Near-global three-dimensional hail signals
detected by using GPM-DPR observations

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Abstract

This study proposes a method of detecting three-dimensional hail distribution by using the Global Precipitation Measurement (GPM) Dual-frequency Precipitation Radar (DPR) products in combination with the atmospheric temperature from a reanalysis product. In this study, the hail class contains hailstones, high-density graupel, and small frozen droplets. The radar reflectivity at the Ku-band (ZKu) and dual-frequency ratio (DFR) values are examined for hydrometeor classification at the five atmospheric temperature ranges in comparison to the ground radar product in a test hail-storm case. A simple model assuming binary collision for the riming process, which represents a significant reduction in the number concentration with the conservation of the mass concentration, explains well the hail signals on the scatter plot of ZKu and the DFR. This study determines the thresholds of the ZKu and DFR values at each temperature range based on the simple model. Furthermore, this study evaluates the thresholds in 74 hail-storm cases and proposes two filters to remove melting snow and rain contamination below the freezing level. The five-year dataset of the GPM-DPR observations shows that hail is widely distributed over oceanic convergence zones as well as continental convective regions. Most oceanic hail layers are found to be thin (i.e., less than 1500 m) and confined near the freezing level. Therefore, such hail signals have been potentially missed by ground observations. An additional filter removes such thin hail...
layers and effectively works to detect only deep hail storms, specifically, over continental regions.

**Keywords** GPM; satellite observation; hail; ice cloud microphysics; severe storm
1. Introduction

The ice cloud microphysical characteristics of hail, high density, large size, and fast terminal velocity, are unique compared to those of other ice hydrometeors, such as snow and light graupel. Ice embryos are produced by the glaciation of rain droplets over the freezing level and then develop by accretion with supercooled rain droplets (e.g., Browning, 1963; Wisner et al., 1972; Rasmussen and Heymsfield, 1987). Early hail particles are sustained around the freezing level by strong updrafts and continue growing to reach up to a few centimeters in diameter. Large hail particles finally fall to the ground with very fast terminal velocities and are often reported as a disaster event (e.g., Allen et al., 2017; Groenemeijer et al., 2017).

The roles of hail in severe storms have been missed in conventional weather forecasting simulations. Recently, numerical studies reported that storm simulations with hail modeling represent stronger radar echoes than those without hail modeling (e.g., Milbrandt and Morrison, 2013; Lang et al., 2014). Nevertheless, most cloud microphysical models used for numerical weather prediction do not yet predict hail (e.g., Wilson and Ballard, 1999; Saito et al., 2006; Seifert and Beheng, 2006; Thompson et al., 2008; Morrison et al., 2009).

Thus, the investigation of hail has been under-developed.

One of the reasons for such microphysical settings without hail is the infrequent occurrence and local nature of hailstorms. Complicated cloud microphysics for hail
assuming three-phase equilibrium within the melting layer was just loaded for most simulation cases. Now, the World Climate Research Programme (WCRP) states that weather and climate extremes are one of the grand challenges to be assessed (https://www.wcrp-climate.org/grand-challenges/grand-challenges-overview). With the advancement of computers, extreme events under climate change can be assessed using regional climate models across regions (Giorgi et al., 2019). At the same time, the horizontal resolution of the global climate model has become finer than 10 km to capture local convective cloud systems (Satoh et al., 2019; Stevens et al., 2019; Stevens et al., 2020).

Hail modeling is to be standardized for simulating severe storms in both regional and global climate models. Accordingly, the modeling community needs hail observations to evaluate hail modeling on a global scale.

Hail information has been historically collected by ground observatories. The microphysical characteristics of hail have been directly sampled with, e.g., hail pads; the occurrence and geographical distribution of hail are reported from local weather forecast offices (e.g., Storm Data). Such hail reports have been utilized for developing empirical hail maps by region to estimate the potential risk due to hail hazards (e.g., Allen et al., 2015). At the same time, ground remote sensing techniques using ground radar networks have been established for detecting hail signals in severe storms (e.g., Lenning, 1998; Heinselman and Ryzhkov, 2006; Park et al., 2009; Dolan et al., 2013). On the other hand, such observations
are concentrated in populated areas in a few developed countries, and hence, the occurrence frequency and climatological distribution of hail has not been examined in most regions.

Satellite remote sensing has recently been used for hail detection assuming severe storm signals as proxies of hail. Cecil (2009) developed a hail-detection method for space-borne passive microwave imager observations for the first time. The method was applied to 12-year products of the Tropical Rainfall Measurement Mission (TRMM) to determine the climatological distribution of hail-storms (Cecil and Blankenship, 2012). After the launch of the Global Precipitation Measurement (GPM) core satellite, the hail-detection method was improved by using the GPM Microwave Imager (GMI) and Dual-frequency Precipitation Radar (DPR) sensors (Mroz et al., 2017). Le and Chandrasekar (2021) found that the vertical structure of the dual-frequency ratio near the freezing level had information to separate graupel and hail from snow. Their approaches were designed to estimate hail probabilities in atmospheric columns, and hence, the three-dimensional hail distribution has not yet been demonstrated in most regions.

This study aims to detect near-global three-dimensional hail signals by using DPR. The vertical structure of hail is evidently informative for weather forecasting (e.g., the utility of the Next Generation Weather Radar (NEXRAD) system by National Oceanic and Atmospheric Administration (NOAA)) and is crucially needed for model evaluation.
Hydrometeor classification in global-scale modeling has a short history. For example, liquid/solid phase discrimination has been evaluated after the launch of the Cloud-Aerosol Lidar and Infrared Pathfinder Satellite Observations (CALIPSO) (e.g., Hu et al., 2010; Cesana et al., 2015). Evaluation methods for snow, graupel, and hail classes have not yet been established, and hence, satellite hydrometeor classification products for such classes meet the demand from the climate modeling community. This study separates dense ice particles and snow categories based on the optical properties and defines the former as hail. Thus, in this study, hail potentially contains high-density graupel and small frozen droplets, but light-density graupel and aggregates will be excluded.

In Section 2, the theoretical basis of DPR analysis is introduced, and the cloud microphysical characteristics of hail on DPR signals are examined. In Section 3, the dataset for the near-global analyses is described. In Section 4, the near-global three-dimensional distribution of hail signals is presented. A summary is given in Section 5.

### 2. Theoretical basis for hail detection

#### 2.1 The radar reflectivity factor at Ku and Ka bands

The radar reflectivity factor $z_e$ represents the strength of radar backscatter from hydrometeors and is calculated as follows:
\[ z_{e,j} = \frac{\lambda^4}{\pi^2 |K_j|^2} \int_0^\infty n_j(D)\sigma_b(D, \lambda) dD, \]

\[ K_j = (m_j^2 - 1)/(m_j^2 + 2), \]

where \( m \) is the complex refractive index of a medium \( j \) (here, liquid water, ice, or a mixture of them are assumed), \( n(D) \) is the particle size distribution (PSD), and \( \sigma_b \) is the backscattering cross section at wavelength \( \lambda \) [mm]. Here, \( \sigma_b \) is calculated according to Mie-theory, and \( |K|^2 = 0.93 \) was conventionally used for precipitation radar (e.g., Atlas and Ludlam, 1961). The Marshall-Palmer (MP) (Marshall and Palmer, 1948) distribution and Gunn-Marshall (GM) (Gunn and Marshall, 1958) distributions are assumed for rain and snow PSDs, respectively:

\[ n_j(D) = N_{0,j}\exp(-A_jD), (j = r,s) \] (2)

\[ N_{0,r} = 8000 \ [m^{-3} \text{mm}^{-1}] \text{ and } A_r = 4.1P^{-0.21} [\text{mm}^{-1}]. \]

\[ N_{0,s} = 3800 \ P^{-0.87} [m^{-3} \text{mm}^{-1}] \text{ and } A_s = 2.55P^{-0.48} [\text{mm}^{-1}]. \]

where \( D \) is the diameter for liquid droplets [mm] (melted diameter for ice particles) and \( P \) is the precipitation rate [mm hr\(^{-1}\)]. The subscripts \( r \) and \( s \) indicate rain and snow, respectively.

