Turbulent Diffusion from a Patchy Surface into the Boundary Layer

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Abstract

Turbulent diffusion from a small surface source into the boundary layer is studied by means of a wind-tunnel experiment. On the tunnel floor, a strip-shaped heater was placed perpendicularly to the mean flow. We regarded the heat as a passive scalar, and measured the air temperature in the boundary layer over the tunnel floor. The passive scalar, advected on the mean flow, diffuses in the floor-normal direction. For the diffusion of the passive scalar throughout the boundary layer, we find a characteristic length scale in the mean-flow direction. This length scale, which is always twice the boundary-layer thickness regardless of the other experimental parameters, would be useful when we consider the atmospheric boundary layer over a patchy surface.

1. Introduction

Let us consider diffusion of a passive scalar from a surface source into a turbulent boundary layer. The source size is not much greater than the thickness of the boundary layer. This often occurs in the atmospheric boundary layer, both for diffusion of heat and for diffusion of matter, because the land surface is often patchy. We would like to understand the overall character of such turbulent diffusion throughout the boundary layer.

The simplest two-dimensional situation is that the surface temperature is \( T_s = T_0 \) at \( x \leq 0 \) and \( x > d_z \), and \( T_s = T_1 \) at \( 0 < x \leq d_z \). The air temperature at \( x \leq 0 \) is also \( T_0 \). Here \( T_0 \) and \( T_1 \) are constants with \( T_0 < T_1 \), and \( x \) is measured in the mean-flow direction. The temperature difference \( T_1 - T_0 \) is sufficiently small, the outer-flow velocity \( U_0 \) is sufficiently large, so that the convection is negligible. The boundary layer is stationary and does not change markedly above around the surface source from \( x = 0 \) to \( d_z \). Then the problem is to obtain the time-averaged temperature \( T(x, z) \) in the boundary layer, where \( z \) is the height from the surface.

From the similarity law for diffusion (Monin and Yaglom 1971; Townsend 1976; Landau and Lifshitz 1987; Schlichting and Gersten 2000), the time-averaged temperature \( T \) is determined as

\[
\frac{T - T_0}{T_1 - T_0} = f\left(\frac{x}{d_z} \cdot \frac{z}{d_z} \cdot \frac{U_0 d_z}{v} \frac{v}{\chi_{mol}}\right). \tag{1}
\]

Here \( d_z \) is the boundary-layer thickness, \( v \) is the kinematic viscosity, and \( \chi_{mol} \) is the molecular diffusivity. Since the Reynolds number \( U_0 d_z / v \) is large, the overall structure of a turbulent boundary layer is independent of the kinematic viscosity \( v \), and hence of the Reynolds number \( U_0 d_z / v \). The turbulent diffusion dominates over the molecular diffusion that reflects the Prandtl number \( v/\chi_{mol} \). Thus the air temperature is de-
termined by the positional and geometrical parameters $x/d_z$, $z/d_z$, and $d_z/d_x$ at most.

The existing theoretical works are not satisfactory. Sutton (1943) obtained an analytical solution for the equation of the time-averaged temperature $T$:

$$ U \frac{\partial T}{\partial x} = \frac{\partial}{\partial z} \left( \chi_{\text{trb}} \frac{\partial T}{\partial z} \right). $$

The assumption was that the mean velocity $U$ scales as $z^m$, and the turbulent diffusivity $\chi_{\text{trb}}$ scales as $z^{1-m}$. These scaling laws are mere approximations. It was also assumed for simplicity that the boundary-layer thickness $d_z$ is infinite (for similar works, see Monin and Yaglom 1971; Brutsaert 1982). Townsend (1965, 1976) resorted to a different assumption. Since Eq. (2) is linear, the solution for the present problem is a superposition of the solutions for two types of surface conditions: (i) $T_x = T_0$ at $x \leq 0$, and $T_x = T_1$ at $x > 0$ and (ii) $T_x = 0$ at $x \leq d_x$, and $T_x = T_0 - T_1$ at $x > d_x$. The temperature distribution downstream of such a discontinuity tends logarithmic. Hence the solution was searched among distribution functions that correspond to a sum of two logarithmic functions near the surface. However, in the present problem, the source size is small, $d_x \ll d_z$. The logarithmic distribution is achieved only asymptotically far downstream of the temperature discontinuity (Antonia et al. 1977).

