Development of a Three-Dimensional Spectral Linear Baroclinic Model and its Application to the Baroclinic Instability Associated with Positive and Negative Arctic Oscillation Indices

Hiroshi L. TANAKA

Center for Computational Sciences, University of Tsukuba, Tsukuba, Japan

and

Sawaka SEKI

Life and Environmental Sciences, University of Tsukuba, Tsukuba, Japan

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Abstract

In this study, a linear baroclinic model (LBM) is developed from a three-dimensional (3D) spectral primitive equation model. With this LBM, we investigate the linear stability problem for various zonally varying basic states on a sphere. For a zonal climate basic state, we confirm that the traditional Charney and dipole Charney modes appear as the most dominant unstable modes in the synoptic to planetary scales.

For a zonally varying basic state, we find that these unstable modes are modified by the regionality of the local baroclinicity of the basic state. Given the zonally varying barotropic basic state, we find that the barotropically most unstable standing mode appears to be the Arctic Oscillation (AO) mode. In this study, the eigensolution of the LBM is regarded as a generalized extension at the 3D normal mode at the motionless atmosphere to those of an arbitrary climate basic state.

As an application of the LBM, various zonally varying basic states associated with the positive and negative AO indices are substituted into the LBM to find the response of the baroclinic eddies. According to the result, the positive feedback dominates in the Atlantic sector for positive AO index because of the presence of enhanced double-jet structure.

When the AO index is negative, the eddy momentum flux converges in the mid-latitudes to shift the subtropical jet poleward in the Atlantic and Pacific sectors because of the intensified baroclinic instability. The positive feedback operates in a different way in the Atlantic and Pacific sectors depending on their double or single westerly jets. It is concluded that the baroclinically unstable modes are modified by the positive/negative AO index, so that the induced local eddy momentum flux shows a positive feedback to the AO.

Keywords arctic oscillation; linear baroclinic model; baroclinic instability; barotropic instability; normal mode

1. Introduction

A three-dimensional (3D) spectral primitive equation model was constructed by Kasahara (1977) by using vertical structure functions in the vertical and Hough harmonics on the sphere. The solutions to the primitive equations linearized for a basic state at rest are referred to as 3D normal modes, which can construct an orthonormal basis for the nonlinear 3D spectral primitive equation model. In such a 3D spectral model, all linear terms such as pressure
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standing unstable modes shows a similar structure to the linear dynamical system, they found that the barotropic spectral model linearized about a nonzonal basic state in Tanaka and Kung (1989).

The fastest growing mode at the synoptic scale is identified as Charney mode (Charney 1947), and another type of Charney mode with a meridional dipole structure is found to dominate at a planetary scale.

The analysis of the baroclinic instability for a zonally varying basic state was pursued by Frederiksen (1982) in the quasi-geostrophic framework.

He showed that the zonal asymmetry of the basic state reorganizes the synoptic-scale baroclinic waves to yield the Pacific and Atlantic storm tracks. He also found a blocking-like unstable mode with a meridional dipole structure among a number of unstable solutions. These unstable modes indicate a life cycle with changing 3D structure combined with the exponential growth as an unstable solution. In addition, he found a unique standing unstable mode with a geographically fixed 3D structure. However, the physical interpretation of the dipole modes and standing mode was less clear compared with that of synoptic-scale unstable Charney modes.

The standing unstable mode was further investigated by Tanaka and Matsueda (2005) by using a barotropic spectral model linearized about a nonzonal climate basic state.

With an inclusion of the scale-dependent diffusion to the linear dynamical system, they found that the standing unstable mode shows a similar structure to the Arctic Oscillation (AO) proposed by Thompson and Wallace (1998). It is a barotropically unstable standing mode, but the growth rate becomes about zero by introducing a surface friction in the form of Rayleigh friction. Because the eigensolution has a zero eigenvalue with zero frequency and growth rate, the governing matrix of the linear system becomes singular, and the AO mode appears to be resonant to the arbitrary quasi-steady forcing.

The AO is an atmospheric annular pattern dominating in the Northern Hemisphere, especially during winter. The AO is defined as a leading empirical orthogonal function (EOF) mode of the wintertime sea-level pressure (SLP). The score time-series is referred to as the AO index (AOI), which indicates the strength of the AO. When the AOI is positive, a low-pressure anomaly appears in the polar region north of 60°N and a high-pressure belt appears in the mid-latitudes surrounding the low-pressure anomaly. Thus, the polar jet near 60°N is intensified for the positive AOI having a barotropic structure.

One of the factors that intensifies the AO is an interaction with the baroclinic eddies by the zonal wave interactions. Limpasuvan and Hartmann (2000) and Lorenz and Hartmann (2003) showed that the zonal-wave interactions intensify the zonal wind, which is a relevant characteristic of the AO. Tanaka and Tokinaga (2002) conducted a linear instability analysis for the atmospheric basic state with the extremely positive and negative AOI. They found that the basic state with the positive AOI tends to excite the baroclinic modes, which transport more eddy momentum polerward to intensify the polar jet. Such a baroclinic instability appearing in planetary scale was called as polar mode M₄.

On the other hand, the ordinary baroclinic instability appearing in the synoptic scale is known as a Charney mode (M₃) showing a single amplitude maximum in mid-latitude. The Charney mode with a meridional dipole structure in geopotential is called as a dipole Charney mode (M₄). The positive feedback between the AO and baroclinically unstable modes is further investigated by Seki et al. (2011) for the wide range of the AOI. As the AOI becomes large positive, the structure of M₄ is modified to that of M₃, and the ridge axes of the M₄ change to tilt from northeast to southwest. The eddy momentum is then transported more poleward than the ordinary situation to intensify the polar jet. These results explain that the baroclinically unstable modes interact with the AO through positive feedback by modifying the features of eddy momentum transport.

The previous studies by Tanaka and Tokinaga (2002) and Seki et al. (2011) adopted the zonally symmetric basic state for the linear stability analysis because of the limitation of the computational resources. Thus, the experiment does not include the wave-wave interactions, and the atmospheric variable does not feel the local effects of the large mountain ranges or large-scale land-sea distribution. Eichelberger and Hartmann (2007) show that the leading mode of
variability describes the latitudinal shifting of the eddy-driven jet when the jets are well separated. Chang and Fu (2002; 2003) find that the AO is well correlated with the storm tracks, especially in the Atlantic sector, where the westerly tends to acquire the double-jet shape.

Although there are many studies on the variability in the zonal-eddy interactions associated with the locality of the atmospheric state, the spatial relationship between the AO and baroclinic instability has not been studied theoretically for the nonzonal basic state.

