NOTES AND CORRESPONDENCE

Evidence of Trend in Return Levels for Daily Windrun in New Zealand

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Abstract

Annual maxima of daily windrun is modeled statistically with respect to year for nineteen locations spread throughout New Zealand, and for one location in the Cook Islands. The generalized extreme value distribution is fitted to each data set to describe both the mean and the variability for each location and to predict statistically their behavior in the coming decades. We find evidence of significant trend in wind extremes for eight of the locations in New Zealand considered: six of these were decreases with percentage change in the median of annual maximum windrun ranging from 16–41% over the period of records, while the remaining two were increases with percentage change ranging from 20–24%. We provide estimates of 10, 100 and 1000 year return levels for daily windrun, and describe how they vary with the latitude of the locations. We also establish statistical upper bounds for maximum daily windrun for some of the locations in New Zealand: four locations in the North Island and one in the South Island.

1. Introduction

The T year return level of a variable is the level exceeded on average only once in T years. Thus, for example, the 2 year return level is the median of the distribution of the variable recorded annually. The variable studied here is the annual maximum of daily windrun. We estimate its 2, 10, 100 and 1000 year return levels for nineteen locations in New Zealand and one location in the Cook Islands.

Estimation of return levels is usually based on the three extreme value distributions—

Gumbel, Fréchet and negative Weibull—discovered by Fisher and Tippett (1928) for the large sample behavior of maxima. In this paper we use the generalized extreme value distribution. It was developed by Jenkinson (1955), by combining the three extreme value distributions; see also Hosking et al. (1985), Galambos (1987) and Davison and Smith (1990).

Little has been published on return levels for New Zealand wind data. Two references that we are aware of are: De Lisle (1965) and Revfeim and Hessell (1984), the former provides an analysis of maximum wind gusts for thirty three stations in New Zealand, while the latter suggests an alternative extreme value model with ‘physically meaningful parameters’ and illustrates its use for wind gust data from four stations around New Zealand. Applications of

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extreme value theory to overseas wind data include Jenkinson (1955) and Ross (1987). Farago and Katz (1990) give a review of applications of extreme value theory to climate data.

The main objective of this paper is to explore the wind data for trends. Trends in wind data could be caused by a variety of reasons such as climate change, multidecadal natural climate fluctuations, site movement, site exposure and changes in observational procedure. Based on moving averages of pressure differences, Royal Society of New Zealand (1988) find evidence of cyclical behavior of wind flow in New Zealand, with 1935–1980 being one cycle; but, as the data set considered only runs from about 1934 to about 1984 this is conjectural. If data sets covering two or more such cycles were available, then one could model wind flow as sums of sinusoids as done for rainfall by Tabony (1981), Kane (1988), Kane and Trivedi (1988) and Kane and Teixeira (1991). In an investigation of sampling error due to short records, Peterka (1992) investigates trends in extreme wind for twenty nine stations in the midwest of the United States. For earlier work relating to trends in wind for the United States, see Boger (1968) and Santomau (1970). In an analysis of thirty eight years of wind data from three sites along the South African east and south coasts, Schumann (1992) finds that the northernmost site (Durban, at about 30-degrees-S) differs markedly from the two southern sites (Port Elizabeth and Cape Town, at about 34-degrees-S). However, he identifies no clear trends in the wind speeds and suggests use of longer time series. In the study of worldwide ship data sets for surface wind speeds, Ward (1992) reports that, during the period 1949–1988, there is a mean increase in wind speed of about 16% due to changes in observational practice on board ships. Kumar and Fernandes (1994) show that surface wind speeds over the Western, Central, Eastern and Southern Arabian sea regions increased markedly for the period from 1948 to 1972. Inoue and Bigg (1995) produce significant evidence to suggest that the trade winds along the northern flank of the South east Pacific High, and in the eastern/central equatorial Pacific, have been gaining strength since the 1950s. Schiesser et al. (1997) observe a negative regional trend for Northern Switzerland to suggest that the period before