There are numerous experiments for a release of heat or gas at the surface of a turbulent boundary layer. However, for our own problem, i.e., turbulent diffusion of a passive scalar from a small surface source throughout the boundary layer in the two-dimensional configuration, directly relevant experiments are not so numerous. Since these experiments tend to focus on the vicinity of the surface source, the overall character of the turbulence diffusion remains uncertain (section 4).

We propose that the diffusion throughout a turbulent boundary layer is characterized by the relaxation length $l_x$, a length scale in the mean-flow direction. To the extent of our knowledge, such a proposal has not been made so far. The relaxation length is estimated on dimensional grounds in section 2. An experiment is conducted to confirm the presence of the relaxation length, and obtain its actual value. The experiment is described in section 3.

The results are discussed in section 4. We conclude with remarks for the application to the atmospheric boundary layer in section 5.

2. Dimensional analysis

Turbulent diffusion throughout a boundary layer is dominated by largest and strongest eddies. Their circulation velocities are of the order of the outer-flow velocity $U_0$. Their sizes are of the order of the boundary-layer thickness $d_z$. The corresponding turbulence diffusivity $\chi_{\text{trb}}$ is

$$ \chi_{\text{trb}} \propto U_0 d_z \quad (3) $$

(Monin and Yaglom 1971; Landau and Lifshitz 1987). There is no significant diffusion from the turbulent boundary layer into the outer laminar flow. The relaxation timescale $\tau$, i.e., a characteristic timescale for the diffusion of a passive scalar throughout the boundary layer (Landau and Lifshitz 1987), is

$$ \tau \propto \frac{d_x^2}{\chi_{\text{trb}}} \propto \frac{d_z}{U_0} \quad (4) $$

This is just the turnover time of the largest eddies. With the typical value of the mean velocity for the whole boundary layer $U_{\text{typ}}$, the timescale $\tau$ yields the relaxation length $l_x$, i.e., a characteristic length scale in the mean-flow direction:

$$ l_x \propto U_{\text{typ}} \tau \propto \frac{U_{\text{typ}}}{U_0} d_z. \quad (5) $$

We assume that the mean velocity $U$ scales only with the outer-flow velocity $U_0$ (see below). Then the factor $U_{\text{typ}}/U_0$ is constant. The relaxation length $l_x$ is determined by the boundary-layer thickness $d_z$ alone.

We comment on the turbulence diffusivity $\chi_{\text{trb}}$. Since largest eddies dominate the diffusion, it is not local. Then Eq. (2), based on the local temperature gradient $\partial T/\partial z$, is not exactly valid. The turbulent diffusivity $\chi_{\text{trb}}$ in Eqs. (3) and (4) should be regarded as a quantity defined for the temperature distribution smoothed over a large scale.

We also comment on the scaling of the mean velocity $U$ with the outer-flow velocity $U_0$. The usual assumption is the scaling with the friction velocity. However, the friction velocity is defined near the surface. Throughout a boundary layer, whether its surface is smooth or rough, the better assumption is that the mean
velocity $U$ and the velocity fluctuation $u$ scale with the outer-flow velocity $U_0$ (Townsend 1976; Perry and Schofield 1973; Perry and Abell 1977; Zagarola and Smits 1998).