The purpose of this study is to develop a general linear baroclinic model (LBM) from the 3D spectral primitive equation model. The nonlinear equations are linearized about various nonzonal basic states such as positive and negative AOI. The local change in baroclinic-barotropic instability is theoretically investigated for various phases of the positive and negative AOI. The LBM developed in this study enables us to conduct a linear stability analysis with the zonally varying basic state, so that we can investigate the regionality of the positive feedback and spatial structure of the baroclinically unstable modes.

2. Methodology and data

2.1 Governing equations

The LBM developed in this study is based on a spectral primitive equation expanded by the 3D normal mode functions (Tanaka and Kung 1989). A system of primitive equations in a physical space includes the horizontal equations of motions, hydrostatic equation, thermodynamic energy equation, equation of continuity, and equation of state. Next, these six equations are reduced to three prognostic equations of horizontal motions and thermodynamics with three dependent variables \(u, v, \phi'\). Here \(u\) and \(v\) are the zonal and meridional components of the horizontal velocity \(V\).

The variable \(\phi'\) is a deviation of the local isobaric geopotential from the reference state geopotential \(\phi_0\) that is related to the reference state temperature \(T_0\) by the hydrostatic relation. By using a matrix notation, these primitive equations in a spherical coordinate of longitude \(\lambda\), latitude \(\theta\), pressure \(p\), and time \(t\) are written as follows:

\[
\mathbf{M} \frac{\partial \mathbf{U}}{\partial t} + \mathbf{L} \mathbf{U} = \mathbf{N} + \mathbf{F},
\]

where

\(\mathbf{U}\): a dependent variable vector

\[
\mathbf{U} = \begin{bmatrix} u \\ v \\ \phi' \end{bmatrix}
\]

\(\mathbf{M}\): a linear matrix operator with a vertical differentiation

\[
\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\frac{\partial}{\partial p} \frac{p^2}{\gamma R} \frac{\partial}{\partial p} \end{bmatrix}
\]

\(\mathbf{L}\): a linear matrix operator with a horizontal differentiation

\[
\mathbf{L} = \begin{bmatrix} 0 & -2\Omega \sin \theta & \frac{1}{\cos \theta} \frac{\partial}{\partial \lambda} \\ 2\Omega \sin \theta & 0 & \frac{1}{a} \frac{\partial}{\partial \theta} \\ \frac{1}{\cos \theta} \frac{\partial}{\partial \lambda} & \frac{1}{\cos \theta} \frac{\partial}{\partial \theta} & 0 \end{bmatrix}
\]

\(\mathbf{N}\): a nonlinear term vector

\[
\mathbf{N} = \begin{bmatrix} -\mathbf{v} \cdot \nabla u - \omega \frac{\partial u}{\partial p} + \frac{\tan \theta}{a} \mathbf{uv} \\ -\mathbf{v} \cdot \nabla v - \omega \frac{\partial v}{\partial p} - \frac{\tan \theta}{a} \mathbf{uv} \\ \frac{\partial}{\partial p} \left( \frac{p^2}{\gamma R} \mathbf{v} \cdot \nabla \phi' + \frac{\omega p}{\gamma} \frac{\partial}{\partial p} \left( \frac{\partial \phi'}{\partial p} \right) \right) \end{bmatrix}
\]

\(\mathbf{F}\): a diabatic term vector

\[
\mathbf{F} = \begin{bmatrix} F_v \\ \frac{\partial}{\partial p} (pQ) \\ \frac{\partial}{\partial p} (c_p \gamma) \end{bmatrix}
\]

The nonlinear term vector includes advection and curvature terms. The diabatic term vector \(\mathbf{F}\) includes the zonal \(F_v\) and meridional \(F_r\) components of frictional forces and a diabatic heating rate \(Q\). The symbol \(a\) denotes the earth’s radius, \(\Omega\) is the angular speed of the earth’s rotation, \(R\) is the specific gas constant, \(c_p\) is the specific heat at a constant pressure, \(\omega\) is the vertical \(p\)-velocity, and \(\nabla\) is the horizontal del-operator.

The static stability parameter \(\gamma\) is given by

\[
\gamma(p) \equiv \frac{RT_0(p)}{c_p} - p \frac{dT_0(p)}{dp},
\]

where \(T_0(p)\) is a function of \(p\) alone. Refer to Table 1.
for the symbols and variables used in the model equations in this study.

Next, (1) is transformed into the wavenumber space using an expansion in 3D normal mode functions in a resting atmosphere $\Pi_{nlm}(\lambda, \theta, p)$. Here the suffixes represent the following: $n$ is the zonal wavenumber, $l$ is the meridional wavenumber, and $m$ is the vertical wavenumber. The 3D normal mode functions are orthonormal because they are constructed by Hough harmonics (horizontal normal modes), $\Theta_{nl} \exp (i\eta \lambda)$, where $i$ is an imaginary unit, and vertical structure functions (vertical normal modes), $G_m(p)$:

$$\Pi_{nlm}(\lambda, \theta, p) = G_m(p) \Theta_{nl} \exp (i\eta \lambda).$$  

(8)

Taking the advantage of the Hough functions, our 3D spectral model is constructed using only the Rossby modes without the fast-moving gravity modes. The dependent variables $U$ and external forcing $F$ can be expanded into the wavenumber space with (8):

$$U(\lambda, \theta, p, r) = \sum_{n=\infty}^{N} \sum_{l=0}^{L} \sum_{m=0}^{M} w_{nlm}(r) X_n \Pi_{nlm}(\lambda, \theta, p),$$  

(9)

$$F(\lambda, \theta, p, r) = \sum_{n=\infty}^{N} \sum_{l=0}^{L} \sum_{m=0}^{M} f_{nlm}(r) Y_n \Pi_{nlm}(\lambda, \theta, p).$$  

(10)

The wavenumbers $n$, $l$, and $m$ are truncated at $N$, $L$, and $M$, respectively. The scaling matrices $X_n$ and $Y_n$ include a phase speed of gravity waves in shallow water ($\sqrt{gh_n}$) with the equivalent height $h_n$:

$$X_n = \begin{bmatrix} \sqrt{gh_n} & 0 & 0 \\ 0 & \sqrt{gh_n} & 0 \\ 0 & 0 & gh_n \end{bmatrix}$$  

(11)

$$Y_n = \begin{bmatrix} 2\Omega^2 \frac{gh_n}{\Delta} & 0 & 0 \\ 0 & 2\Omega^2 \frac{gh_n}{\Delta} & 0 \\ 0 & 0 & 2\Omega \end{bmatrix}$$  

(13)

The expansion coefficients $w_{nlm}$ and $f_{nlm}$ are the functions of time alone.