the 1940s was windier than the following decades. They explain that this could be the result of the whole westwind belt moving slightly further north, where a deepening of cyclones has been observed in recent times. Keckhut et al. (1999) investigate long-term changes of wind data, using the United States rocketsondes at six selected sites at northern tropical and subtropical locations (from 8 degrees S to 34 degrees N): the method used is based on a multifunction regression analysis that allows for a continuous linear trend, for natural variability, and for sudden changes of the mean due to successive instrumental improvements. Van den Broeke (2000) notes that annual cycles of wind speed at some Antarctic stations have changed—with increased wind speed in the solstitial months and a decrease in the equinoctial months—due to significant weakening of the semi-annual oscillation; see also Portnyagin et al. (1993).

None of these model the *variability* as well as the mean of wind extremes—although simulations giving changing variability and a discussion of gale and extreme wind predictions using a global circulation model may be found in Sections 6.4 and 6.5.6 of Houghton et al. (1996). The novelty of this paper is that it provides a method for modeling both the mean, and the variability of wind extremes. The methodology is described in Section 3. Section 2 describes the datasets. The corresponding results on return levels for daily windrun are given in Section 4 (with discussion).

2. The data

The data consists of annual maximum of the daily windrun in km, that is variable eighteen of the New Zealand climate database (Penney 1997). See also Fouhy et al. (1992). The data sets cover nineteen locations in New Zealand spread from Leigh to Winton and one location in the Cook Islands.

The locations of the nineteen New Zealand stations are shown in Fig. 1. The station names, years of data, latitude and longitude are given in Table 1.

For each of the nineteen stations it would be of interest to know which instruments were used, for what periods, the heights of the instruments, what changes have occurred to the instruments and the recording procedures and

Fig. 1. Locations of the stations.

how the exposure has changed due to site change or growth of trees or buildings. However, there is no database available giving the answers to these questions. Fouhy et al. (1992) answer these questions more or less for a selec-

tion of stations—only three of which are among our nineteen stations. We give some details from Fouhy et al. (1992) on these three stations.

- Station name: Hamilton at Ruakura (years of data: 1965–1992).

Instruments and records: A Robinson anemometer was installed with a Munro cup counter on a 6 m pole in 1936. The cup anemometer was replaced in 1980, 1983 and 1989. Records were missing for December of 1983 and October of 1990.

Site exposure: A 20 m oak to the west may have provided some shelter. In 1987, some change in exposure occurred because of tree growth.

- Station name: Wellington at Kelburn (years of data: 1928–1995).

Instruments and records: A Dines pressure tube anemometer was installed between 1928 and 1930, the mast being 6.5 m above the roof of an office which was 7.77 m above the ground. The anemometer was overhauled in 1951 and 1960. In 1959, it was reading 3 knots too low at 80 knots. A cup generating anemometer and a remote wind vane were installed in 1961. The Dines pressure tube

Table 1. The data sets used.

Station Name	Years of Data	Latitude	Longitude
Cook Island at Rarotonga Airport	1971–1996	–10.883	–165.833
Leigh	1967–1996	–36.283	174.817
Woodhill Forest	1948–1989	–36.750	174.433
Auckland at Oratia	1948–1978	–36.917	174.617
Waihi	1946–1989	–37.383	175.833
Hamilton at Ruakura	1965–1992	–37.783	175.283
Whatawhata	1971–1997	–37.800	175.150
Rotoehu Forest	1938–1986	–37.540	176.310
Gisborne at Manutuke	1971–1990	–38.683	177.917
Wanganui at Cooks Garden	1971–1998	–39.933	175.050
Palmerston North	1956–1997	–40.350	175.617
Pahiatua at Mangamutu	1938–1981	–40.450	175.800
Wallaceville	1972–1997	–41.133	175.050
Wellington at Kelburn	1928–1995	–41.283	174.767
Grassmere Salt Works	1971–1998	–41.430	174.090
Lincoln	1891–1987	–43.650	172.483
Mt Cook at The Hermitage	1971–1995	–43.430	170.060
Winchmore	1950–1998	–43.833	171.717
Waimate	1931–1981	–44.430	171.030
Winton	1971–1997	–46.333	168.267

anemometer was replaced by an electric anemometer in 1966. The anemometer malfunctioned in 1971, was stiff then loose in 1973 (meaning that its rotation with wind speed was too slow and too fast at times), read low in 1976, and read low in 1977 for high winds. Adjustments were made. The vane and the cup anemometers were reported damaged in 1979. A cup counter was replaced in 1983. Records were missing for 1961–1963 and 1965–1966.

