Time-Height Distribution of Water Vapor Derived by Moving Cell Tomography During Tsukuba GPS Campaigns

Wataru NOGUCHI

Radio Science Center for Space and Atmosphere, Kyoto University, Uji, Japan

Takayuki YOSHIHARA

Electronic Navigation Research Institute, Mitaka, Japan

Toshitaka TSUDA

Radio Science Center for Space and Atmosphere, Kyoto University, Uji, Japan

and

Kazuro HIRAHARA

Graduate School of Environmental Studies, Nagoya University, Nagoya, Japan

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Abstract

Observations using a dense network of 75 Global Positioning System (GPS) receivers were carried out in Tsukuba, Japan, in October–November 2000 and July–September 2001. We have applied a tomography technique on the slant path propagation delay of the GPS signals to derive the temporal and spatial distribution of water vapor. In particular, we have employed the moving cell method, which was originally developed by Seko et al. (2000) for a beta-mesoscale precipitation system associated with the Baiu front. Because we are interested in local-scale phenomena, we tried to determine the water vapor distribution with a time resolution of ten minutes. The tomographic analyses resolve the water vapor variations during the meteorological events studied here.

1. Introduction

One of the useful capabilities of ground-based GPS meteorology is to estimate the amount of water vapor from the propagation delay along GPS signal paths in a moist atmosphere. Precipitable water vapor (PWV), which is the integrated water vapor in a vertical column over

a unit area, has been estimated accurately from the zenith path delay of the GPS signals, and the results have been verified by simultaneous radiosonde observations (e.g., Otani et al. 2000). Furthermore, the residual of individual slant path delays relative to the mean zenith delay is now used to study the azimuthal anisotropy of water vapor distribution.

In this study we use the slant path water vapor (SWV), which is defined as the total water vapor along a GPS ray path. We have conducted several experiments employing a number of GPS receivers in 1995–2001, as sum-

Corresponding author: Toshitaka Tsuda, Radio Science Center for Space and Atmosphere, Kyoto University, Gokasyo, Uji, Kyoto 611-0011, Japan.
E-mail: tsuda@kurasc.kyoto-u.ac.jp
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Table 1. Local-scale water vapor measurements with a dense GPS receiver network

Period	Location	Number of receivers	Horizontal extent (km)
14 Nov~15 Nov 1995	Shigaraki	24	7
22 Oct~25 Oct 1997	Nagoya	12	1
14 Oct~13 Nov 2000	Tsukuba	75	15
17 Jul~2 Sep 2001	Tsukuba	75	15

marized in Table 1. A dense network with 75 receivers was established in a square area of about 15 km $\times$ 15 km in Tsukuba in 2000–2001, where the average spacing of the receivers was about 1–2 km. In addition, other meteorological instruments were also used, including three radiometers, radiosondes, and a lidar. The details of the Tsukuba campaigns are described elsewhere in this special issue (Shoji et al. 2004). We have obtained a large number of SWV data from the Tsukuba campaigns, and in this study we determine the time-height distribution of water vapor by means of a tomography technique.

Using the Shigaraki campaign data in 1995, Hirahara (2000) applied tomography analysis to local-scale phenomena within a horizontal domain of about 15 km. However, because of the slow motion of GPS satellites, the time resolution was limited to about two hours, so the results only described the slowly varying features of the water vapor distribution. In this study, we try to improve the time resolution of the tomography analyses by using the moving cell technique, which was first applied to the meso-beta-scale phenomena in a Baiu front study by Seko et al. (2000).

2. Water vapor tomography using moving cells

The delay (ΔL) of the GPS signal in the neutral atmosphere can be related to the refractive index n . The effects of the atmosphere can be divided into two terms: the delay and the bending of radio waves. Ichikawa et al. (1995) reported that, for GPS satellites with the eleva-

tion angle higher than about 15 degrees, the bending term is negligible compared to the delay.

Hereafter, instead of the refractive index n , we use the refractivity N , written as:

$$N \equiv (n - 1) \times 10^6 \quad (1)$$

The refractivity N can be expressed as:

$$N = K_1 \frac{P_d}{T} + K_2 \frac{P_v}{T} + K_3 \frac{P_v}{T^2} \quad (2)$$

$$= K_1 \frac{P_d}{T} + \left(K_2 + \frac{K_3}{T} \right) \frac{P_v}{T} \quad (3)$$

where T , P_d and P_v are the temperature, dry atmospheric pressure and the water vapor partial pressure, respectively, and K_1 , K_2 and K_3 are empirical coefficients (Thayer 1974). The second term in (3) is called the wet refractivity. From (1) and (3), we solve for ΔL :

$$\Delta L = 10^{-6} \int \frac{K_1 P_d}{T} ds + 10^{-6} \int \left(K_2 + \frac{K_3}{T} \right) \frac{P_v}{T} ds \quad (4)$$

The first term can be estimated from the surface pressure (Elgered et al. 1991). Therefore, given the measured delay ΔL , we can calculate the second term in (4), which is called the wet delay.