The orthonormality of the 3D normal mode functions are given by an inner product $\langle , \rangle$ defined by the mass integral over the whole atmospheric domain:

$$\langle \Pi_{nlm}, \Pi_{n'l'm'} \rangle = \frac{1}{2\pi \rho} \int_0^{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Pi_{nlm} \Pi_{n'l'm'} \cos \theta \lambda \phi d\phi dp = \delta_{nl'} \delta_{l'm'},$$  

(14)

where $^*$ is a complex conjugate, and $\delta_{ij}$ is a Kronecker delta.

To transform the primitive equations from the physical space into wavenumber space, the inner product (14) is applied for the primitive equations (1) and 3D normal mode functions (8). Thus,

$$\left\langle M \frac{dU}{dt} + LU - N - F, Y_n^* \Pi_{nlm} \right\rangle = 0.$$  

(15)

By substituting (9) and (10) into (15), applying the nondimensional time $\tau (\equiv 2\Omega t)$, and reconstructing each term using the property of the Hough harmonics (Kasahara 1976), we finally get a 3D spectral primitive equation as follows.

$$\frac{dW_i}{d\tau} + i\sigma W_i = -i \sum_{j=1}^{K} \sum_{k=1}^{K} r_{ijk} W_j W_k + f_i \quad (i = 1, 2, 3, \cdots, K).$$  

(16)

The first term on the left-hand side of (16) represents the time differentiation of the expansion coefficient. The second term on the left is a linear normal mode for the resting atmosphere represented by the imaginary unit $i$ and eigenfrequencies of Laplace’s tidal equation $\sigma$. On the right-hand side, the first term represents the nonlinear mode-mode interactions, which include double summations of quadratic terms of the state variables weighted by the interaction coefficients $r_{ijk}$. The second term represents the external forcing including the diabatic process.

The suffixes $i$, $j$, $k$ are abbreviations for combinations of 3D wavenumbers $nlm$, $n'l'm'$, respectively. $K$ is a dimension of the system (16) and equals to $K = (2N + 1)(L + 1)(M + 1)$. The real interaction coefficients $r_{ijk}$ are evaluated by the triple product of the 3D normal mode functions.

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### Table 1. Symbols, definitions, and variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>time (s)</td>
</tr>
<tr>
<td>$u$</td>
<td>zonal wind (m s$^{-1}$)</td>
</tr>
<tr>
<td>$v$</td>
<td>meridional wind (m s$^{-1}$)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>geopotential (m$^2$ s$^{-2}$)</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature (K)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>vertical $p$-velocity (Pa s$^{-1}$)</td>
</tr>
<tr>
<td>$F_n$</td>
<td>zonal component of frictional force</td>
</tr>
<tr>
<td>$F_v$</td>
<td>meridional component of frictional force</td>
</tr>
<tr>
<td>$Q$</td>
<td>diabatic heating rate</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>angular speed of earth’s rotation</td>
</tr>
<tr>
<td>$a$</td>
<td>radius of the earth</td>
</tr>
<tr>
<td>$c_p$</td>
<td>specific heat at constant pressure</td>
</tr>
<tr>
<td>$R$</td>
<td>specific gas constant of dry air</td>
</tr>
<tr>
<td>$h_m$</td>
<td>imaginary unit</td>
</tr>
<tr>
<td>$\rho$</td>
<td>radius of the earth</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>specific gas constant of dry air</td>
</tr>
</tbody>
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The orthonormality of the 3D normal mode functions are given by an inner product $\langle , \rangle$ defined by the mass integral over the whole atmospheric domain:

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The first term on the left-hand side of (16) represents the time differentiation of the expansion coefficient. The second term on the left is a linear normal mode for the resting atmosphere represented by the imaginary unit $i$ and eigenfrequencies of Laplace’s tidal equation $\sigma$. On the right-hand side, the first term represents the nonlinear mode-mode interactions, which include double summations of quadratic terms of the state variables weighted by the interaction coefficients $r_{ijk}$. The second term represents the external forcing including the diabatic process.

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coefficients of the LBM. In equation of the LBM. In April 2013 H. L. TANAKA and S. SEKI

\[ r_{jk} = \frac{1}{2\pi p f} \int_0^\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[ \begin{array}{c} U_i \\ V_i \\ Z_i \end{array} \right]^{\tau} P \begin{bmatrix} (n_i U_k + \tan \theta V_k) \cos \theta \\ (n_i V_k + \tan \theta U_k) \cos \theta \\ n_i Z_k \cos \theta \end{bmatrix} \begin{bmatrix} -P_i \frac{dU_k}{d\theta} + P_j U_k \\ -P_i \frac{dV_k}{d\theta} + P_j V_k \\ -P_i \frac{dZ_k}{d\theta} + P_j Z_k \end{bmatrix} \begin{bmatrix} U_j \\ V_j \\ \sigma Z_j \end{bmatrix} \cos \theta d\lambda d\theta dp. \]

(17)

Here \( P_i \) to \( P_j \) are the functions of \( p \) evaluated by the triple products of the vertical structure functions (Tanaka and Terasaki 2005). When the analytical vertical structure functions are used, an analytical vertical integral is possible for these functions. In this study, we applied the spectral expansion method by Kasahara and Puri (1981) for the computation of the vertical normal modes. The important feature of \( r_{jk} \) is that the interaction coefficient can only exist under the triad condition satisfying \( n_i = n_j + n_k \) for the zonal wavenumbers. We adopt the following boundary conditions: 1) There are no kinematic winds at the lower boundary of the atmosphere. 2) The atmospheric total energy is finite under the given upper boundary.

2.2 Development of the linear baroclinic model

Next, we linearize (16) by introducing a perturbation method to develop the linear baroclinic model (LBM). The independent variables \( w \) and external forcing \( f \) are divided into the time-independent basic state \((\overline{w}, \overline{f})\) and small perturbations superimposed on the basic state \((\overline{w}, \overline{f})\), the same symbols as the original variables are used for convenience), and substituted into (16). After disregarding the nonlinear perturbation terms, we obtain a linear spectral primitive equation model.

\[ \frac{dw_i}{d\tau} + i\sigma w_i = -i \frac{\xi}{\sum_{j=1}^K (r_{jk} + r_{kj})} \sum_{j=1}^{K} (r_{jk} + r_{kj}) \overline{w_j} + f_i, \]

(18)

where the modal index \( k \) is used for the basic state, and \( i \) and \( j \) for the perturbations that interact with the basic state. The linear equation (18) is the governing equation of the LBM. In (17), the interaction coefficients \( r_{jk} \) need to satisfy the triad combination for the zonal wavenumber \( n_i = n_j + n_k \). Thus, if the basic state \( \overline{w} \) is zonally symmetric (\( n_k = 0 \)), \( r_{jk} \) is required to satisfy \( n_i = n_k \).