The backscattering cross section strongly depends on the refractive indices of the medium and the size parameter \((x=\pi D/\lambda)\) according to Mie theory (e.g., Atlas and Ludram, 1961; Herman and Battan, 1961). Thus, \( Z (= 10\log_{10}(z_e)) \) and \( Z \) differences between two channels have information about representative sizes and hydrometeor phases.

Liao and Meneghini (2011) (hereafter LM11) proposed a method to identify the
hydrometeor phases and bulk densities of snow particles using the radar reflectivity factors
at the Ku (frequency of 13.6 GHz or wavelength $\lambda = 2.2$ cm) and Ka (frequency of 35.5 GHz
or $\lambda = 8.5$ mm) bands (ZKu and ZKa, respectively). Figure 1 shows the relationships of ZKu
and the dual frequency ratio (DFR = ZKu - ZKa) calculated for snow with various bulk
densities, for solid ice, and for rain according to Mie theory (cf. Appendix A for details). Based
on the scatter-plot of ZKu and the DFR, nondense sponge-like ice particles are
characterized by smaller ZKu and larger DFR. LM11 performed a demonstration experiment
to investigate the relationship between ZKu and the DFR in actual snow events and found
that rain, snow, and mixed-phase classes are clustered in different regions on the ZKu-DFR
diagram (shaded regions in Fig. 1). Recently, Akiyama et al. (2019) showed that the
measured radar reflective factor at the Ku band (ZKum) and the measured dual frequency
ratio (DFRm) in a hail storm case have distinctively different values from those in a heavy
snowfall case.

This study utilizes the ZKu-DFR diagram for detecting hail. On the other hand, Liao and
Meneghini (2016) concluded that partitioning between the rain class and the mixed-phase
class, which mainly consisted of melting snow in their cases, is difficult using only ZKu and
the DFR, although the ZKu-DFR diagram has the potential to classify dense ice
hydrometeors. In the following subsections, simple models to characterize melting snow and
hail (or dense graupel) are presented for hydrometeor classification.
2.2 Melting snow on the ZKu-DFR diagram

In the case of LM11 (cf. Fig. 1), the ZKu and the DFR values for mixed-phase particles are mostly explained by melting snow in the melting layer. According to Yokoyama and Tanaka (1984), the melting layer in stratiform rain systems is characterized by constant mass fluxes through phase change as follows:

\[ N_s v_{t,s} = N_m v_{t,m} = N_r v_{t,r}, \]

where \( N \) is the number concentration and \( v_t \) is the terminal velocity. The subscript \( m \) indicates melting snow. This model implicitly assumes that the number concentration of melting snow decreases as the melting volume fraction \( f \) increases through the assumption that \( v_{t,r} > v_{t,s} \).

Given an initial condition of snow particles above the melting layer, the transition of ZKu and the DFR of falling particles in the melting layer are calculated by combination with the melting layer model (Eq. (3)) and Mie theory (e.g., Liao and Meneghini, 2005). Assuming the GM PSD for snow above the melting layer, the PSD of rain after melting (hereafter this class is referred to as melted snow) no longer follows the MP PSD. Note that \( f \) is calculated for individual snow particles as a function of the particle diameter and falling distance from the freezing level, and hence, the mass-weighted mean melting volume fraction \( \bar{f} \) is analyzed in the following paragraphs to understand the bulk characteristics of the optical properties.

Figure 2 shows four examples of the transition from snow (blue dashed line) to melted 9
snow (red solid line) by melting snow (black lines) on the ZKu-DFR diagram. In addition, rain
curves with the MP PSD (orange solid line) and modified gamma PSDs fitted to various
types of surface precipitation (orange dashed lines, Tokay and Short, 1996) are
superimposed. According to the melting layer model, ZKu and the DFR increase as $\bar{f}$
increases up to 0.5 (cf. black dashed lines in Fig. 2). An increase in ZKu in the melting layer
is known as the bright band and is the typical characteristic of stratiform rain systems.
Subsequently, ZKu and the DFR decrease toward the end of melting (cf. black solid lines in
Fig. 2). Melted snow (red line) has a larger DFR for a given ZKu than rain (orange solid and
dashed lines) does. This indicates that liquid droplets just below the melting layer have
distinctively smaller number concentrations (larger mean volume diameter) compared to that
of surface precipitation.

The rain class for ZKu > 30 dBZ (red shaded region on the right) and mixed-phase class
(green shaded region) fall along the envelope of the melting snow (for $\bar{f} > 0.5$) and melted
snow curves. Such classes inevitably accompany snow class, and hence, the key point to
identifying melting snow class is the bimodal structure of the snow signals and the melting
snow/melted snow signals on the ZKu-DFR diagram near the freezing level.

Note that the refractive indices for melting snow strongly depend on the mixing methods
of liquid and ice water. Liao and Meneghini (2005) intensively investigated the mixing
condition of melting snow and found that a stratified-sphere model reproduced the bright
band well in a stratiform rain event. This study used Bruggeman’s mixing formulation as a reasonable simplification method (Liao and Meneghini, 2005).

2.3 Hail on the ZKu-DFR diagram

This study introduces ZKu and the DFR signals that result from the cloud microphysical characteristics of hail growth to detect hail. Hail particles grow mainly by the accretion of liquid droplets. Assuming binary collision for the accretion, a collision of a liquid droplet with a hail particle results in a larger single hail particle. Thus, rapid accretion results in a significant reduction in the total number concentration with the conservation of the total mass concentration. This phenomenon, called collisional growth, can be represented by the evolution of the particle size distribution. Now, the MP distribution for rain (Eq. (2)) is assumed for frozen droplets as the initial condition of hail. Given the MP distribution for coexisting rain and hail particles, the process of halving the total number concentration \( n_{tot} \) with the invariant total mass concentration \( m_{tot} \) approximately corresponds to the process of modifying the assumed size distribution parameters as \( N_0 \rightarrow 0.4N_0, A \rightarrow 0.8A \) because

\[
\begin{align*}
n_{tot} & \propto N_0 / A \\
m_{tot} & \propto N_0 / A^4
\end{align*}
\]

Figure 3 shows the collisional growth curve of hail starting from the MP distribution (Eq. (2)) with various precipitation rates. Interestingly, rain for ZKu > 30 dBZ and mixed-phase classes also fall along the collisional growth curves. This means that the envelopes of the
melting snow curves (Fig. 3) and the collisional growth curves are similar to each other. Snow in the melting layer is assumed to lose its number concentration under the constraint condition of Eq. (3). As a result, the number concentration of melting snow/melted snow is as small as that of hail. Figure 3 indicates that hail signals derived from the ZKu-DFR diagram are potentially contaminated by the melting snow class just below the freezing level.