For the tomography analysis, the volume of the atmosphere above the GPS receiver network is divided into rectangular parallelepiped cells. We assume that the refractivity is constant in each cell, so that (4) can be discretized as:

$$\Delta L = 10^{-6} \int N ds = 10^{-6} \sum N_k \Delta s_k \quad (5)$$

Fig. 1. A schematic diagram of the tomography analysis with the moving cell method

where N_k is the refractivity in the k -th cell, and Δs_k is the path length in that cell. We express N_k in each cell as a combination of the horizontally averaged refractivity $\bar{N}(z)$ and the residual N'_k as $N_k = \bar{N}(z) + N'_k$. Eq. (5) is now rewritten as:

$$\Delta L - \int \bar{N}(z) dz = 10^{-6} \sum N'_k \Delta s_k \quad (6)$$

This is the observational equation, where the unknowns are the set of N'_k .

At one epoch, the number of equations given by (6) becomes the same as pairs of GPS receivers and GPS satellites. Note that a GPS receiver usually detects four to ten GPS satellites at one epoch. However, the total number of the equations may not be sufficient enough to resolve the three-dimensional distribution of water vapor in local-scale phenomena. Therefore, Hirahara (2000) stacked all the GPS data to determine the distribution in the time window for the tomography analysis during which GPS satellites moved and the sufficient path coverage became available. Because the motion of the GPS satellites is slow, the time window was set to be two hours.

We assume that the cell at each height moves at the speed of the background horizontal wind, without changing the refractive index value inside the cell. This method is, therefore, called the moving cell method, and is illustrated in Fig. 1. Using this technique, the number of paths that run through the cells increases. As a result, we can determine the tomography solution with a higher temporal resolution.

The moving cell method was successfully applied by Seko et al. (2000) for analyzing the

water vapor distribution within a regional scale, i.e., with a horizontal extent of about 100 km every 1 hour, where the horizontal and vertical resolution was 14 km and 1 km, respectively. Note that Seko et al. (2000) did not consider the height variation of the horizontal wind velocity, and the cells at all altitudes were assumed to move at about 40 km/h (11 m/s), which was taken from the observed motion of the precipitation area.

We apply this technique for a local-scale tomography with the dense receiver network observation within the horizontal domain of about 15 km. The stacked observational equations, (6) can be written as:

$$\mathbf{d} = \mathbf{G}\mathbf{m} \quad (7)$$

where $\mathbf{d}$ is the data vector, $\mathbf{G}$ is the coefficient matrix and $\mathbf{m}$ is the unknown vector. We solve (7) using the damped least squares method (Hirahara 1977; Menke 1984), which is commonly used in seismic tomography. This method requires minimization of the following form:

$$\|\mathbf{d} - \mathbf{G}\mathbf{m}\|^2 + \mathbf{m}^T \mathbf{\Theta} \mathbf{m} \quad (8)$$

where $\mathbf{\Theta}$ is the diagonal damping matrix and $\mathbf{m}^T$ represents the transpose of $\mathbf{m}$. $\mathbf{\Theta}$ is expressed as:

$$\mathbf{\Theta} = \begin{pmatrix} \sigma_d^2/\sigma_{m_{o1}}^2 & & & \\ & \sigma_d^2/\sigma_{m_{o2}}^2 & & \\ & & \ddots & \\ & & & \sigma_d^2/\sigma_{m_{ok}}^2 \end{pmatrix} \quad (9)$$

in which individual elements are the ratio of the variance of data σ_d^2 and a priori variance $\sigma_{m_{oi}}^2$ of each parameter. Finally, we get the solution ($\mathbf{m}^*$) as:

$$\mathbf{m}^* = (\mathbf{G}^T \mathbf{G} + \mathbf{\Theta})^{-1} \mathbf{G}^T \mathbf{d} \quad (10)$$

It is noteworthy that the spatial resolution and the reliable range of the tomography results significantly depend on the damping factor, which is defined by the ratio of the a priori data variance to the a priori parameter variance $\sigma_d^2/\sigma_{m_{ok}}^2$. When the damping factor is small, the resolution improves, but the reliable range becomes worse, and vice-versa for a larger damping factor. Therefore, it is important to obtain an adequate damping factor by consid-

Table 2. Parameters for tomography analysis (simulation and August 2001)

Sampling rate	t_{int}	30 s
Number of satellites	N_{sat}	8 Satellites
Number of receivers	N_{sta}	75 receivers
Cell size (E-W × N-S × Height)		1.5 × 1.5 × 1.0 km
Number of cells (E-W × N-S × Height)		12 × 12 × 8
Estimation time	t_{int}	600 s

ering the required resolution and accuracy. After repeating the tomography analysis with various damping factors ranging from 0.1 to 300, we have empirically found that the damping factor should be within 10–50, and we finally chose it to be 30. Note that we assumed that the damping factor in each cell is constant, assuming the data quality is homogeneous.