Site exposure: Around 1948, when additional rooms were built on the top floor of the office, the exposure of the anemometer began to deteriorate. This was further aggravated by growth of pine trees, 15–20 m high, about 20 m to the north of the office. Trees and bushes were cut back in 1949 and 1959.

- Station name: Lincoln (years of data: 1891–1987).

Instruments: A Robinson's anemometer was present in 1913 and may have been in use for thirty years. The instruments currently in use are: a Munro cup counter anemometer and a Lambrecht recorder installed in 1981 and 1985 (The Lambrecht is an accurate recorder, whereas the Robinson and the Munro are considered more qualitative than quantitative. The Munro is considered inaccurate below 5 knots.).

Site exposure: Trees had grown considerably by 1915. Rows of pines seriously affected exposure by 1927 and needed topping. In August of 1927, the anemometer pole was found not vertical. In December of 1942, the mast was found not steady and enclosure closed in by hedges. Two rows of poplar windbreaks were noted in 1967 but not expected to affect the enclosure for several years. Exposure of the Robinson cup anemometer had been deteriorating in 1972. The deterioration caused by the poplars was noted in 1975.

Site change: The site moved 1 km in 1943, approximately 0.5 km in 1962 and 0.2 km in 1975. In 1943, a marked increase in windrun was expected from the site change.

In summary, we have illustrated some of the problems with the data due to change of instrumentation, site change, change of building and change of exposure due to growth of trees or their trimming.

3. Methodology

The method that we use allows both the mean and the variability of wind extremes to change with time. We model the change polynomially with time as polynomial powers have the appeal of simplicity and flexibility to describe a wide variety of behavior—including effects of climate change, site exposure and changes in observational procedure such as instrumental improvements. The model is fully parametric and has had some success with temperature and rainfall data—see Withers and Nadarajah (2000a, 2000b). Thus, it is reasonable to expect that parameters extracted from the historical data can be used for extrapolation into the future for obtaining extreme values such as 1000 year windruns. However, the model is by no means perfect: we shall ignore effects of factors such as multidecadal climate oscillations, and site movements which may not be represented by the model.

In terms of the standardized time variable

$$x_N = N/(n + 1),$$

the model chosen for the annual maximum windrun, Y_N for years $N = 1, \dots, n$ is

$$Y_N = \mu_N(\alpha) + \sigma_N(\beta)e_N, \quad (1)$$

where either

$$\sigma_N(\beta) = \exp\left(\sum_{i=0}^p \beta_i x_N^i\right)$$

and

$$\mu_N(\alpha) = \sum_{i=0}^p \alpha_i x_N^i,$$

giving a model denoted $L(m)$, or

$$\sigma_N(\beta) = \exp\left(\sum_{i=0}^p \beta_i x_N^i\right)$$

and

$$\mu_N(\alpha) = \exp\left(\sum_{i=0}^p \alpha_i x_N^i\right),$$

giving a model denoted $E(m)$. Note that the models $L(0)$ and $E(0)$ are equivalent. Here, $\mu_N(\alpha)$, the mean of the process, and $\sigma_N(\beta)$, the variability or scale of the process, have been

parameterized in such a way to model a variety of trend behavior. The case $m = 1$ corresponds to linear trends, and $m = 2$ to quadratic trends. The case $m = 0$ corresponds to no trends. The noise or residuals, $\{e_1, \dots, e_n\}$, are assumed to be independent with the generalized extreme value distribution