3. Experiment

The experiment was done in a wind tunnel of the Meteorological Research Institute. Its test section had the size 300, 80, and 80 cm, respectively, in the streamwise ($x$), spanwise ($y$), and floor-normal ($z$) directions. A thin electric heater, with the length $d_x=80$ cm, was placed at about 150 cm downstream of the entrance to the test section, as shown in Fig. 1. The windward edge of the heater corresponds to the origin $x=z=0$ cm. The heater was on an aluminum plate, i.e., heat conductor, which was kept apart from the tunnel floor. Upstream and downstream of the heater, the tunnel floor was covered with styrofoam plates, i.e., heat insulator. On the heater and styrofoam plates, there were cubic blocks, i.e., roughness elements. Those at $x \leq -75$ cm were large, and triggered turbulence. Their spacing was $\delta x = \delta y = 10$ cm. The blocks at $x > -75$ cm were small, and sustained the turbulence. Their length of side was 1 cm. Their spacing was $\delta x = 8$ cm and $\delta y = 10$ cm.

Several measurements were conducted, as listed in Table 1. In the run A, the incoming-flow velocity, which is regarded as the outer-flow velocity $U_0$, was 2 m s$^{-1}$. The streamwise width of the heater was $d_x=7.5$ cm. The side length of the large blocks at $x \leq -75$ cm was $L=3.0$ cm. One of these parameters was changed in the other runs. The run B had $d_x=15.0$ cm. The run C had $U_0=1$ m s$^{-1}$. The run D had $U_0=3$ m s$^{-1}$. The run E had $L=1.5$ cm. The run F had $L=2.0$ cm.

As mentioned above, we placed large blocks at $x \leq -75$ cm, in order to trigger turbulence. This is usual for an experiment of a boundary layer in a small wind tunnel, where the fetch is too short for the natural development of boundary-layer turbulence. The presence of the large blocks is nevertheless not serious. Our results in section 4 are independent of the side length of the large blocks $L$.

![Fig. 1. Side view of our experimental setup. The coordinates $x$, $y$, and $z$ are in the streamwise, spanwise, and floor-normal directions. The origin $x=z=0$ cm is at the windward edge of the heater.](image)

Table 1. Summary of the air temperature $T_0$, the heater temperature $T_1$, the outer-flow velocity $U_0$, the boundary-layer thickness $d_z$ at $x=0$ cm, and the relaxation length $l_x$ in the runs A–F.

<table>
<thead>
<tr>
<th>Run</th>
<th>$T_0$ [°C]</th>
<th>$T_1$ [°C]</th>
<th>$U_0$ [m s$^{-1}$]</th>
<th>$d_z$ [cm]</th>
<th>$l_x$ [$d_z$]</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>11.0</td>
<td>38.1</td>
<td>2</td>
<td>11.7</td>
<td>1.7</td>
<td>Standard run$^*$</td>
</tr>
<tr>
<td>B</td>
<td>9.8</td>
<td>38.4</td>
<td>2</td>
<td>11.7</td>
<td>2.1</td>
<td>$d_x=15.0$ cm</td>
</tr>
<tr>
<td>C</td>
<td>11.5</td>
<td>42.5</td>
<td>1</td>
<td>10.1</td>
<td>1.9</td>
<td>$U_0=1$ m s$^{-1}$</td>
</tr>
<tr>
<td>D</td>
<td>12.4</td>
<td>57.7</td>
<td>3</td>
<td>11.6</td>
<td>1.7</td>
<td>$U_0=3$ m s$^{-1}$</td>
</tr>
<tr>
<td>E</td>
<td>12.2</td>
<td>40.2</td>
<td>2</td>
<td>7.7</td>
<td>2.0</td>
<td>$L=1.5$ cm</td>
</tr>
<tr>
<td>F</td>
<td>12.2</td>
<td>40.8</td>
<td>2</td>
<td>8.8</td>
<td>1.9</td>
<td>$L=2.0$ cm</td>
</tr>
</tbody>
</table>

$^*$The outer-flow velocity $U_0=2$ m s$^{-1}$, the heater width $d_x=7.5$ cm, and the block size $L=3.0$ cm (see text).
We used a cold-wire thermometer to measure the air temperature, and a hot-wire anemometer to measure the streamwise velocity. These two instruments had the same single-wire probe, i.e., platinum-plated tungsten wire with 5 μm diameter and 1.25 mm sensitive length. The probe was attached to a traversing system. When the hot-wire anemometer was used, the heater temperature was set to be the same as the incoming-flow temperature, in order to avoid noise due to the fluctuation of the air temperature. Thermocouples were also used to monitor the surface temperatures of the heater and styrofoam plates at several points.