Fig. 2. Tomography analysis of the refractivity distribution data using a model atmosphere

3. Test of the method using an idealized refractivity distribution

In order to verify the moving cell technique, we solved a case in which refractivity (and SWV) data were given from an idealized atmosphere, though other conditions were taken from the Tsukuba campaign. In this simulation, we referred to positions of the GPS satellites around 14:00 local time (JST) on 1 Aug 2001, when each receiver detected eight GPS satellites. We also used the coordinates of the 75 GPS receivers deployed during the Tsukuba campaigns. The wind velocity profile was taken from the Doppler radar observation. The sampling rate of the SWV data was 30 seconds.

In the following we discuss the required time interval, which must be chosen long enough to allow a sufficient number of observational equations. This requires examination of the number of observational equations and unknown parameters.

The total number of observational equations, n_{eq} becomes:

$$n_{eq} = n_{sta} \times n_{sat} \times t_{int}/\Delta t \quad (11)$$

Fig. 3. Fluctuating components of the refractivity from the tomography analysis at 13:50~14:40 JST on 1 Aug 2001

where n_{sat} is the number of GPS satellites being seen, n_{sta} is the number of receivers, t_{int} is the duration for stacking, and Δt is the sampling rate. This n_{eq} must be much larger than the number of cells. The number of cells is related to the cell size and the extent of the estimation area. The cell size should be large enough for at least one ray to always pass through it.

We investigated the wind velocity profile at 14:04 JST on 1 Aug 2001 from the Doppler radar at the Meteorological Research Institute (MRI), and found that the maximum wind velocity was 27.2 m/s at 8 km. Considering the background wind speed, we set the horizontal cell size as $1.5 \text{ km} \times 1.5 \text{ km}$, and the stacking time as ten minutes. Table 2 shows the parameters employed in the analysis procedure below.

Under these conditions, we tested the tomography analysis method with a simple example. We constructed SWV data assuming a refractivity distribution in which all cells had zero values except one located at a height of 2–3 km, which had a refractivity of $N = 20$. In order to obtain as many observational equations as possible, we also used SWV data for the slant paths at low elevation angles that intersect the side boundary of the estimation area. In this case, the refractivity outside the analysis area is taken from the background refractivity $\bar{N}(z)$.

The result is shown in Fig. 2. The refractivity in the cell given as $N = 20$ is inferred as $N = 18.23$. This simulation confirms that the moving cell technique allows us to analyze refractivity distribution with a good time resolution (ten minutes).

4. Tomography analyses using Tsukuba campaign observations

We have selected two cases from the Tsukuba campaigns in 2000 and 2001, respectively, and applied the tomography analysis with the moving cell method.

4.1 Case 1: Results on 1 August 2001

We analyzed the data around 14:00 JST on 1 Aug 2001, when the Tsukuba area had a thunderstorm. From 14:00 to 14:20 JST, heavy rain occurred, so considerable variations in the refractivity were expected. The parameters for the tomography analysis are shown in Table 2.

The horizontally averaged refractivity was obtained using the radiosonde launched at 08:30 JST. Note that this profile was observed six hours before the tomography analysis; therefore, we may expect some changes of the profile within six hours. This may change slightly the mean profile that is assumed to be horizontally invariant over the area of the interest at each height. But, the deviation from the mean derived by the tomography analysis is not affected. Table 3 shows wind velocity at MRI observed by Doppler radar. We adopted these values as the horizontal motion of the cells at each height.

Figure 3 shows the results of the fluctuating components of the refractivity distribution at the height of 0–4 km during 13:50–14:40 JST. There are five independent results. The large refractivity values at 3–4 km appeared at 14:00–14:10, continued until 14:20–14:30, and disappeared at 14:30–14:40. The analyzed behavior is consistent with a thunderstorm in the area. The enhanced refractivity region moved southeastward, with a direction and speed that is consistent with the radar-derived horizontal winds. For example, in the 3–4 km height range the large refractivity area moved about 3 km southeast between 14:00–14:10 and 14:10–14:20, which corresponds to the horizontal speed of about 5 m/s. During this period, the eastward and northward horizontal wind velocity was 3.8 m/s and 3.9 m/s, respectively, from the radar observation (see Table 3). That is, the horizontal wind speed was 5.5 m/s in the southeastward direction.