$$F_\xi(x) = P(e_N \leq x) = \exp\{-(1 + \xi x)^{-1/\xi}\} \quad (2)$$

for $1 + \xi x > 0$, where ξ is referred to as the shape parameter. The constraint $1 + \xi x > 0$ implies that for $\xi < 0$, maxima are bounded above by $\mu_N - \sigma_N/\xi$, and for $\xi > 0$, maxima are bounded below by $\mu_N - \sigma_N/\xi$. The particular case of (2) for $\xi = 0$,

$$F_0(x) = \exp\{-\exp(-x)\}$$

for $-\infty < x < \infty$, is known as the EV1 or Gumbel distribution, while the cases $\xi > 0$ and $\xi < 0$ are known as the EV2 or Frechet and the EV3 or negative Weibull distributions, respectively. Maximum rainfall and floods usually follow the EV2 distribution with $\xi > 0$ (Withers and Nadarajah 2000a), while temperature extremes usually follow the EV3 distribution with $\xi < 0$ (Withers and Nadarajah 2000b). However—as we shall see in the next section—wind maxima can follow both types of distribution.

4. Results and discussion

4.1 Fitting

Fitting the model, (1), means obtaining estimates of the unknown vectors α , β and the unknown shape parameter ξ . We used the maximum likelihood estimates—see Appendix A for details—denoted $(\hat{\alpha}, \hat{\beta}, \hat{\xi})$.

For each of the twenty data sets we fitted the following models by the method of maximum likelihood: L(0) with $\xi = 0$ —denoted below by ‘Gumbel’, L(0), L(1), L(2), E(1) and E(2). We then applied the likelihood ratio test, between successive models to yield the so-called p-value—see again Appendix A for details. Based on these p-values, we discriminated the models in the following manner to determine the one giving the best fit: if the p-value for the Gumbel versus L(0) is greater than 0.05, then conclude that Gumbel provides the best fit; if this is not the case and the p-value for L(0) versus L(1) is greater than 0.05, then conclude that L(0) provides the best fit; if this is not the case and the

p-value for L(1) versus L(2) is greater than 0.05, then conclude that L(1) provides the best fit which implies that there is a significant linear trend; and, if this is not the case and the p-value for L(2) versus L(3) is greater than 0.05, then conclude that L(2) provides the best fit which implies that there is a significant quadratic trend.

Higher order polynomials are often better at describing a data set, but their projections into the future tend to fluctuate too wildly—and in the case of variability, to either shrink too quickly or expand too quickly. This was often found to be the case with fitting the cubic models, L(3) and E(3), to the data, thus those results were discarded.

The main results of the fits are shown in Figs. 2–9. The figures go from North to South, with the best models on the right. In each set of four plots, the top two refer to the first two models listed and the bottom two to the last two models listed. For those data sets, where L(0) or the Gumbel is the best model, no figure is given. For those data sets, where L(1) or E(1) is the best model, we plot the return levels for Y_N for the four models: Gumbel, L(0), L(1) and E(1). For those data sets, where L(2) or E(2) is the best model, we plot the return levels for the four models: L(1), L(2), E(1) and E(2). The figure captions also include the p-value of the likelihood ratio test (see Appendix A) of the models on the left versus those on the right. In each figure, the 2 year (median), 10 year, 100 year and 1000 year return levels of daily wind-run are plotted, i.e.

$$\mu_N(\hat{\alpha}) + \sigma_N(\hat{\beta})F_{\hat{\xi}}^{-1}(q)$$

for $q = 0.001, 0.01, 0.1, 0.5, 0.9, 0.99, 0.999$, where $(\hat{\alpha}, \hat{\beta}, \hat{\xi})$ is the maximum likelihood estimate. In addition, we plot the upper bound of maximum daily windrun,

$$\mu_N(\hat{\alpha}) - \sigma_N(\hat{\beta})/\hat{\xi},$$

whenever $\hat{\xi} < 0$. The median ($q = 0.5$) and the upper bound are the solid lines. The other return levels are drawn as the dotted and dashed lines. Note that $F_{\hat{\xi}}^{-1}(\exp[-1]) = 0$ so that $\mu_N(\alpha)$ is the $\exp(-1) \approx 37\%$ quantile.