Next, the dependence of the hail signals on the atmospheric temperature is examined. Hail growth regimes change from wet to dry as the atmospheric temperature decreases and the accreted liquid water content decreases (e.g., Lesins and List, 1986); the change results in decreases in the hail density at colder atmospheric temperature ranges. Macklin (1962) experimentally determined that the hail density $\rho_h$ decreases as the atmospheric temperature decreases as follows:

$$\rho_h = 0.11 \left( - \frac{rv_0}{T_s} \right)^{0.76}. \tag{4}$$

where $r$ is the radius of the collected liquid droplet [μm], $v_0$ is the impact speed at the collision [m s\(^{-1}\)], and $T_s$ is the ice surface temperature [°C]. The impact speed generally decreases as the altitude (atmospheric temperature) increases (decreases) because lighter and slower particles are more likely to be lifted up. Based on Fig. 3 from Macklin (1962), the typical values of $\rho_h$ of 0.800, 0.600, 0.400, 0.250, and 0.100 are assumed at the atmospheric temperature ranges $T_a \geq 273$, $273 > T_a \geq 263$, $263 > T_a \geq 253$, $253 > T_a \geq 243$, and $T_a < 243$, respectively. Figure 4 shows the changes in the collisional growth curves on the ZKu-DFR.
diagram with various hail bulk densities $\rho_h$ from 0.916 g cm$^{-3}$ to 0.100 g cm$^{-3}$. Smaller ZKu values and larger DFR values of hail signals are expected as the atmospheric temperature (altitude) decreases (increases).

In this study, two characteristics of ZKu and the DFR signals are focused on to separate melting snow/melted snow and hail. One characteristic is that melting snow and melted snow mainly exist at atmospheric temperatures warmer than 273 K and that snow dominates just above the freezing level. The other characteristic is that ZKu and the DFR of hail can develop extremely as convection develops (e.g., the collisional growth curves with $P > 10$ mm hr$^{-1}$ in Fig. 3). Based on the abovementioned characteristics, this study utilizes (1) thresholds of ZKu and the DFR, (2) atmospheric temperature, and (3) filtering to reduce misclassification for hail detection. The thresholds are determined compared to the NEXRAD ground radar observations in Section 4.1, and filters to remove melting snow and rain contamination are proposed in Section 4.2.

### 3. Dataset

This study used the attenuation-corrected radar reflectivity ($z\text{FactorCorrected}$) at the Ku and Ka bands rather than the measured radar reflectivity ($z\text{FactorMeasured}$) at the Ku and Ka bands to keep the consistency between the data analyses and the theoretical basis (cf. Section 2). A five-year dataset from March 2014 to December 2018 was analyzed: the GPM-DPR level 2 products with version 05A from March 2014 to December 2017 and with version
05B during 2018. In addition, the atmospheric temperature $T_a$ from the Japanese 55-year Reanalysis (JRA-55) products (Kobayashi et al., 2015) was used. The reanalysis data were collocated to the GPM-DPR orbit data by using the nearest neighbor method at the nearest time step.

Note that this study assumed that attenuation correction works well even in intense storm cases despite of large uncertainties and ambiguities in the attenuation correction at Ku and Ka bands (e.g., Mroz et al., 2018). Analyses using the attenuation-corrected radar reflectivity potentially contain errors originated from the assumptions used for attenuation correction. Previous researches avoided the issue by using the measured radar reflectivity factor (e.g., Mroz et al., 2017; 2018; Akiyama et al., 2019; Le and Chandrasekar, 2021). In addition, multiple scattering effects to ZKu and ZKa (e.g., Battaglia et al., 2016) are not removed from the product. However, the multiple scattering effects are alleviated by the path-integrated attenuation correction method used for satellite products (Seto and Iguchi, 2007; 2015; Seto et al., 2021).

In this study, precipitation columns that contain ZKu values larger than a lower limit (10 dBZ was chosen here (cf., Hou et al., 2014)) below the freezing level were sampled (hereafter, this database is referred to as DBRain) because rain droplets below the freezing level are the essential source of hail growth above the freezing level. In addition, subsets of the DBRain dataset that are very likely to contain hail signals are prepared to determine the
thresholds of hail signals on the ZKu-DFR diagram. The subsets DB273K, DB268K, and DB258K contain precipitating columns whose echo-top temperature is observed at atmospheric temperatures colder than 273 K, 268 K, and 258 K, respectively. Here, the echo top is defined as a ZKu value greater than or equal to 40 dBZ. The 40 dBZ Ku band reflectivity height above the freezing level is known to be a good indicator of hail (Mroz et al., 2017) and hence is used just for reference. Note that this study did not use typePrecip and flagBB of the GPM-DPR level 2 standard product (Awaka et al., 2016; Le et al., 2016) to classify melting snow, hail, and convective systems.

This study used the hydrometeor classification data from the NEXRAD level 3 products (Park et al., 2009) for the evaluation of the hail thresholds based on the GPM-DPR. Specifically, ice crystals (CR), dry snow (DS), wet snow (WS), graupel (GR), big drops rain (BD), light and moderate rain (LR), heavy rain (HR), and a mixture of rain and hail (HA) were classified. In this study, LR+HR+BD is referred to as rain for simplicity. The NEXRAD consists of the polarimetric WSR-88D radars at 159 sites over the United States of America. Scenes are observed with a beam width of 1.0° at six elevator angles of 0.5°, 0.9°, 1.5°, 1.8°, 2.4°, and 3.4°. In the analyses, the data were converted into latitude-longitude coordinates with a 0.0067 degree resolution by the NOAA Weather and Climate Toolkit version 4.3.1. The matching method between the GPM-DPR and the NEXRAD is described in Appendix B.
4. Results

4.1. Definition of hail thresholds on the ZKu-DFR diagram

First, the ZKu-DFR diagram near the freezing level was analyzed because hail is frequently observed at this level (Dolan et al., 2013). Figure 5 shows the joint probability density function (JPDF) of the ZKu and DFR values within the atmospheric temperature range from 273 to 263 K. In the DBRain (Fig. 5a), two major modes, the light ice mode (upper left) with $\rho_s \sim 0.2$ g cm$^{-3}$ and the rain mode (bottom), are distinct, and the heavy ice mode with $\rho_s \sim 0.4$ to 0.8 g cm$^{-3}$ is slightly observed between the light ice mode and rain mode.

In contrast, in the DB273K database (Fig. 5b), ZKu values larger than 30 dBZ are more frequently observed, and the rain mode merges with the heavy ice mode at ZKu values from 30 to 40 dBZ and DFR values from 0 to 5 dBZ. The DFR values in the merged mode drastically increase at ZKu values larger than 40 dBZ and peak at approximately nine to ten. The shape of the merged mode follows melting snow/melted snow curves or collisional growth curves in the ZKu-DFR diagram. Given an atmospheric temperature colder than 273 K, the merged mode represents that rain droplets begin to freeze and hail/graupel grows by accretion in convective clouds. On the other hand, the light ice mode is distinctive, and hence, well-organized stratiform clouds are also contained in DB273K.