Figure 2 shows the vertical profiles of the average $U$ and the root-mean-square fluctuation $\langle u^2 \rangle^{1/2}$ of the streamwise velocity at $x = 0$ cm. The velocity is normalized by the outer-flow velocity $U_0$. The height $z$ is normalized by the boundary-layer thickness $d_z$ at $x = 0$ cm. The velocity profiles are common for the runs A and B.

![Figure 2: Vertical profiles of the average $U$ and the root-mean-square fluctuation $\langle u^2 \rangle^{1/2}$ of the streamwise velocity. The velocity is normalized by the outer-flow velocity $U_0$. The height $z$ is normalized by the boundary-layer thickness $d_z$ at $x = 0$ cm. The velocity profiles are common for the runs A and B.](image)

4. Results and discussion

Figure 3 (upper panel) shows the two-dimensional distribution of the temperature increase $\delta T$ in the run A. It is evident that the region of significant temperature increase grows in the course of diffusion.

Figure 4 shows the vertical profiles of the temperature increase $\delta T$ at $x = 0.64, 0.96$, and $1.71d_z$ in the run A. On this semi-logarithmic plot, the data points make up straight lines, so far as they are above the noise level. The temperature increase $\delta T$, at each of the $x$ positions, is an exponential function of the height $z$: $\delta T \propto \exp(-z/h_z)$. Since an exponential law implies the presence of a characteristic scale, it follows that the vertical profile of the temperature increase $\delta T$ is characterized by the scale

$$w_{nc} = [g \beta h_z(T_1 - T_0)]^{1/2}$$

(Schlichting and Gersten 2000). Here $g$ is the gravitational acceleration, $\beta = 0.003{\text{C}}^{-1}$ is the thermal expansion coefficient, and $h_z \ll 1$ cm is the scale height for the distribution of the air temperature above around the heater (see below, Fig. 5). We obtain $w_{nc} \ll 0.1$ m s$^{-1}$ or $w_{nc} \ll 0.1U_0$, which is below the average $U$ and the root-mean-square fluctuation $\langle u^2 \rangle^{1/2}$ of the streamwise velocity at the height $h_z$ (Fig. 2). Thus the convection is negligible.
height $h_z$, under which the heat is well mixed with the air. The exponential law above the noise level is also seen at all the other $x$ positions in the run A, and at all the $x$ positions in the runs B–F. The correlation coefficient on the semi-logarithmic plot is always greater than 0.96.

Figure 5 shows the streamwise profiles of the scale height $h_z$. The scale height $h_z$ and the streamwise displacement $x$ are normalized by the boundary-layer thickness $d_z$. The profiles are the same among all the runs as a result of the similarity law for diffusion (section 1). Moreover, on this semi-logarithmic plot, the data points make up straight lines with high correlation coefficients (>0.98). The scale height $h_z$ is an exponential function of the streamwise displacement $x$: $h_z \propto \exp(x/l_x)$. This exponential law implies that the streamwise growth of the scale height $h_z$ is characterized by a length scale in the mean-flow direction $l_x$. We regard $l_x$ as the relaxation length in Eq. (5), a characteristic length scale in the mean-flow direction for the diffusion throughout the boundary layer. It should be noted that the relaxation timescale in Eq. (4), which yields the relaxation length, was also defined from the exponential factor for turbulent diffusion (Landau and Lifshitz 1987). The $l_x$ value is twice the boundary-layer thickness $d_z$ (Table 1). If we extrapolate the growth curve, the scale height $h_z$ is equal to the boundary-layer thickness $d_z$ at $x = (3.4–3.7) \times l_x$.