4.2 Return levels

Traditionally return levels were calculated on Gumbel paper, that is using the Gumbel

Fig. 2. Annual maximum windrun, in km, for Woodhill Forest. Fitted models: Gumbel (top left), $L(1)$ (top right), $L(0)$ (bottom left), $E(1)$ (bottom right). Statistical significance: for $L(0)$ versus $L(1)$ p -value = 0.00003; for $L(0)$ versus $E(1)$ p -value = 0.0003. Line types: ——— for 2-year windrun and the upper bound; for 10-year windrun; - - - - for 100-year windrun; - . - . for 1000-year windrun.

model. But this model gave the best fit only for some of the twenty stations. For Leigh and Hamilton at Ruakura, we found that the model, $L(0)$, gave the best fit, but with $\xi \neq 0$. The corresponding return levels for these two stations were significantly higher than what would have been obtained under the Gumbel model. This shows that the use of Gumbel paper (traditional method) can seriously underestimate return levels, as shown by Pearson and Henderson (1998) and Withers and Nadarajah (2000a).

With regard to change of return levels with respect to time, there was evidence of significant trend for eight of the twenty locations considered. The station names are: Woodhill Forest, Whatawhata, Rotoehu Forest, Palmerston

North, Pahiatua at Mangamutu, Wellington at Kelburn, Winchmore and Winton. The predominant pattern of change was a decrease in mean and an increase in variability. For Woodhill Forest, Whatawhata, Pahiatua at Mangamutu, Wellington at Kelburn, Winchmore and Winton the linear models, $L(1)$ and $E(1)$, gave the best fits—see Figs. 2, 3, 6, 7, 8, and 9. For Rotoehu Forest and Palmerston North the quadratic models, $L(2)$ and $E(2)$, gave the best fits—see Figs. 4 and 5. The negative y -axis values in some of the figures show the weakness of (1) in extrapolating too far into the future. These trend patterns must be treated with caution, because of possible effects of change in site, site exposure and observational practices. Un-

Fig. 3. Annual maximum windrun, in km, for Whatawhata. Fitted models: Gumbel (top left), L(1) (top right), L(0) (bottom left), E(1) (bottom right). Statistical significance: for L(0) versus L(1) p -value = 0.0002; for L(0) versus E(1) p -value = 0.0003. Line types as in Fig. 2.

fortunately, information on these changes is very limited as discussed in Section 2. In fact, we have this information available only for one of the eight stations—Wellington at Kelburn—for which site exposure changed in 1948, 1949 and 1959 and instruments/records changed in 1951, 1959, 1960, 1961, 1966, 1971, 1973, 1976, 1977, 1979 and 1983. But in Fig. 7 the general pattern of the data, or the downward trend suggested by the model, does not appear to have been affected by these changes. Thus, we may conclude that the trend for Wellington at Kelburn is genuinely due to climatic effects. This does not mean that the trend patterns for the other seven stations can also be genuine: Whatawhata and Winton appear to show enormous downward and upward trends, respectively, that are unlikely causes of the climate.

To give a simple quantitative impression of the trend patterns we estimated

- the slope, $100\alpha_1/(n+1)$, representing the magnitude of change in mean annual maximum windrun per century;
- and, the percentage of change in the median of annual maximum windrun over the period of records

for the linear model, L(1), for all eight locations. The estimates and associated standard errors are given in Table 2. There are six negative trends and two positive trends.