Because of the limitation of calculation resources, Tanaka and Tokinaga (2002) and Seki et al. (2011) employed the zonally symmetric basic state and conducted the linear stability analysis in such a highly idealized case. However, needless to say, the real atmosphere is zonally asymmetric, associated with the regionality of the westerly or the effect of land-sea distribution for the atmosphere. In this study, the LBM allows \( n_k \) to have all zonal wavenumbers within its truncation, so that we can conduct a linear stability analysis with a zonally asymmetric basic state.

Because the negative zonal wavenumbers represent the complex conjugates of the positive zonal wavenumbers, we can rearrange in the matrix form for \( n_i \geq 0 \) and \( K_1 = (N + 1) (L + 1) (M + 1) \):

\[ \frac{d\mathbf{w}}{d\tau} + i\mathbf{D}\mathbf{w} = -i\mathbf{B}\mathbf{w} - i\mathbf{C}\mathbf{w}^* + \mathbf{f}, \]

(19)

where

\[ \mathbf{w} = (w_1, \cdots, w_N)^T, \quad \text{for } n_i \geq 0 \]

\[ \mathbf{f} = (f_1, \cdots, f_N)^T, \quad \text{for } n_i \geq 0 \]

\[ \mathbf{D} = \text{diag}(\sigma_1, \cdots, \sigma_N), \quad \text{for } n_i \geq 0 \]

\[ \mathbf{B} = \sum_{k=1}^K (r_{jk} + r_{kj}) \overline{w_k}, \quad \text{for } n_j \geq 0 \]

\[ \mathbf{C} = \sum_{k=1}^K (r_{jk} + r_{kj}) \overline{w_k}, \quad \text{for } n_j < 0. \]

2.3 Eigenvalue problems

To perform the linear stability analysis for the zonally asymmetric basic state, a complex wave-type solution is substituted into (19). Because an inviscid and adiabatic eddy will be examined, we disregard \( \mathbf{f} \) for perturbations.

The solution may be assumed in the form:

\[ \mathbf{w}(\tau) = \xi \exp(\nu\tau), \]

(25)

where \( \mathbf{w}_R \) is the real part and \( \mathbf{w}_I \) is the complex part of \( \mathbf{w} \). \( \xi \) and \( \xi \) are the structure vectors of the solution and \( \nu \) is the eigenvalue of the frequency.

Because both matrices \( \mathbf{B} \) and \( \mathbf{C} \) in (19) become complex full matrices, they are also split into real and imaginary parts. Then, the eigenvalue problem becomes

\[ \nu \left[ \begin{array}{c} \xi \\ \xi \end{array} \right] = \left[ \begin{array}{cc} \mathbf{B}_R + \mathbf{C}_I & \mathbf{B}_R - \mathbf{C}_R + \mathbf{D} \\ -\mathbf{B}_R - \mathbf{C}_R - \mathbf{D} & \mathbf{B}_I - \mathbf{C}_I \end{array} \right] \left[ \begin{array}{c} \xi \\ \xi \end{array} \right]. \]

(26)

The complex matrices \( \mathbf{B} \) and \( \mathbf{C} \) are determined by the
basic state \( \bar{w}_b \), and \( D \) is a constant matrix containing the Laplace’s tidal frequency. Therefore, the eigenvalues (\( \nu \)) and eigenvectors (\( \xi, \zeta \)) may be solved using a standard real-valued matrix solver. The real part of the eigensolution is expressed by

\[
\begin{bmatrix} \bar{w}_b \\ \bar{w}_l \end{bmatrix}(t) = 2\exp(\nu_b t) \begin{bmatrix} \xi_b \\ \xi_l \end{bmatrix} \cos(\nu_l t) - \begin{bmatrix} \xi_l \\ \xi_l \end{bmatrix} \sin(\nu_l t). \tag{27}
\]

The real part of the eigenvalue \( \nu_b \) represents the growth rate, and the complex part \( \nu_l \) represents the frequency. An eigenmode with a positive growth rate is called an unstable mode.

In this study, an eigenmode with the largest growth rate \( \nu_b \) is regarded as the fastest growing baroclinic instability mode. The eigenvalue problem (26) was first solved by Tanaka and Kung (1989) only for the system of zonal wavenumber 1. The present study has extended the system to full interactions with all zonal wavenumbers.

As is evident from the solution, the eigenmode has a life cycle in its structure with the period determined by \( \nu_l \). As a special case when the eigenvalue is real, the structure is geographically trapped with no life-cycle except for the positive and negative sign of the eigenvector. The mode is called a standing mode, which appears to be most interesting for the study of the Arctic Oscillation as discussed in this study.

### 2.4 Data

The data used in this study are from the National Centers for Environmental Prediction (NCEP) and National Center for Atmospheric Research (NCAR) reanalysis of four-times-daily zonal wind, meridional wind, and geopotential height on constant pressure levels since 1950 (Kalnay et al. 1996). For the 3D spectral model, the wavenumbers are truncated at \( N = 20, L = 20, M = 6 \), with a boundary condition of an equatorial wall, solving for the Northern Hemisphere.

The vertical structure functions are calculated for 24 vertical layers by the mean of the spectral method (Kasahara and Puri 1981), and only the lower order vertical modes up to \( M \) are used in this study because the higher order modes have a problem of aliasing.

Figure 1a illustrates the 30-year mean zonal-mean zonal wind for December to February (DJF) during 1971 to 2000, which will be used as a basic state for the linear stability analysis.

The structure is reconstructed by the truncated basic state of \( \bar{w}_b \).

For the Arctic oscillation index (AOI), we used the NCEP/NCAR reanalysis since 1950, computed with the barotropic component of the expansion coefficient (Tanaka 2003).

To represent the basic state modified by the AO, we added (or subtracted) the anomalies of dependent variables regressed onto the normalized AOI to (or from) the climatology.

