59$), which nearly coincides with the value for the ER equation ($v\Phi_{\text{in}} = 2.5$), although both $v$ and $\Phi_{\text{in}}$ vary with shape anisotropy of crystals. This fact implies that the value of $v\Phi_{\text{in}}$ is almost constant against the $W/L$ ratio of crystals; for such a case, the $\eta_m/\eta_{\text{m}}$: $\Phi$ relation is a function only of $\Phi_m$ and the $\eta_m/\eta_{\text{in}}$ ratio can be predicted by combining the KD equation and the $\Phi_{\text{in}}$:aspect ratio. For confirmation, more datasets including both viscosity measurements and quantitative textural analyses of crystal-bearing magmas are needed.

**The effect of crystals on yield stress**

In Figure 3, the obtained $\tau_y$ is shown against $\Phi$. We were able to detect finite yield stress at $\Phi$ larger than 0.133 and $\tau_y$ monotonously increased with increasing $\Phi$, indicating that the critical crystallinity for onset of yield stress ($\Phi_c$), was lower than 0.133. In fact, $\Phi_c$ is considered to be the lowest $\Phi$ to form a pervasive crystal network and the network resists shear stress to develop yield stress. Because the initially formed network is fragile and easily destroyed by shear, we can detect yield stress at $\Phi$ in some degree larger than $\Phi_c$ (e.g., Saar et al., 2001). We tried to constrain $\Phi_c$ precisely by comparing the $\tau_y$ with an empirical equation (e.g., Zhou et al., 1995; see caption of Fig. 3), however this failed due to large errors in the regression parameters and the small number of $\tau_y$ data in this study.

The upper limit value of $\Phi_c (= 0.133)$ is significantly lower than that calculated for randomly oriented uniform isotropic particles (0.22 for cubic and 0.285 for sphere; Saar et al., 2001). It is similar to that for randomly oriented rectangular prisms with a $W/L$ ratio similar to the mean of the ratios of crystals in the basalt ($\Phi_c = 0.10$–0.15 for rectangular prisms with $W/L$ ratio of 0.1–0.2; Saar et al., 2001); this fact is contrary to our expectation because $\Phi_c$ for the basalt is expected to be higher than the value reported by Saar et al. (2001). One reason is that plagioclase crystals in the basalt were parallelly arranged to a moderate degree (Ishibashi, 2009). Numerical investigation of Walsh and Saar (2008) showed that $\Phi_c$ for crystals that are arranged in parallel should be higher than that for randomly oriented particles. Another reason is that the sizes of crystals vary considerably in the basalt. Although the effect of crystal size variation on $\Phi_c$ remains unclear, $\Phi_c$ seems to increase as the size variation of crystals increases. As the crystal size variation increases, $\Phi_{\text{in}}$ increases; therefore, the effective volume fraction of crystals, $\Phi/\Phi_{\text{in}}$, decreases. Decrease of $\Phi/\Phi_{\text{in}}$ makes it difficult to form a pervasive connection of crystals at the same $\Phi$, resulting in an increase of $\Phi_c$. Therefore, the coincidence between the upper limit value and the numerical one indicates that the effects of parallel arrangement and size variation were offset by other effects that tend to decrease $\Phi_c$.

We inferred that the effect of decreasing $\Phi_c$ is derived from variation in shape anisotropy of crystals. Saar et al. (2001) showed numerically that $\Phi_c$ decreases drastically with increasing shape anisotropy of crystals. Some examples are those for oblate particles, $\Phi_c \approx 0.015$ at $W/L$ ratio = 0.02, and $\Phi_c \approx 0.15$ at $W/L$ ratio = 0.2, which implies that only a few crystals with low $W/L$ ratios can form a pervasive network without numerous crystals with high $W/L$ ratios; that is, crystals with lower $W/L$ ratios contribute to yield stress development more effectively. For such a case, crystals that are effective for yield stress development actually have $W/L$ ratios lower than the mean. Consequently, $\Phi_c$ of randomly oriented crystals with variable shape anisotropy are necessarily lower than that of uniformly shaped crystals even when the mean $W/L$ ratios are the same. For our case, the effect of variation in shape–anisotropy of crystals was (perfectly or partially) offset by the effects of parallel arrangement and size variation, resulting in an upper limit value of $\Phi_c$ similar to the calculated value for randomly oriented crystals. This suggests the shape–anisotropy distribution of crystals is a

![Figure 3. Yield stresses ($\tau_y$) are shown against crystallinity ($\Phi$). Bars represent estimation errors of $2\sigma$. The solid curve represents the least-squares fitting curve corresponding to an empirical equation of Zhou et al. (1995) ($\tau_y = A((\Phi/\Phi_c)^1)\eta^p$, where $A$ and $p$ are constants). The parameter values are $\Phi_{\text{in}} = 0.45, \Phi_c = 0.021, A = 2.95 \times 10^{-4}$ (Pa), and $p = 0.285$, respectively.](image-url)
critical factor for onset of yield stress.

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