In the DB268K and DB258K databases, the light ice mode ($\rho_s =$0.1 to 0.4) disappears.
In addition, the rain mode is narrowly observed around DFR = 0 dBZ and ZKu = 15 to 25 dBZ. This indicates that slowly falling snow is not sustained near the freezing level and that a number of small rain droplets are continuously transported above the freezing level in stronger convection. In the DB258K database, the JPDF falls within the collisional growth curves from $P = 10$ to $P = 100$ mm hr$^{-1}$. This study expects that hail signals are partially represented by the JPDF in the DB258 database.

An intense hail-storm, which was simultaneously observed by the NEXRAD ground radar observations and the GPM-DPR over Texas, USA, at 22:25 UTC on 26 May 2015 (cf. Iguchi et al., 2018), was used to evaluate the hail signals. By referencing the NEXRAD level 3 hydrometeor classification product, GPM grids that contained at least one graupel or hail pixel were sampled (details are described in Appendix B).

Figures 6a-e show the ZKu-DFR diagram from the DB258K database at various atmospheric temperature ranges. The hydrometeor classes from the NEXRAD level 3 product are also superimposed. In general, the mode of the JPDF shifts to the left and upper sides of the ZKu-DFR diagram as the atmospheric temperature decreases, and the shift almost follows the transition of $\rho_h$ from 0.800 to 0.100 g cm$^{-3}$ (cf. Eq. (4) and Fig. 4). In addition, the hail class is commonly observed on the right side of the collisional growth curve with $P = 10$ mm hr$^{-1}$.

The thresholds of the hail signals on the ZKu-DFR diagram are determined by the
collisional growth curve with $P = 10$ mm hr$^{-1}$ as the upper boundary of the DFR values and the solid ice curve as the lower boundary of the DFR values at each ZKu value. In addition, the upper and lower limits are set for the DFR values. Furthermore, the boundary values are approximated with simpler functions, and the thresholds are finally defined as follows:

<table>
<thead>
<tr>
<th>Collisional growth curve</th>
<th>DFR $\leq C_1 \times $ZKu + $C_2$,</th>
<th>$T_{a,\text{low}} \leq T_a &lt; T_{a,\text{high}}$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid ice curve</td>
<td>$0.0032 \times (\text{ZKu} - 3.0)^2 + 0.2 \leq \text{DFR}$,</td>
<td></td>
</tr>
<tr>
<td>Upper and lower limits</td>
<td>$C_3 \leq \text{DFR} \leq C_4$,</td>
<td></td>
</tr>
</tbody>
</table>

The constants $C_1$, $C_2$, $C_3$, and $C_4$, which are set at each atmospheric temperature range, are summarized in Table 1. The thresholds of ZKu and the DFR for hail detection are shown in Fig. 7. Note that the collisional growth curve is derived from a very simple model assuming binary collision, and hence, its accuracy is to be improved. On the other hand, the accuracy of the hail thresholds can be adjusted by appropriately choosing $P$ values for the collisional growth curve.

Now, the threshold method is evaluated by estimating the detection errors. The probability of missing hail ($P_{\text{miss}}$), rain contamination ($P_r$), snow contamination ($P_s$), and graupel contamination ($P_g$) are defined as follows:
\[
\begin{align*}
P_{\text{miss}} &= \frac{N_{h\text{miss}}}{N_h}, \\
N_{h\text{miss}} &= N_h - N_h, \\
P_j &= \frac{N_{j,\text{in}}}{N_{\text{thr}}}, \quad (j = r, s, g),
\end{align*}
\]

where \( N_{h\text{miss}} \) is the number of hail pixels that are not detected by the thresholds, \( N_h \) is the number of hail pixels, \( N_{\text{thr}} \) is the total number of pixels included in the thresholds, and \( N_{j,\text{in}} \) (\( j = r, s, g \)) is the number of rain (\( r \)), snow (\( s \)), or graupel (\( g \)) pixels that are included in the thresholds. At atmospheric temperatures warmer than 273 K, hail signals are potentially contaminated by rain droplets near the solid ice curve and are occasionally missed below the solid ice curve (Fig. 7a). In this test case, the probabilities of the former error (\( P_r \)) and the latter error (\( P_{\text{miss}} \)) are 3.8% and 12.7%, respectively. Similarly, at atmospheric temperatures colder than 273 K, the hail signals are potentially contaminated by snow (Figs. 7b-7e), but \( P_s \) is sufficiently small. At atmospheric temperatures colder than 253 K, it is difficult to distinguish the light ice mode, which lies along the \( \rho_s = 0.1 \) to 0.2 line, from the heavy ice mode, which lies along the collisional growth curve, because the two modes are almost merged (Figs. 6d and 6e). However, the collisional growth line with \( P = 10 \text{ mm hr}^{-1} \) separates the graupel and hail classes from the snow class well. Finally, more than 80% of the hail is expected to be captured in most hail layers.

Note that the hail thresholds (Eq. (5)) potentially misclassify other categories at ZKu values less than 40 dBZ. Mroz et al. (2017) reported that the probability of hail occurrence decreases when the radar echo above the freezing level is less than 40 dBZ. Similarly, hail
classification in the NEXRAD product is suppressed by the radar-echo threshold of 40 dBZ to avoid misclassification (Park et al., 2009). In the following section, the hail thresholds are further evaluated in various storm cases.

4.2. Evaluation of the hail thresholds

a. Dataset for evaluation

The hail thresholds (Eq. (5)), which were determined in the single event, are evaluated in various events. The NEXRAD sites in central North America (from 30° N to 40° N and from 105° W to 90° W) are chosen. This study picks up hail events that satisfied two conditions: (i) the hail thresholds satisfied at least one GPM-DPR range gate below 1000 m above ground level and (ii) ZKu values equal to or more than 40 dBZ were observed above the freezing level. As a result, 74 events at 29 NEXRAD sites were chosen during 2014. The NEXRAD level 3 products are provided every 5 minutes. For matching the NEXRAD product with the GPM level 2 product, the NEXRAD observation within 3 minutes of the GPM observation is picked up. In addition, this study analyzes observation pixels that are not farther than 150 km from each radar site. The hail events used for the analyses are summarized in the supplemental information. The matching method between the GPM-DPR observations and the NEXRAD level 3 product is the same as in the previous analysis (cf. Appendix B). Figure 8 shows the frequency of dominant hydrometeor classes from the NEXRAD level 3 product within the analysis range in the chosen cases. Hail is mostly

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observed at atmospheric temperatures from 25 to -10℃. In the atmospheric temperature range, the thresholds for hail detection on the ZKu-DFR diagram are potentially contaminated by wet snow or rain (cf. Section 2).

To examine the contamination rates, the frequencies of hail, graupel, wet snow, and rain classes are plotted on the ZKu-DFR diagram at each atmospheric temperature range (Fig. 9). Here, the wet snow class in the NEXRAD product corresponds to melting snow. The thresholds for hail detection (Eq. (5)) capture well a major portion of the hail signals (70% - 80%) all over the atmospheric temperature ranges. Thus, the ZKu-DFR diagram works sufficiently to detect hail even though ZKu and ZKa signals from the GPM-DPR satellite in hail-storms are known to be strongly affected by multiple scattering effects and horizontal inhomogeneity (Mroz et al., 2018).