Leaving the eight locations aside, we have twelve locations for which there is no significant evidence of trend. Hamilton at Ruakura and Lincoln are the only two locations amongst these twelve for which we have site and instru-

Fig. 4. Annual maximum windrun, in km, for Rotoehu Forest. Fitted models: L(1) (top left), L(2) (top right), E(1) (bottom left), E(2) (bottom right). Statistical significance: for L(1) versus L(2) p -value = 0.0000002; for E(1) versus E(2) p -value = 0.0000002. Line types as in Fig. 2.

ment related information (see Section 2). For Hamilton at Ruakura changes to instruments, and site exposure appear to have occurred in the 1980s (1980, 1983, 1987, and 1989). However, on inspecting the dataset, which has only a few data points in the 1980s, we see little sign that the changes had significant impact on the conclusion of no trend. For Lincoln there were site movements as well as changes to site exposure and instruments. The site movements in 1943, 1962 and 1975 appear to have caused marked and sudden increases in the wind extremes. But our model is not versatile enough to accommodate these effects—as mentioned in Section 3. If these sudden changes had been corrected in some way then it is possible that we might have observed a significant trend for Lincoln.

Figure 10 plots estimates of the shape parameter, ζ (denoted ξ), and the 10, 100 and 1000 year return levels for daily windrun against latitude (in degrees) for the model, L(0), for the nineteen New Zealand stations, running from South on the left to North on the right. The plots also show 95% confidence intervals for the return levels. From Wellington to Auckland going northwards the return levels appear to be decreasing, then they appear to increase. The peaks are around -42 and -36 degrees latitude. This is consistent with De Lisle (1965)'s analysis of maximum wind gusts, where he finds that the 20-year return levels peak for the Cook Strait area, together with the north of Northland (It is well-known that wind gusts in the Cook Strait are the highest in New Zealand.).

Fig. 5. Annual maximum windrun, in km, for Palmerston North. Fitted models: L(1) (top left), L(2) (top right), E(1) (bottom left), E(2) (bottom right). Statistical significance: for L(1) versus L(2) p -value = 0.0000001; for E(1) versus E(2) p -value = 0.0000000. Line types as in Fig. 2.

A further point is that twelve out of the nineteen estimates for the shape parameter, ζ , are below the line $\zeta = 0$. When tested for significance of these estimates, we found that only five out of these twelve are significantly less than zero. Thus, windrun maxima must have a statistical upper bound for these five stations. The station names, as well as estimates of the corresponding upper bound, with their standard errors are given in Table 3. As in Fig. 1 except for Winton, all of them are located in the North Island. The geographical distinction between the North and South Islands could explain this.

5. Conclusions

We have investigated nineteen locations in New Zealand, and one in the Cook Islands for

evidence of trend in wind extremes. For Winton, Wellington at Kelburn, Rotoehu Forest, Whatawhata, Winchmore and Woodhill Forest, we have shown that wind extremes have decreased by 16%, 21%, 26%, 27%, 28% and 41%, respectively. For Palmerston North and Pahiatua at Mangamutu, we have shown that wind extremes have increased by 20% and 24%, respectively. For the remaining locations, we have found no evidence to suggest any change.

The observed changes or the lack of them could have been the result of factors such as site change, site exposure and instrument change. But we have been unable to investigate this in full because of lack of information. Nevertheless, for Wellington at Kelburn, it appears that the observed downward trend may be the result of climate change, while, for Lincoln, it

Fig. 6. Annual maximum windrun, in km, for Pahiatua at Mangamutu. Fitted models: Gumbel (top left), L(1) (top right), L(0) (bottom left), E(1) (bottom right). Statistical significance: for L(0) versus L(1) p -value = 0.03; for L(0) versus E(1) p -value = 0.03. Line types as in Fig. 2.

Table 2. Slopes and percentage changes in maximum windrun for the model L(1).

Station Name	Slope (km/century) and Standard Error	Percentage Change in Median of Annual Maximum Windrun (over period of records)
Woodhill Forest	−904 (162)	−41
Whatawhata	−694 (147)	−27
Rotoehu Forest	−400 (136)	−26
Palmerston North	+327 (231)	+20
Pahiatua at Mangamutu	+306 (108)	+24
Wellington at Kelburn	−320 (53)	−21
Winchmore	−543 (99)	−28
Winton	−692 (190)	−16

Fig. 7. Annual maximum windrun, in km, for Wellington at Kelburn. Fitted models: Gumbel (top left), L(1) (top right), L(0) (bottom left), E(1) (bottom right). Statistical significance: for L(0) versus L(1) p-value = 0.0000006; for L(0) versus E(1) p-value = 0.0000007. Line types as in Fig. 2.

appears that the lack of trends may have been influenced by site movements.