For example, adding three times anomaly (Fig. 1b) to the climatology (Fig. 1a), we can construct the basic state for the AOI + 3\( \sigma \) (\( \sigma \) standard deviation). In this way, zonal means of the the zonally varying basic states for the AOI from -3 \( \sigma \) to +3 \( \sigma \) are represented (Figs. 1c and 1d). Because of its zonally varying character, the basic states are different at each longitude.

For instance, the distributions of the zonal wind tend to have a double-jet shape in the Atlantic sector and a single-jet shape in the Pacific sector in the climatology as compared in Figs. 2a and 2b. The structures regressed on the AOI also vary for each longitude as in Figs. 2c and 2d. The eigenmodes are affected by these localities embedded in the basic states, as will be confirmed in the later section.

### 3. Linear stability analysis with the LBM

#### 3.1 Baroclinic instability for the zonal basic state

The linear stability analysis is performed first for the zonal basic state.

The basic state for January rather than DJF is used here to compare with the results by Seki et al. (2011). When the basic state is zonal, the term \( \bar{w}_b \) has nonzero values only for the wavenumber \( n_i = 0 \). In this case, the triad conditions for the zonal wavenumbers \( n_i = n_j + n_k \) allow interactions only for \( n_j = n_k \). Then, the conjugate term \( C \) vanishes and the term \( B \) becomes block diagonal in (19), which is reduced to

\[
\frac{dw}{dt} + iDw = -iBw. \tag{28}
\]

For this problem, we can find the solution in a much simpler form as follows:

\[
w(r) = \xi \exp(-i\nu t). \tag{29}
\]

The substitution of (29) into (28) yields an eigenvalue problem:

\[
\nu \xi = (D + B)\xi. \tag{30}
\]

Because the matrix in the right-hand side has real-valued entries and is decoupled for each zonal wavenumber, we can solve the linear stability problem for each zonal wavenumber with the diagonal block of dimension \((L + 1)(M + 1)\). The real and complex parts of the eigenvalue \( \nu_R \) and \( \nu_I \) describe the frequency and growth rate, respectively, for this case. A positive \( \nu_I \)
means an unstable mode that travels by the phase speed of \( c = \nu / n \), exhibiting a fixed structure determined by the eigenvector \( \xi \) without the life cycle as in (27).

Note that the solution includes the 3D normal modes of Hough harmonics. When the basic state is at rest, the matrix \( B \) vanishes because the basic state \( \mathbf{\pi}_b \) is zero.

For this case, the system is represented by the diagonal matrix \( D \) as for the harmonic oscillator, and the eigenvalue \( \nu \) becomes Laplace’s tidal frequency \( \sigma \). Therefore, the problem (28) is recognized as an extension of the 3D normal mode from the resting atmosphere to a zonal basic state. Here the modes are all neutral for the motionless basic state because there is no energy in the basic state. However, the modes can draw available potential energy to grow as an unstable normal mode for the zonal climate basic state.

The baroclinic instability for the zonal basic state was first solved by Tanaka and Kung (1989) as an eigenvalue problem of the linearized primitive equations on a sphere.

The unstable modes are presented as a function of the zonal wavenumber \( n \).

The fastest growing mode of the growth rate 0.4 day \(^{-1}\) appears at the zonal wavenumber 8 with its phase speed of 10° day \(^{-1}\), which is identified as the Charney mode of the baroclinic instability. The geopotential structure indicates a maximum in the mid-latitudes with a westward vertical phase tilt. The growth rate of the Charney mode decreases to nearly zero at planetary scale, and the alternative baroclinic

Fig. 1. Latitude-height structures of (a) zonal wind climatology \((\text{m s}^{-1})\), (b) zonal wind anomaly regressed onto the AOI \((\text{m s}^{-1})\), (c) the basic state of zonal wind \((\text{m s}^{-1})\) for the AOI +3.0 \( \sigma \), and (d) the basic state of zonal wind \((\text{m s}^{-1})\) for the AOI −3.0 \( \sigma \). The climate data are averaged for DJF from 1971 to 2000 in the Northern Hemisphere. The solid (dotted) lines indicate positive (negative) values.
instability called the dipole Charney mode dominates at the zonal wavenumber 3 and 4. Here the structure shows a meridional dipole in the geopotential structure. The same linear stability analysis was solved by Tanaka and Tokinaga (2002) to find a tripole Charney mode as the third-order Charney-type baroclinic instability with a meridional tripole structure.

In this study, the baroclinic instability for a zonal basic state is compared for the solutions of the general form and the zonal form to confirm the methodology of the LBM in the general form. Figure 3 plots the scatter diagram of the complex-valued eigenvalues, i.e., spectrum, obtained as the solution for the LBM (26) for the zonal basic state, as in Fig. 1a. The abscissa denotes the frequency $\nu_I$ and the ordinate denotes the growth rate $\nu_R$ in dimensionless values. According to the result, we find a dominant unstable Charney mode $M_C$ as connected by a solid line at the lower frequency range at 0.1, which corresponds to the period $1/(2\nu_I) = 5$ day.

For the zonal wavenumber 8, the period corresponds to the phase speed $360^\circ/(8 \times 5 \text{ day}) = 9^\circ \text{ day}^{-1}$. The peak growth rate of 0.04 corresponds to $4\pi\nu_R = 0.5$ day$^{-1}$ in a dimensional value.

The growth rate and frequency coincide with the values in previous studies.

Because the system matrix governing the instability (26) is real, the complex conjugate of the solution is also the solution of the system. Therefore, the scatter
The linear stability analysis using the 3D spectral primitive equation model is now extended to a zonally varying basic state compiled as the 30-year mean January climate of the observed atmosphere. The solutions are recognized as a generalization of the 3D normal modes from the resting atmosphere to a general 3D basic state on the sphere. The basic state has three meteorological variables of zonal wind, meridional wind, and geopotential deviation from the global mean on the isobaric surface.

In general, when the wind and mass fields are not in geostrophic balance, gravity waves are generated rapidly, associated with the geostrophic adjustment. Because of this imbalance problem, a linear stability problem has been difficult to solve in the primitive equation, and only the quasi-geostrophic equation has been solved as an eigenvalue problem.

The difficulty is overcome by the 3D normal mode method because we can effectively eliminate the unimportant gravity waves by means of a truncation of the basis functions.

If desired, we can include Kelvin modes or other important gravity modes depending on the purpose of the study. In this study, we are interested in the Arctic oscillation as an atmospheric unstable normal mode on a realistic 3D basic state. Therefore, gravity modes are eliminated from the beginning of the analysis.