Nonnegligible frequency of spotty hail signals is found outside the hail thresholds; hail signals are missed at DFR values smaller than the solid ice curve (the lower boundary of the DFR values), especially at atmospheric temperatures warmer than 273 K. The former source of the missing hail signals is difficult to capture because such signals are not organized. The latter source of the missing hail signals is strongly contaminated by rain. The wet snow class (melting snow) is also observed within the hail thresholds, especially below the freezing level.

This study utilizes the vertical structure of ZKu and the DFR to reduce rain and melting
contamination below the freezing level. To characterize the contaminated precipitation systems, two subsets of the NEXRAD-GPM matching database are prepared: the HR (heavy rain) and HG (hail/graupel) subsets contain radar profiles in which at least one rain and hail/graupel layer is observed within the hail thresholds below the freezing level, respectively. The sample sizes of the HR and HG subsets are 3000 and 3731 during 2014, respectively.

b. Melting snow filter

Melting snow signals are characterized by light snow particles above the freezing level and heavy dense particles just below the freezing level (cf. Section 2.2). These characteristics are clearly observed in Figs. 9j and 9k. Hence, contaminated columns are excluded (the melting snow filter) when the following two criteria are satisfied:

(i) The hail base temperature $T_{hb}$ is warmer than 273 K.

(ii) Snow signals dominate at atmospheric temperatures from 273 K to 263 K.

Here, the hail base is defined by the lowest layer that satisfies the hail threshold (Eq. (5)).

For example, the vertical profiles of the ZKu-DFR diagrams near the freezing level are calculated in the cases of $T_{hb}$ ranging from 273 K to 283 K (Fig. 10). The bimodal structure of the JPDF is clearly observed at atmospheric temperatures from 263 K to 273 K (Figs. 10a and 10c) and is found to be separated by the constant bulk density curve with $\rho_s = 0.5 \text{ g cm}^{-3}$. At atmospheric temperatures from 273 K to 283 K, a transition from the light ice mode to
the heavy ice mode is observed (Figs 10b and 10d). These are typical characteristics of the melting layer. Hereafter, this study uses the constant bulk density curve with \( \rho_s = 0.5 \text{ g cm}^{-3} \) as the threshold to isolate the snow class at the atmospheric temperature range. The curve is approximated as follows (see Figs. 10a and 10c):

| DFR > 0.005 \times ZKu^2 - 0.2. | 273 \text{ K} > T_a \geq 263 \text{ K} | (7) |

Note that the curve crosses the hail thresholds at higher ZKu values. Therefore, the snow mode (hereafter GPM-snow class) is defined on the ZKu-DFR diagram when both Eq. (7) and the lower boundary for the DFR (DFR \( \geq 0.8 \times ZKu - 23 \)) are satisfied. The GPM-snow class matches the light ice mode of the dry snow class from the NEXRAD products well (see Fig. 9n).

Interestingly, the melting layer is commonly observed among the HR and HG subsets (Figs. 10a, 10c). Thus, the hail signals below the freezing level are enormously contaminated by melting snow. For safety, this study assumes that GPM-DPR range gates that satisfy the hail thresholds below the freezing level are assumed to be melting snow when 50% of the GPM-DPR range gates at atmospheric temperatures from 263 K to 273 K are classified as GPM-snow. The melting snow filter substantially reduces melting snow contamination (see Table 2). In particular, melting snow contamination is detected in approximately 95% of columns in the HG subset when the hail base is observed at atmospheric temperatures from 273 K to 283 K.
Note that an alternative melting layer flag was proposed by Awaka et al. (2016) and Le et al. (2016) and is provided by the GPM-DPR level2 standard product. Their techniques use the difference between the measured DFR (DFRm) above the melting layer and DFRm below the melting layer to detect the location and thickness of the melting layer (cf. Fig. 2).

In contrast, the melting snow filter proposed in this study is designed to capture dry snow over the melting layer (cf. Fig. 10 and Eq. (7)). Thus, the melting snow filter does not deal with the vertical structure of the melting layer (bright band) but sufficiently removes melting snow contamination both in convective and stratiform precipitation systems.

c. Heavy rain filter

This study hypothesizes that hail signals below the freezing level result from the deep hail layer above the freezing level. In general, hail particles, which are detectable at temperatures warmer than 273 K, are large so as not to completely dissipate by sublimation and melting. Such larger hail particles require stronger updrafts, and hence, deep hail layers are expected to be formed above the freezing level. In previous studies, some vertically integrated parameters, which represent the hail layer depth above the freezing level (e.g., Lenning et al., 1998; Mroz et al., 2017), are similarly utilized for hail detection.

This study uses a threshold ratio $R_{thr}(k) = \frac{L_{thr}(k)}{L(k)}$ to separate hail from rain. Here, $L(k)$ is defined by the layer number from the freezing level to a certain level $k$, and $L_{thr}(k)$ is
defined by the number of layers that satisfy the hail thresholds (Eq. (5)) from the freezing
level to a certain level \( k \). Thus, \( R_{\text{thr}} \) represents an index of the hail layer depth.

Figure 11 shows the vertical profiles of \( R_{\text{thr}} \) sorted by the hail base temperature. \( R_{\text{thr}} \) values become larger as the hail base temperature becomes warmer in the HG subset. This indicates that the hail layer deepens as the hail base temperature increases, as expected. In contrast, the vertical profiles of \( R_{\text{thr}} \) are less sensitive to the hail base temperature in the HR subset. Therefore, \( R_{\text{thr}} \) values are distinctive between the HR and HG subsets when the hail base is observed at atmospheric temperatures warmer than 283 K. In most hail storm cases, hail layers are continuously distributed from the freezing level to the top of the hail layers. Thus, approximately 60% (30%) of the storm population has a hail top temperature colder than -20°C in the HG (HR) subset (Fig. 11a).

The precipitation columns are assumed to be uncontaminated by rain when \( R_{\text{thr}} > 0.8 \) at a certain threshold temperature \( T_{\text{thr}} \) (heavy rain filter). In other words, storms with \( R_{\text{thr}} \leq 0.8 \) at \( T_{\text{thr}} \) are excluded from the analysis, and hence, the storm population decreases after this process. Hereafter, this process is referred to as the heavy rain filter. The dependence of the population ratio on the threshold temperature \( T_{\text{thr}} \) is examined (Fig. 12). The population ratio is the ratio of the storm population with the heavy rain filter to the storm population without the filter. The rain contamination decreases as \( T_{\text{thr}} \) decreases in all storms. On the other hand, hail signals are more likely to be missed at lower \( T_{\text{thr}} \) values.
The hail/graupel class and the rain class are indistinguishable below the freezing level when the hail base temperatures are observed from 273 K to 283 K. In this case, hail or graupel completely melts just below the freezing level, and hence, hail, graupel, and rain classes coexist within a very narrow temperature range. However, the case is not frequently observed in the NEXRAD-GPM matching database (shown in the following subsection), and hence, this study is not focused on reducing rain contamination in this case.