4.2 Case 2: Results on 25 November 2000

We also analyzed the Tsukuba data from midnight to 1:00 JST on 25 Nov 2000. Note that the water vapor profile was observed with a Vibrational Raman lidar during this period. Figure 4 shows the time-height section of the water vapor for one hour. The water vapor density increased at 2–3 km and 4–5 km. The lidar results are used for a comparison with the tomography solution.

We used the analysis parameters listed in Table 4. As we found the background north-south wind velocity was nearly zero from the radiosonde observations at MRI, we took a larger cell size along that direction. The average refractivity and the background wind ve-

locity are taken from a radiosonde observation at 23:30 JST on 24 November 2000. Figure 5 shows the horizontal distribution of the refractivity at 0–5 km that is independently analyzed every ten minutes during 0:00–0:50 JST. A very consistent smooth variation can be seen in the time series. Increase in the refractivity appeared at 2–3 km and 4–5 km from the tomography analysis, which is generally consis-

tent with the Raman lidar observations at MRI. At 2–3 km and 4–5 km, the results in Figure 5 are compared with the simultaneous Raman lidar observations at MRI in Figs. 6 and 7, respectively. Note that we converted the analyzed refractivity to the water vapor density (P_w), using the temperature and pressure profiles observed with the radiosondes.

It is important to note that in the moving cell method the individual cells move through the analyzed area following the background mean winds; therefore, the time variations of P_w over a single location can be delineated within the time duration necessary for the tomography analysis (ten minutes). In Figs. 6 and 7 the analyzed P_w values are plotted with the time resolution of one minute.

At 2–3 km, the P_w values agree well between the independent determinations from tomography and the lidar. Moreover, the time variations of P_w were consistent. That is, the tomography analysis shows a gradual increase of P_w , and the enhancement at 0:40 JST, which is also seen in the lidar results. However, at the 4–5 km altitude, the time variations in the first half of the analyzed period do not agree well, though the absolute values of P_w in the latter half are consistent.

Fig. 4. Water vapor density at 25 Nov 2000 from lidar observations at MRI

Fig. 5. Fluctuating components of the refractivity from the tomography analysis at 0:00–0:50 JST on 25 Nov 2000

Table 3. Wind velocity (m/s) observed by Doppler radar at MRI

Time (JST)		13:56	14:04	14:18	14:26	14:33
0–1 km	Eastward Wind	0.997	0.962	0.273	−1.194	1.101
	Northward Wind	−5.104	−5.720	−5.293	−4.701	−4.621
1–2 km	Eastward Wind	−0.998	−0.843	0.963	−0.045	−0.921
	Northward Wind	−2.882	−2.565	2.143	−1.699	−1.719
2–3 km	Eastward Wind	5.098	−5.641	−4.214	−3.773	−3.937
	Northward Wind	−0.245	0.310	2.298	−1.956	1.144
3–4 km	Eastward Wind	8.325	9.649	−5.825	−3.929	−3.068
	Northward Wind	−0.648	0.152	3.790	2.995	2.156

Table 4. Parameters for tomography analysis (November 2000)

Sampling rate	t_{int}	30 s
Number of satellites	N_{sat}	8 Satellites
Number of receivers	N_{sta}	75 receivers
Cell size (East-Weat × North-South × Height)		$1.5 \times 2.0 \times 1.0$ km
Number of cells (E-W × N-S × Height)		$16 \times 8 \times 8$
Estimation time	t_{int}	600 s

One possible reason for this disagreement is that the positions of the GPS satellites during this analysis period were not ideal; that is, the number of available GPS satellites was as small as four to five, causing a smaller number of rays to pass through the analyzed volume. Furthermore, more satellites (three to four) were seen on the west side, with less data on the east side. With fewer independent data, the result is negatively impacted. This may result in the inaccurate determination of p_w with the tomography analysis.

5. Summary

We have applied a tomography analysis using the moving cell method to the high-resolution GPS network at Tsukuba in order to estimate water vapor distribution with high temporal resolution. We first tested the method using an idealized distribution with ten-minute resolution. This simulation confirms that the moving cell technique works well on an idealized case. We then analyzed actual data from the Tsukuba campaigns. In the first example, the results agree with the meteorological situation, which contained a thunderstorm.

Fig. 6. Time variation of the water vapor density at 2–3 km over MRI from the tomography analysis (solid line) and the lidar observation (broken line)

Fig. 7. The same as Fig. 6 except for the results at 4–5 km

Five independent solutions indicated that an enhanced refractivity field moved southeastward following the background horizontal winds. In the second example, we compared the tomography results with the simultaneous lidar observations and found reasonably good agreement. Thus, the moving cell tomography is a promising method for retrieving local-scale refractivity distributions.

However there are some issues to be resolved. We need to investigate the effects of our assumptions, such as uniform refractivity in each cell and uniform background winds over the entire area.

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