We have produced evidence to support earlier findings that the geographical distribution of extreme wind has peaks for the Cook Strait area, together with the north of Northland.

We have established that maximum daily windrun for Leigh, Woodhill Forest, Palmerston North, Wellington at Kelburn and Winton may not exceed 1388 km, 969 km, 1001 km, 1522 km and 1131 km, respectively.

Appendix A

Here we comment on maximum likelihood estimation and likelihood ratio tests.

Let $\theta = (\alpha, \beta, \zeta)$. The likelihood of θ , denoted $L_n(\theta)$, is defined as the joint density of $(Y_1, \dots, Y_n)$ evaluated at the realized observations, i.e.

$$L_n(\theta) = f(y_1, \dots, y_n; \theta).$$

Thus, using equations (1) and (2), we can write

Table 3. Upper bounds for maximum daily windrun for the model L(0).

Station Name	Upper Bound (km) and Standard Error
Leigh	1388 (76)
Woodhill Forest	969 (50)
Palmerston North	1001 (35)
Wellington at Kelburn	1522 (236)
Winton	1131 (116)

Fig. 8. Annual maximum windrun, in km, for Winchmore. Fitted models: Gumbel (top left), L(1) (top right), L(0) (bottom left), E(1) (bottom right). Statistical significance: for L(0) versus L(1) p-value = 0.000008; for L(0) versus E(1) p-value = 0.00001. Line types as in Fig. 2.

$$L_n(\theta) = \exp \left\{ - \sum_{N=1}^n \left(1 + \xi \frac{y_N - \mu_N(\alpha)}{\sigma_N(\beta)} \right)^{-1/\xi} \right\} \\ \times \prod_{N=1}^n \frac{1}{\sigma_N(\beta)} \left(1 + \xi \frac{y_N - \mu_N(\alpha)}{\sigma_N(\beta)} \right)^{-(1/\xi+1)}.$$

The principle of maximum likelihood estimation is to estimate θ by the value that maximizes $L_n(\theta)$. There are many numerical algorithms that maximize functions of several variables: we used the E04JAF routine from the NAG Fortran library to perform the maximization.

Now to the likelihood ratio test. Suppose we have $\theta = (\theta_1, \theta_2)$, where θ_2 is a vector of length k , and that we wish to test the hypothesis that $\theta_2 = \mathbf{0}$. Let

$$l_n = \sup_{\theta_1} L_n(\theta_1, \mathbf{0}) / \sup_{\theta} L_n(\theta_1, \theta_2).$$

The likelihood ratio test of level approximately $1 - \alpha$ (usually, 0.95) is to accept the hypothesis if $-2 \log l_n \leq \chi_k^2(1 - \alpha)$, where $\chi_k^2(1 - \alpha)$ is the $1 - \alpha$ quantile of the χ_k^2 distribution. For testing $L(m)$ given $L(m+1)$, $k = 2$ so that l_n is approximately uniformly distributed on $(0, 1)$, and hence the p-value, a measure of the level of significance of the test, is identical to l_n . The test accepts the hypothesis that $L(m+1)$ is not a significant improvement on $L(m)$ if the p-value is greater than α .

For further details on maximum likelihood estimation, and likelihood ratio tests see Wald (1943) and Cox and Hinkley (1974).

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Fig. 9. Annual maximum windrun, in km, for Winton. Fitted models: Gumbel (top left), L(1) (top right), L(0) (bottom left), E(1) (bottom right). Statistical significance: for L(0) versus L(1) p -value = 0.0002; for L(0) versus E(1) p -value = 0.0002. Line types as in Fig. 2.

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Fig. 10. Estimates of the shape parameter, ζ , and the return levels (10, 100 and 1000 year) versus latitude for the model $L(0)$.

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