d. Alternative solid ice curve

The sensitivity of the hail thresholds to hail detection is examined in this subsection. In particular, the solid ice curve is expected to differ by rain systems. This study assumes the gamma distribution for convective rain systems compiled by Tokay and Short (1996) for the alternative solid ice curve. Its formulation is to be referred to the case 1 in Table 2 in the original paper. In the same manner as Eq. (5), the alternative solid ice curve is approximated as follows:

\[
0.0032 \times (Z_{Ku} - 3.0)^2 - 2.0 \leq \text{DFR}. \quad (8)
\]

Figure 13 shows the difference in the coverage of the hail thresholds with the alternative solid ice curve. The thresholds with the alternative solid ice curve capture more hail signals than the thresholds with the original solid ice curve do. In particular, \( P_{\text{miss}} \) improves at atmospheric temperatures warmer than 263 K. On the other hand, the alternative solid ice curve gets closer to the rain curve with the MP-PSD at warmer temperatures. As a result,
the expanded thresholds more frequently contain other hydrometeors (graupel and rain) and

$P_h$ decreases particularly below the freezing level. These results show that the missing rate

and the contamination rate are incompatible.

e. Filtering test

Figure 14 shows the vertical profile of $N_{thr}$, $P_{miss}$, $P_{snow}$, $P_{ws}$, and $P_{rain}$ in the NEXRAD-

GPM matching database. Here, the analysis results with the hail thresholds with the default

solid ice curve and no filters are indicated as CTL. The melting snow filter ($F_{ML}$), heavy rain

filter with $T_{thr} = -10$ °C and $T_{thr} = -20$ °C ($F_{r10}$, $F_{r20}$, respectively), and the alternative solid

ice curve (ALT) are tested against the CTL. The hail signals from the NEXRAD products are

captured well by the hail thresholds at atmospheric temperatures colder than 20°C and are

missed more as the stronger filter is used, especially below the freezing level (Figs. 14b;14f).

The hail thresholds with the alternative solid ice curve work well to reduce $P_{miss}$ below the

freezing level without filtering (Fig. 14f) but substantially overestimates $N_{thr}$ compared to the

NEXRAD hail profile (Fig. 14e) due to misclassification (Fig. 14h). As a result, the hail

thresholds with the alternative solid ice curve need stronger filtering than the hail thresholds

with the default solid ice curve, as expected. Most hail signals above the freezing level detect

the hail class or the graupel class, whereas rain contamination dominates at atmospheric

temperatures warmer than 20 °C. The use of the heavy rain filter modestly reduces the rain
contamination below the freezing level, and the removal efficiency of the heavy rain filter is small between $T_{thr} = -20 \, ^\circ C$ and $T_{thr} = -10 \, ^\circ C$ (Figs. 14d; 14h).

Finally, the $N_{thr}$ profile from the CTL with the $F_{ML}$ and $F_{r10}$ filters is comparable to the hail profile from the NEXRAD product (Fig. 14a). Thus, this study applies the CTL thresholds with the $F_{ML}$ and $F_{r10}$ filters to analyze the near-global three-dimensional hail distribution. For reference, the reliability of hail detection is compiled at each temperature range and each pair of ZKu and the DFR based on the NEXRAD-GPM matching database (Appendix C).

Note that wet snow is minor in the NEXRAD-GPM matching database (Fig. 14d). However, in fact, the melting snow filter substantially removes the contaminated pixels from the GPM observations (Table 2). These results indicate that wet snow is potentially populated but is misclassified as other classes in the NEXRAD-GPM matching database. Since the vertical resolution of the NEXRAD product is coarse (1 km to 3 km) compared to the fine vertical structure of the melting layer (~1 km), it is difficult to dominantly classify wet snow within a GPM-DPR footprint in the NEXRAD-GPM matching database. On the other hand, GPM-DPR has finer vertical resolution (125 m) and, hence, frequently detects melting layer (wet snow) signals in stratiform rain systems. Therefore, the melting snow filter is not excluded, although the melting snow filter slightly increases $P_{miss}$ (Fig. 14b).

5. **Near-global three-dimensional distribution of hail signals**

The near-global distribution of the hail signals is analyzed from March 2014 to
December 2018. The frequency of hail signals \( f_{\text{hail}} \) is normalized by the GPM-DPR observations at each 1.25° × 1.25° latitude-longitude grid. The hail thresholds (Eq. (5) and Table 1), which are discretized by the 10 K temperature range, result in discretized hail distribution in the global analysis. Therefore, the coefficients used for the hail thresholds are linearly interpolated by the atmospheric temperature to derive the smooth hail distribution as follows:

\[
C_j = C_j^{k-1} + \frac{C_j^k - C_j^{k-1}}{10} (T_a - T_a^{m, k-1}) \quad (j = 1, 2, 3, 4)
\]

\[
T_a^{m, k-1} \leq T_a < T_a^{k, m, mid},
\]

\[
T_a^{k, m, mid} = \frac{T_a^{k, low} + T_a^{k, low}}{2}.
\]

The hail detection procedures are summarized schematically in Fig. 15.

The near-global hail distribution with the \( F_{\text{ML}} \) filter is shown in Fig. 16a. The hail signals are widely distributed along the Intertropical Convergence Zone, South Pacific Convergence Zone, South Atlantic Convergence Zone, and storm track regions in the midlatitudes. The heavy rain filter removes the hail signals below the freezing level, which are potentially contaminated by large rain droplets (Figs. 16c and 16d). The use of the heavy rain filter reduces the hail signals to one-third over the tropics and half over the midlatitudes.

The frequent occurrence of hail signals over the oceans is a unique feature illustrated by the GPM-DPR observations. The hail signals are specifically concentrated near the freezing level (Fig. 16d). It is found that 52% and 33% of the hail signals are confined within a single GPM-DPR layer (Δz = 125 m) over the oceans and land, respectively. In addition,
The 90th percentile of the hail signals has layer thicknesses of 1500 m and 6750 m over the oceans and land, respectively. Such narrow distributions of hail and frozen droplets, especially over the oceans, were directly detected by past observations using video sondes. Takahashi (2006) compiled more than 200 video sonde profiles over the east Asian monsoon region and revealed that frozen droplets frequently existed near the freezing level and that their mass concentrations were dominant over the tropical open ocean. These observations indicate that frozen droplets can be generated just above the freezing level even in moderate convection over the oceans. Therefore, Fig. 16c shows the geographical distribution of hail, which frequently contains relatively small dense ice particles. Such ice particles in the free troposphere have been missed by past ground observations using hail pads and ground hail reports.

The vertical resolution of measuring instruments is crucial to detect hail signals because frozen droplets are narrowly distributed around the freezing level. Assuming the S-band radar of the NEXRAD product with a beam width of 1.0°, the vertical resolution of the ground radars is coarser than 1500 m at distances farther than 100 km from a radar site (cf. Appendix B). Thus, most hail signals, specifically over the oceans, are potentially missed by such ground observations.

Finally, this study tries to test an additional filter to pick up hail signals that can reach the ground surface. The new filter is the same as the heavy rain filter but the filter is applied
regardless of the hail base temperature (Fig. 15). Since only the deep hail signals pass the filter (Section 4.2.c), precipitating columns that contain only small frozen droplets confined around the freezing level are excluded. As a result, this procedure clearly removes hail signals over the oceans (Fig. 17).