Figure 5 plots the scatter diagram of the complex-valued eigenvalues obtained as the solution for the
LBM (26) for the zonally varying basic state of the 30-year mean January data. The abscissa denotes the frequency $\nu_I$, and the ordinate denotes the growth rate $\nu_R$ in dimensionless values. According to the result, we can identify a dominant unstable Charney mode $M_C$ at the lower frequency range of about 0.1, which corresponds to a period of 5 days.

Because the system matrix is not decoupled in zonal wavenumbers, one unstable mode has all contributions from different zonal wavenumbers. Thus, we cannot argue the property of each zonal wavenumber, nor we can calculate the phase speed of the unstable mode. Instead, all transient modes have a life cycle described by (27). Note that there are some standing modes with real eigenvalues, although the growth rate is small.

Figure 6 illustrates the life cycle of the most unstable mode with a growth rate of 0.27 day$^{-1}$ and a period of 4.5 days. The phase of the life cycle is denoted by the angle of $0^\circ$, $60^\circ$, $120^\circ$, and the $180^\circ$, in Fig. 6a to Fig. 6d, respectively. Note that Fig. 6d has just the opposite sign of Fig. 6a. The reader can track the movement of troughs and ridges for one cycle in the figure, then the next cycle will start from the adjacent troughs and ridges. By tracking the life cycle of troughs and ridges, a reader can see the structural change not only for one sector of the life cycle but also for all longitudinal excursions around the mid-latitudes by tracking six life cycles. The baroclinic waves are most amplified over the Atlantic Ocean, showing the wedge shape of troughs and converging eddy momentum flux about $60^\circ$ N to intensify the polar jet. A southward excursion of a trough is observed over the Far East from Siberia to Japan.

Figure 7 illustrates the most unstable standing mode appearing in the LBM for the nonzonal January basic state. The growth rate is only 0.04 day$^{-1}$ and the period is infinity. The modal structure is geographically locked by the existence of the complex conjugate term $C$. There is a large negative anomaly in the polar region with four troughs extending toward $45^\circ$E, $150^\circ$E, $120^\circ$W, and $30^\circ$W. The structure of the standing mode
can have the opposite sign of the solution because of the property of the eigenvector. This standing mode is connected to the standing mode in the barotropic basic state in the next.

3.3 Barotropic instability for a zonally varying basic state

The linear stability analysis for the nonzonal basic state represented by (26) of the LBM may be solved as a barotropic model, using only the barotropic component $m = 0$ for the state variable $w_0 = w_{0\text{ref}}$. The
matrix size to be solved for the barotropic model is reduced substantially with the new matrix dimension of $K_0 (N + 1) (L + 1)$. The property of the solution is identical to the baroclinic model (27), and the solution becomes a standing mode when the eigenvalue is real. Figure 8 illustrates the barotropic height of the climate basic state. The barotropic height is a vertical average of the geopotential height deviation from the global mean on the isobaric surface. The figure is reconstructed from the expansion coefficient of the basic state $\bar{w}_k$ using (9).

Figure 9 plots the scatter diagram of the complex-valued eigenvalues obtained as the solution for the barotropic model with the zonally varying basic state of the 30-year mean January data. It is worth noting that the most unstable mode is a standing mode at $\nu_1 = 0$ (see star symbol in Fig. 9). The growth rate is only $0.004 (= 0.05$ day$^{-1}$), which is supported by the barotropic instability from the nonzonal component of the jet stream. The barotropic height distribution is illustrated in Fig. 10a. There is a negative anomaly over the Arctic and three positive anomalies appear at Europe, Japan, and north America. It is important to note that this barotropic instability is the origin of the Arctic oscillation mode $\text{MAO}$, as is verified later. The second unstable standing mode (Fig. 9 triangle) is illustrated in Fig. 10b. There is a pair of positive and negative anomalies over Greenland and North Siberia. This mode may correspond to the Arctic dipole mode, which causes the transpolar sea ice drift suggested by Wu et al. (2006).

The most unstable barotropic mode $M_{10}$ was extensively investigated by Tanaka and Matsueda (2005) by means of the singular value decomposition (SVD) technique as well as the eigenvalue problem.
They introduced a linear damping represented by a hyper diffusion for the viscosity and Rayleigh friction for the surface friction:

$$d_i w_i = - k_D c_i^{-4} w_i - \nu_s w_i$$

(EVP). A diffusion coefficient $k_D$ is set as $k_D (2 \Omega a_s^4) = 2.7 \times 10^4 \text{ m}^4 \text{ s}^{-1}$ and a linear damping coefficient $\nu_s$ is set as $1.5 \times 10^{-3}$. Here $c_i$ is the phase speed of the Hough mode representing the 3D scale dependency.

The result of the scatter diagram of the spectrum is plotted in Fig. 11. All solutions are neutral or damping modes. The Arctic oscillation mode $M_{AO}$ appears as the standing mode $\nu_i = 0$, and the growth rate $\nu_R$ becomes zero by the inclusion of linear damping. It is found that the AO mode $M_{AO}$ in Fig. 10a is modified to the barotropic height, as illustrated in Fig. 12. There is a negative height anomaly over the Arctic, surrounded by a belt of positive height anomaly in the mid-

![Fig. 10](image)

Barotropic height (in arbitrary unit) of (a) the AO mode and (b) second mode in the Northern Hemisphere. The solid (dotted) lines indicate positive (negative) values.

![Fig. 11](image)

Scatter diagram of the growth rate ($\nu_R$) and frequency ($\nu_I$) for the barotropic January basic state with a linear damping. The star indicates the AO mode.

![Fig. 12](image)

Barotropic height (in arbitrary unit) of the AO mode with the linear damping for the January basic state. The solid (dotted) lines indicate positive (negative) values.
latitudes with two positive centers at the North Atlantic and North Pacific. This feature of \( M_{AO} \) coincides with the result of EVP and SVD by Tanaka and Matsueda (2005), and the observational analysis by Thompson and Wallace (1998). The scale-dependent diffusion modifies the structure of the standing barotropic mode to the structure of the Arctic oscillation, and the surface friction shifts the growth rate to zero maintaining the same structure of the eigenvector. In this study, we find that the origin of the AO mode \( M_{AO} \) is the most unstable barotropic instability mode for the nonzonal barotropic basic state.

Because the linear dynamical system becomes singular by the appearance of the zero eigenvalue mode, this mode is called a singular eigenmode of the AO. According to the SVD analysis of the steady solution in response to steady forcing, the AO mode with the zero eigenvalue becomes resonant for arbitrary quasi-steady forcing to cause an excitation of the AO structure. All climate models exhibit a common structure of the AO as the most dominant internal variability (see Ohashi and Tanaka 2009). The result supports that the AO is a dynamical eigenmode of the atmosphere that can be resonantly excited by any quasi-steady forcing.