Judging from the same results for the runs A and B, the streamwise source size $d_x$ is unim-
important, so far as $d_x$ is small, $d_x \lesssim d_z$, although the similarity law for diffusion includes the parameter $d_x/d_z$ (section 1). If $d_x/d_z$ were greater, its effect should be discernible. The extreme example is the formation of an internal boundary layer, the thickness of which depends on the streamwise source size $d_x$ (see Antonia et al. 1977).

The exponential laws obtained for Figs. 4 and 5 are zeroth-order approximations. We might obtain more complicated laws if we made a more detailed analysis. However, such complicated laws are not necessary, because we are interested in the overall character of turbulent diffusion throughout a boundary layer.

The existing experiments have revealed the exponential or quasi-exponential law for the vertical distribution of the temperature increase $\delta T$ (Monin and Yaglom 1971; Townsend 1976; see also Bara et al. 1992; Kastner-Klein and Fedorovich 2002). However, the exponential law for the streamwise growth of the scale height $h_z$ and the presence of the relaxation length $l_z$ have not been reported so far. This is probably because the existing experiments tend to focus on the vicinity of the surface source.

Figure 3 (lower panel) shows the two-dimensional distribution of the root-mean-square temperature fluctuation $\langle t^2 \rangle^{1/2}$. The temperature fluctuation $\langle t^2 \rangle^{1/2}$ has the same order of magnitude as the temperature increase $\delta T$. With an increase of the streamwise displacement $x$ from the heater, the height at which $\langle t^2 \rangle^{1/2}$ is maximal is increasingly large. This height coincides with the scale height $h_z$ of the temperature increase (dotted line). The temperature fluctuation $\langle t^2 \rangle^{1/2}$ is maximal at the interface between the airs that have and have not been well mixed with the heat (Townsend 1976; Antonia et al. 1977; see also Bara et al. 1992; Crimaldi et al. 2002). The height of this interface corresponds to the scale height $h_z$.

5. Concluding remarks

The two-dimensional distribution of the temperature increase $\delta T$ revealed that the diffusion of a passive scalar from a small surface source throughout the boundary layer is characterized by a length scale in the mean-flow direction. This relaxation length $l_z$ is twice the boundary-layer thickness $d_z$, regardless of the other parameters. The reason is that the diffusion is dominated by largest eddies in the boundary layer. Their sizes are of the order $d_z$. A consistent feature was found for the two-dimensional distribution of the root-mean-square temperature fluctuation $\langle t^2 \rangle^{1/2}$.

We expect that our results from a wind-tunnel experiment are applicable to the atmospheric boundary layer. This is because, among boundary layers, the properties of largest eddies that dominate the diffusion are expected to be almost universal. Throughout a boundary layer, whether its surface is smooth or rough, the vertical profiles of the mean velocity $U$ and the velocity fluctuation $u$ are almost universal if normalized by the boundary-layer thickness $d_z$ and the outer-flow velocity $U_0$ (Townsend 1976; Perry and Schofield 1973; Perry and Abell 1977; Zagarola and Smits 1998). Only near the surface, its effect is important. The atmospheric boundary layer is no different from a turbulent boundary layer.

The relaxation length $l_z$ would be useful when we consider the atmospheric boundary layer over a patchy surface. This situation is crucial to our understanding of the land-atmosphere interaction (Bonan et al. 1993; Chen et al. 2003, and references therein).
relaxation length $l_x$ corresponds to the length of correlation among the patches. When the separation between two of the patches is greater than the relaxation length $l_x$, their heat and material fluxes are independent. This is because the heat and material have diffused away before the air moves from one patch to the other. However, when the separation is less than the relaxation length $l_x$, the fluxes from the patches are dependent. It would be better to consider the patches as a single source. An example is the oasis effect for evaporation from water surfaces. If the total area of the water surfaces is the same, the evaporation is maximal in the most sparse distribution of the water surfaces. This study reveals that the oasis effect is significant if the scale for the sparseness exceeds the relaxation length $l_x$.

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