This hail map is similar to that obtained by past observational studies validated by ground truth observations (Cecil and Blankenship, 2012; Mroz et al., 2017). Specifically, well-known frequent hail occurrences over the so-called Tornado Alley of the United States are clearly captured (e.g., Allen et al., 2015). In addition, this hail map is similar to the thunder storm map (e.g., Zipser et al., 2006; Peterson, 2019). Thus, this hail map captures the occurrence of intense convective thunder storms that induce severe hail disasters.

Note that filtering to remove contamination over the oceans was not evaluated because of the lack of ground truth observations. Zenith radar observations on observation vessels are to be used for evaluating GPM-DPR products over the oceans in the future. In addition, users should be careful when analyzing specific hail events using this hail product because hail signals near the ground are likely to be contaminated by rain (cf. Fig. 15d). For example, in terms of disaster prevention operations, a heavy rain filter with warmer $T_{thr}$ values is preferable for safety.

6. Summary

This study defined the hail signal using the ZKu-DFR diagram. In terms of cloud
microphysics, the hail signals lie along the collisional growth curve. The ZKu and DFR values for hydrometeor classification were examined at five atmospheric temperature ranges by comparing them to the NEXRAD level 3 product in a test case of hail-storms. The collisional growth curve with $P = 10 \text{ mm hr}^{-1}$ and assumed bulk ice densities separate hail from other hydrometeors well. This study showed effectiveness of the use of the attenuation-corrected radar reflectivity for hydrometeor classification despite of the uncertainties in attenuation correction.

The thresholds of ZKu and the DFR for hail detection were then evaluated in an additional 74 hail-storm events and were found to be contaminated especially by melting snow and rain. This study proposed the melting snow filter and heavy rain filter to reduce contamination using the vertical profile of the ZKu and DFR values in storms.

Finally, the near-global three-dimensional hail distribution was derived by using the GPM-DPR products in combination with the atmospheric temperature from the JRA-55 reanalysis product. The GPM-DPR observations indicate that hail is widely distributed over oceanic convergence zones as well as continental convective regions. Specifically, hail is frequently observed in a thin layer near the freezing level. High-resolution space-borne radar is capable of capturing such hail signals. The new hail dataset can be utilized for model evaluation, detection of the early stage of hailstorm development, and estimation of the potential hail risk at the global scale.
Supplement

Supplement 1. Table S1 summarizes the hail-storm events used for the NEXRAD-GPM matching database.

Acknowledgments

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The GPM-DPR level 2 product was obtained from the Japan Aerospace Exploration Agency (JAXA) G-Portal (https://gportal.jaxa.jp/gpr/?lang=en). The JRA-55 reanalysis product was obtained from the Japan Meteorological Agency (JMA; http://jra.kishou.go.jp/JRA-55/index_en.html). The NEXRAD level 3 product was obtained from the National Oceanic and Atmospheric Administration’s National Climatic Data Center (NOAA’s NCDC; https://www.ncdc.noaa.gov/nexradinv). For the analysis of the NEXRAD level 3 product, the NOAA Weather and Climate Toolkit version 4.3.1 provided by NOAA’s NCDC was used (https://www.ncdc.noaa.gov/wct/). This study was supported by the FLAGSHIP2020 within the priority study 4 (Advancement of meteorological and global environmental predictions utilizing observational “Big Data”) and "Program for Promoting Researches on the Supercomputer Fugaku" (Large Ensemble Atmospheric and Environmental Prediction for Disaster Prevention and Mitigation), which are promoted by the Ministry of Education,
Appendix A. Mie calculation

The backscattering coefficients were numerically calculated following the method used by Herman and Battan (1961). The Mie coefficients were calculated by an efficient solver proposed by Dave (1969) and Wiscombe (1980). A downward recurrence method was used in numerically unstable cases (particularly, cases involving larger size parameters) (Kattawar and Plass, 1967; Lentz, 1976).

We used the refractive indices for liquid water and solid ice at the Ka and Ku bands compiled in the Satellite Date Simulator Unit (Masunaga et al., 2010; Matsui et al., 2014) (see Hufford (1991) and Brussaard and Watson (1994) for the original articles). To derive the refractive indices for sponge-like snow particles, the Hanai-Bruggeman equation (Hanai, 1962) was used to accurately represent a mixed medium of ice and air following Liao and Meneghini (2005). We note that the refractive indices for liquid water are sensitive to temperature, whereas those for ice particles are insensitive to temperature. Therefore, the Mie coefficients of rain were calculated for each temperature range.

Appendix B. Matching the GPM-DPR footprint and the NEXRAD level 3 product

The matching method between the GPM-DPR footprint and the NEXRAD level 3 product
product is explained with an example of the hail-storm case analyzed in Iguchi et al. (2008).

The hail-storm was simultaneously captured by the polarimetric WSR-88D radar located at the KFWS-DALLAS/FTW site at 22:25 UTC on 26 May 2015 and by the GPM-DPR. The issue is the different resolutions of the different observational products. The NEXRAD level 3 data have a horizontal resolution of approximately 0.6 km, and the vertical resolution increases from 1 to 4 km along the beam direction (see Fig. B1). On the other hand, the GPM-DPR level 2 product has a horizontal resolution of approximately 5 km and a vertical resolution of 125 m. To solve this issue, Mroz et al. (2017) resampled the NEXRAD level 1 data at each GPM-DPR footprint and then retrieved the hydrometeor classification. This study selected only all the hydrometeor classes of the NEXRAD level 3 data at each GPM-DPR footprint for simplicity: approximately 50 NEXRAD grids are selected within a 20 km² circle, and the majority class is chosen as the representative class with the exception of GR and HA. GR or HA is chosen as the representative class when at least one GR or HA is sampled within a GPM-DPR footprint. In addition, HA takes priority over GR. The hydrometeor classes from the NEXRAD level 3 data are carefully chosen according to the atmospheric temperatures to avoid contamination. For example, a NEXRAD level 3 grid with a 3 km beam width has an atmospheric temperature variability of more than 15 K from the beam bottom to the beam top. Therefore, hydrometeor classes detected by the NEXRAD level 3 product are likely to be contaminated specifically around the freezing level. In this
study, LR+HR+BD is chosen only when the atmospheric temperature at the top of the beam is warmer than 273 K. Similarly, DS is chosen only when the atmospheric temperature at the bottom of the beam is colder than 273 K.

Appendix C. The reliability flag of hail signals

This study proposes the reliability of hail signals on the ZKu-DFR diagram at each temperature range. The ratio of graupel and hail frequencies to the total frequency is estimated as a hit ratio $R_{hit}$ at each ZKu and the DFR grid on the ZKu-DFR diagram, at which hail is observed by the NEXRAD product at least once. Here, ZKu and the DFR are discretized by 1 and 0.5 intervals, respectively, and then the $R_{hit}$ is estimated after the $F_{ML}$ and $Fr_{10}$ filters are applied to the NEXRAD-GPM matching database during 2014 (Fig. C1).