4. Baroclinic instability under various AO index

4.1 Baroclinic instability modified by the AOI

The modification of the dominant unstable mode is investigated in reference to the AOI varying from \(-3\sigma\) to \(+3\sigma\) shown by Fig. 1 for the DJF mean climate. Figure 13 is the scatter diagram for the growth rate \((\nu_{R})\) and frequency \((\nu_{I})\) in each basic state. The frequency depends on the dominant zonal wavenumber. If the frequency is nearly 0, the zonal wavenumber is close to the planetary scale. In contrast, if the frequency is a value apart from 0, the zonal wavenumber becomes large.

Figure 13 shows that the dominant eigenmode appears in the frequency ranging from \(-0.2\) to \(+0.2\) in an arching shape, similar to those of \( M_{C} \) and \( M_{N} \) in Tanaka and Tokinaga (2002) and Seki et al. (2011).

The present study is unique compared with our former study in that the basic state is nonzonal and all zonal waves of the state variables interact with all zonal waves of the basic state. We selected the two fastest growing modes as the dominant baroclinic instability for each basic state, which are the low-frequency-side unstable mode \( M_{L} \), the star in) and the high-frequency-side unstable mode \( M_{H} \), the square in). While the AOI varies from \(-3\sigma\) to \(+3\sigma\), the \( M_{L} \) and \( M_{H} \) exist continuously (Fig. 14a). Their growth rates tend to increase by about 0.005 as the AOI becomes large positive (Fig. 14b).

Figure 15 illustrates the barotropic height of the \( M_{L} \) and \( M_{H} \) for the AOI from \(-3\sigma\) to \(+3\sigma\). For the climatological basic state (Figs. 15c and 15d), the \( M_{L} \) and \( M_{H} \) have wedge-shaped \( M_{C} \) structures, which prevail in the mid-latitudes.
Comparing the Atlantic and Pacific sectors, the Atlantic baroclinic modes are more prominent and shift poleward than the Pacific baroclinic modes because of the local difference of the basic state (Fig. 2). In this study, the Atlantic sector extends from 300°E to 0°E, and the Pacific sector spans from 150°E to 240°E. In particular, the Atlantic jet axis locates to about 50°N (Fig. 2a); However, the Pacific jet is about 35°N (Fig. 2a). Therefore, the Atlantic baroclinicity is strengthened in higher latitudes than the Pacific baroclinicity to shift the center of the unstable modes significantly poleward. Figure 16 shows the longitude-height cross sections of the geopotential height anomaly of $M_L$ in the Atlantic and Pacific sectors. The latitudes of the cross-sections are located where the center of the $M_L$ exists.

In the climatology (Figs. 16c and 16d), the Atlantic trough (ridge) axis tilts more westward with respect to the height than that of the Pacific trough (ridge). The amplitude maxima are also larger in the Atlantic sector.

When the AOI is large positive (Figs. 15a and 15b), the Atlantic $M_C$ modifies its trough axes tilted from northeast to southwest at the mid-latitudes. The Atlantic amplitude maxima are also shifted more poleward than the Pacific maxima.

Thus, the baroclinic instability in the Atlantic sector is modified to transport the westerly eddy momentum to more polar region, as suggested by Tanaka and Tokinaga (2002) and Seki et al. (2011).

On the other hand, the Pacific $M_C$ becomes weaker as the AOI becomes large positive. The amplitude maxima are also weaker than those in the Atlantic sector. Figure 16a shows that the Atlantic baroclinic instability, which extends into the stratosphere, is amplified much stronger than that of the Pacific (Fig. 16b).

Its zonal scale is also larger than the Pacific sector. Throughout its life cycle, the $M_L$ and $M_T$ maintain the specific structure of the baroclinic instability at the Atlantic sector, so that the interaction between the positive AOI and baroclinic instability mainly appears in the Atlantic sector. This result agrees with that of Chang and Fu (2002; 2003), who reveal that the AOI highly correlates with the storm tracks, especially in the Atlantic sector. This study confirms the result with the LBM. On the other hand, when the AOI is large negative, the $M_C$ appears in the Atlantic and Pacific sectors (Figs. 15c, 15f, 16c, and 16f). Each baroclinic instability has a similar wedge-shaped structure in both sectors.

Therefore, the baroclinically unstable modes excited by the zonally varying basic state have zonally asymmetric interaction with the AOI. The asymmetric interaction is attributed to the asymmetric basic state, specifically the distribution of the zonal wind. With the climatological basic state, the distributions of the $M_C$ depend on the spatial structure of the jet streams that differs locally (Figs. 2a and 2b). Tanaka and Tokinaga (2002) theoretically investigated the response of baroclinic instability to the polar jet, and found the polar mode $M_L$ prevails rather than the $M_T$ or $M_C$.

Eichelberger and Hartmann (2007) revealed that the eddy momentum selectively converges to strengthen the zonal wind when the subtropical and polar jets are well separated, showing a double-jet structure. The present study supports these facts with a linearized model.

4.2 Distribution of the unstable modes associated with the jet stream

In this section, we investigate the main causes of the
Fig. 15. Barotropic height (in arbitrary unit) of (a) $M_l$ in the AOI $+3.0 \sigma$, (b) $M_n$ in the AOI $+3.0 \sigma$, (c) $M_l$ in climatology, (d) $M_n$ in climatology, (e) $M_l$ in the AOI $-3.0 \sigma$, and (f) $M_n$ in the AOI $-3.0 \sigma$ in the Northern Hemisphere. The solid (dotted) lines indicate positive (negative) values.
Fig. 16. Longitude-height cross structure of $M_i$ in (a) the Atlantic sector at 61°N with the AOI $+3.0 \sigma$, (b) as in (a) but for the Pacific sector at 46°N, (c) the Atlantic sector at 55°N with climatology, (d) as in (c) but for the Pacific sector at 43°N, (e) the Atlantic sector at 46°N with the AOI $-3.0 \sigma$, and (f) as in (e) but for the Pacific sector at 43°N. The solid (dotted) lines indicate positive (negative) values.
regionality of the interaction between the AO and baroclinic instability, in terms of the large positive and negative AOI.

The distributions of the eigenmode have a regionality associated with the varying AOI.

One of the reasons is the difference in the structure of the jet stream of the basic state.

Figures 17a and c show the zonal-mean zonal wind in the AOI + 3σ in the Atlantic and Pacific sectors. Similarly Figs. 17b and 17d show the zonal-mean zonal wind anomaly regressed onto the AOI (shade) and zonal-mean westerly eddy momentum flux (u'v', where ('') indicates a deviation from the zonal mean) of the M1 (contour) in the Atlantic and Pacific sectors. When the AOI becomes large positive, the jet stream is modified into a double-jet structure in the Atlantic sector and a single-jet structure in the Pacific sector. Moreover, the M1 converges the eddy momentum flux to intensify the polar jet because of its specific structure (Figs. 15a and 17b). Tanaka and Tokinaga (2002) regarded the M1 as the eigenmode excited by the baroclinicity of the polar jet, so that the intensified polar jet, as seen in Fig. 17a, excites the M1 in the Atlantic sector selectively. On the contrary, there is no specific intensification of the zonal wind in the Pacific sector.

Therefore, when the AOI is large positive, the double-jet structure prevails in the Atlantic sector to strengthen the baroclinicity of the polar jet, so that the M1 is excited as the most unstable eigenmode M1 there.
The \( M_1 \) converges the westerly eddy momentum flux at the higher latitudes to intensify the polar jet. Although this interaction causes the positive feedback between the AO and baroclinic instability, which agrees with Seki et al. (2011), it is found that the feedback is dominant only in the Atlantic sector because of its double-jet structure.

Meanwhile, when the AOI is large negative, the two jets combine to be the strong single-jet structure in both sectors (Figs. 18a and 18c). The westerly eddy momentum converges to shift the subtropical jet poleward (Figs. 18b and 18d), associated with the negative AOI, and the sign of the zonal wind is maintained from because the AOI is negative here. The \( M_c \) is excited by the baroclinicity of the subtropical jet. Thus, when the AOI is negative and low, the baroclinicity of the subtropical jet excites the \( M_c \) to transport the westerly eddy momentum and shift the subtropical jet poleward through the interaction between the negative AOI and baroclinic instability. The single jet structure dominates in the Pacific sector for the positive AOI (Fig. 17c); however, the \( M_c \) does not emerge significantly (Fig. 15a). This is a specific characteristic of this mode. We can find many other unstable modes that amplify actively in the Pacific sector.

5. Summary and conclusion

In this study, a linear baroclinic model (LBM) is developed using the 3D spectral primitive equation model. With this LBM, we can investigate the linear stability problem for various 3D basic state on a sphere. For a zonal climate basic state, we confirmed that the traditional Charney mode \( M_c \) and dipole Charney mode \( M_d \) appear as the most dominant unstable modes in the synoptic to planetary scales.
For a zonally varying basic state, we found that these unstable modes are modified by the regionality of the local baroclinicity of the basic state. Moreover, the zonal asymmetry of the basic state allows the existence of a standing mode with zero frequency, although the growth rate is smaller than the dominant baroclinic instability.

When a zonally varying barotropic basic state is given to the LBM, we found the most dominant unstable mode is the standing mode excited by the barotropic instability of the nonzonal basic state. The structure of the standing mode is geographically fixed showing a negative anomaly over the Arctic, which is surrounded by a positive anomaly in the mid-latitudes. There are three troughs and ridges in the mid-latitudes trapped by the zonal asymmetry of the basic state. Although the standing mode has a fixed structure, the eigenvector allows the structure with opposite signs as an alternative solution. The standing mode appears to be similar to the Arctic oscillation in terms of its structure and behavior. In this study, we demonstrated that the barotropically most unstable standing mode becomes the Arctic oscillation mode, as proposed by Tanaka and Matsueda (2005), by the inclusion of diffusion and surface friction into the linear dynamical system.

In this study, the LBM is regarded as a generalized extension of the 3D normal mode at the motionless atmosphere to arbitrary climate basic states. It is confirmed that the LBM is applicable to the linear barotropic and baroclinic instability problem, and the results are consistent with previous studies.

As an application of the LBM, we investigated the spatial interaction between the AO and baroclinic instability.

Various zonally varying basic states associated with the positive and negative AOI are substituted into the LBM to find the response of the baroclinic eddies. After a linear stability analysis, the eigenmodes with a large growth rate are selected as the dominant unstable modes \( (M_c \text{ and } M_i) \) and examined for their spatial structures.

As a result, when the AOI is large positive, there is a positive feedback between the AO and baroclinic instability, particularly in the Atlantic sector. As the AOI becomes large positive, the \( M_c \) and \( M_i \) modify their \( M_c \) structures into the \( M_i \).

This modified mode transports the westerly eddy momentum to higher-latitudes to intensify the polar jet associated with the AO. On the contrary, in the Pacific sector, the baroclinic instability becomes weak, so that there is no significant convergence of the eddy momentum flux. This is because the positive AO divides the polar jet and subtropical jet in the Atlantic sector and combines the two jets in the Pacific sector. Because the baroclinicity of the polar jet excites the \( M_i \), the \( M_i \) obviously develops only in the Atlantic sector where the polar jet exists. Thus, the \( M_i \) interacts with the positive AO.

Meanwhile, when the AOI is large negative, the \( M_c \) appears as a dominant mode in both Atlantic and Pacific sectors, where the single jet is strongly dominant. The \( M_c \) converges the westerly eddy momentum flux to shift the single-jet to the higher latitudes. The baroclinicity of the subtropical jet ordinarily excites the \( M_c \), so that the interaction between the negative AO and baroclinic instability shifts the subtropical jet poleward through the eddy momentum flux of the \( M_c \).

In conclusion, we find the spatial characteristics of the positive feedback between the positive/negative AOI and modification in the baroclinically unstable modes.

The positive feedback dominates at the Atlantic sector for positive AOI because of the enhanced double-jet structure. On the other hand, when the AOI is negative, the \( M_c \) converges the eddy momentum flux at the mid-latitudes to shift the subtropical jet poleward in the Atlantic and Pacific sectors because of the intensified baroclinic instability.

It is found that the origin of the AO mode is embedded as the most unstable standing mode in the barotropic atmosphere. The zonal asymmetry of the basic state is essential for the existence of the standing mode.

Tanaka and Matsueda (2005) explained the AO as a singular eigenmode using the barotropic model. The LBM used in this study is useful to extend the singular eigenmode theory of the AO under the barotropic-baroclinic interactions. Therefore, the treatment of a proper 3D damping, including the thermal forcing in baroclinic atmosphere, becomes an important subject.

This problem will be reserved as a future study.

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