It is found that the reliability of hail detection is almost larger than 0.9 at atmospheric temperatures colder than 10 °C. In particular, the collisional growth curve with $P = 10$ mm hr$^{-1}$ sufficiently works to determine the lower thresholds of ZKu values. This result extends the typical lower limit of hail thresholds of $Z_{Ku} = 40$dBZ above the freezing level (e.g., Lenning, 1998; Park et al., 2009; Mroz et al., 2017). $R_{hit}$ values are likely to be larger than 0.5 at DFR larger than seven at atmospheric temperatures warmer than 10 °C. One can choose an additional threshold of the DFR for hail detection to reduce contamination as usage.
References


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Fig. 1. ZKu and DFR relationships according to Mie theory assuming rain, solid ice ($\rho_h = 0.916 \text{ g m}^{-3}$), and snow with various bulk densities $\rho_s [\text{g cm}^{-3}]$. Each curve is drawn by linear interpolation of the ZKu and DFR values with precipitation rates $P = 0.1, 0.3, 0.5, 1.0, 3.0, 5.0, 10.0, 30.0, 50.0, \text{and } 100.0 [\text{mm hr}^{-1}]$. The blue, red, and green shaded regions roughly indicate snow, rain, and mixed-phase hydrometeors, respectively, detected by the linear depolarization ratio method at the Ku band in winter snow events from Liao and Meneghini (2011).

Fig. 2. The same as Fig. 1 but ZKu and DFR values from the melting layer model. The transition from snow to melted snow is shown by black lines. Melting conditions with the mass-weighted melting volume fraction $\tilde{f}$ equal to or smaller than 0.5 are shown by black dashed lines, and melting conditions with $\tilde{f}$ larger than 0.5 are shown by black solid lines. The envelop of the initial condition ($\tilde{f} = 0$) and final condition ($\tilde{f} = 1$) of the melting layer model are shown by the blue dashed line and red solid line, respectively. For reference, rain signals with the MP PSD and four typical types of PSD compiled by Tokay and Short (1996) (see Table 1 and Table 2 in the original paper for detail) are shown by orange lines.

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Fig 4. The same as Fig. 3 except for the different assumed bulk densities ($\rho_h = 0.916, 0.8, 0.6, 0.4, 0.25$, and $0.1$ [g cm$^{-3}$]).

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Fig. 8. The frequencies of dominant hydrometeor classes within GPM footprints from the NEXRAD level 3 product within the analysis range during the chosen events.

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Fig. 10. The ZKu-DFR diagram at each temperature range derived from the HG and HR subsets. The thresholds for hail detection (Eq. (5)) are superimposed by thin solid lines. The thick solid line indicates the constant density curve with \( \rho_s = 0.5 \) g cm\(^{-3} \) and the thick dashed line indicates the approximate curve.

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Fig. 11. The mean vertical profiles of the threshold ratio $R_{thr}$ for storms with various hail base temperature $T_{hb}$ ranges. Solid and dashed lines indicate the profiles in the HG and HR subsets, respectively.
Fig. 12. The dependence of the population ratio on the threshold temperature $T_{thr}$ from the HG subset (black lines) and HR subset (red lines). The population ratio is the ratio of the storm population with the heavy rain filter to the storm population without the filter.
Fig. 13. The same as Figs. 9a-9d but with the alternative solid ice curve for the hail thresholds. For reference, the original solid ice curve and rain curve with the MP-PSD are shown by dashed lines and red lines, respectively.
Fig. 14. Vertical profiles of (a, e) $N_{thr}$, (b, f) $P_{miss}$, (c, g) $P_h$ (black lines), the probability of hail or graupel $P_g + P_h$ (red lines), and (d, h) $P_s$ (red lines), $P_{ws}$ (green lines), and $P_r$ (black lines).
lines) from the NEXRAD and GPM-DPR matching datasets. The various line types correspond to the analysis results with various combinations of the filters or no filters. $F_{ML}$, $F_{r10}$, and $F_{r20}$ denote the melting snow filter, and the heavy rain filter with $T_{thr} = -10^\circ$C and $-20^\circ$C, respectively. For reference, the hail occurrence from the NEXRAD product is superimposed by the solid line in (a, e). The left column shows the results with the hail thresholds with default solid ice curve and the right column shows the results with the hail thresholds with the alternative solid ice curve.
Fig. 15. The flowchart of the hail detection procedures.
Fig. 16. The five-year climatology of the near-global distribution of the fraction of the DPR profiles that contain at least one hail layer in a column. The hail frequency $f_{hail}$ is scaled just for visualization. (a) Horizontal map of the hail signals with the $F_{ML}$ filter, and (c) the hail signals with the $F_{ML}$ and $F_{r10}$ filters are shown in the first column. Zonal mean values of the vertical profiles of the hail signals with the same filters are shown in the second column in the same row. The freezing level is shown by a black solid line, and the isotherms are shown by black dashed lines with 10 K intervals in (b) and (d).
Fig. 17. The same as Fig. 16 but with the $F_{ML}$ and $F_{r10}$ filters regardless of the hail base temperature.
Fig. B1. Hydrometeor classes at various elevator angles. The black solid lines indicate the radial changes in the beam widths of 1.0, 2.0, 3.0, and 4.0 km.
Fig. C1. The hit ratio on the ZKu-DFR diagram at each temperature range based on the NEXRAD-GPM matching database within 2014. The hail thresholds are superimposed by solid black lines and sample sizes of the joint histogram are listed in the labels.
Table 1. Parameters used for the hail thresholds at each atmospheric temperature range.

Table 2. Detection rate of the columns that contain melting snow contamination. The detection rate is sorted by the hail base temperature ($T_{hb}$).
Table 1. Parameters used for the hail thresholds at each atmospheric temperature range.

<table>
<thead>
<tr>
<th>Temperature range</th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
</tr>
</thead>
<tbody>
<tr>
<td>273K ≤ Tₐ</td>
<td>0.7</td>
<td>-20</td>
<td>Not used</td>
<td>10</td>
</tr>
<tr>
<td>263K ≤ Tₐ &lt; 273K</td>
<td>0.8</td>
<td>-23</td>
<td>Not used</td>
<td>11</td>
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<tr>
<td>253K ≤ Tₐ &lt; 263K</td>
<td>0.9</td>
<td>-25</td>
<td>Not used</td>
<td>12</td>
</tr>
<tr>
<td>243K ≤ Tₐ &lt; 253K</td>
<td>1.14</td>
<td>-31</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>Tₐ &lt; 243K</td>
<td>1.77</td>
<td>-46</td>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>
Table 2. Detection rate of the columns that contain melting snow contamination. The detection rate is sorted by the hail base temperature ($T_{hb}$).

<table>
<thead>
<tr>
<th>The hail base temperature range</th>
<th>HR</th>
<th>HG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$273 \text{ K} \leq T_{hb} &lt; 283 \text{ K}$</td>
<td>89.5 %</td>
<td>95.1 %</td>
</tr>
<tr>
<td>$283 \text{ K} \leq T_{hb} &lt; 293 \text{ K}$</td>
<td>22.8 %</td>
<td>16.0 %</td>
</tr>
<tr>
<td>$293 \text{ K} \leq T_{hb} &lt; 303 \text{ K}$</td>
<td>18.2 %</td>
<td>7.4 %</td>
</tr>
<tr>
<td>$303 \text{ K} \leq T_{hb}$</td>
<td>6.9 %</td>
<td>0.0 %</td>
</tr>
</tbody>
</table>