NOTE ON KITE AND KITE STRING.

(Second report).

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In my previous paper ¹⁾ on the same subject I have calculated approximately the maximum tension of the kite string for a given wind velocity, and have given an approximate formula for the kite string. At present paper I will give somewhat improved expressions for the tension and form of the string.

Suppose that the wind blows horizontally, and the kite surface which is assumed to be plane and to pass through the center of gravity of the kite makes an acute angle θ with the wind direction. Let α be the angle between the direction of wind and of the resultant wind pressure on the kite and ϕ be the inclination of the main string to the horizontal plane at the kite. Again let us assume that the direction of the resultant pressure R , that of the tension T of the main string at the kite, and that of the weight Mg of the kite lie in one plane. In the position of equilibrium these three forces cancel each other, so that

$$R \cos \alpha = T \cos \phi, \quad \checkmark$$

$$R \sin \alpha = Mg + T \sin \phi,$$

$$\begin{aligned} \cos \alpha &= \frac{x}{R} & x &= R \cos \alpha \\ \sin \phi &= \frac{y}{T} & y &= T \sin \phi \end{aligned}$$

$$\sin \alpha = \frac{y}{R}$$

1). This Journal 29th y. No 9.

$$R a \sin(\pi - \alpha + \theta) = T \{ c \sin(\psi + \theta) - b \cos(\psi + \theta) \}$$

where a is the distance of the center of pressure from the center of gravity of the kite, b the length of the perpendicular drawn to the kite plane from the connecting point of bridle strings, and c the distance of the foot of b from the center of gravity. Here, R , α and ψ are functions of θ . ψ also depends on the angle θ but the change of $\theta + \psi$ is very small. When the dependency of R , α and ψ on θ is known and wind and the weight Mg are given, then we can solve the above three equations, and get θ , ψ and T . Inversely by the close observations of θ , T and ψ we can determine R , α and ψ . In making sufficiently large number of observations with various values of θ , the relation between R , α , ψ and θ will be known.

To determine T , the maximum tension of the string, there is no direct method without an aid of a model in the channel of air current. Rather in the practical use we may determine it from the observation of the string.

The equation of the string given in my previous paper has been approximately obtained under the supposition that the wind pressure on the wire is very small compared with its weight. But by this assumption the limit of the applicability of the formula becomes very narrow, since the wind pressure is in most cases comparable to the weight of the wire and is sometimes in excess of the latter. On this account the following recalculation becomes necessary.

The equations of any plane curve formed by a string under equilibrating forces are

$$dT + F ds = 0$$

$$T \frac{ds}{R} + G ds = 0,$$

where T denotes the tension of the string, F and G are the forces acting on the string of unit length parallel and normal to the elementary length ds , and R is the radius of curvature of the plane curve.

As for the forms of F and G there are many opinions. In the previous paper I put

$$F = -\rho g \sin \theta + \nu v^2 \cos^2 \theta$$

$$G = -\rho g \cos \theta - \mu v^2 \sin^2 \theta.$$

where v is the wind velocity, ρg the weight of the string of unit length, θ the inclination of the string to the horizontal plane and ν , μ are certain coefficients. The coefficients of v^2 in these formulæ form the subject of the present discussion. The wind pressure on an inclined surface has a very complex nature to be put as a function of the wind velocity and the inclination. It depends also on the form and dimension of the surface. This several formulae, showing the relation between them, are found in text books. But it must be remarked that so-called theoretical formulæ of wind pressure on plane plates lost almost their values through the light of the recent brilliant

1). M. G. Eiffel: Installation d'un Laboratoire d'aérodynamique; Mémoires de la société des ingénieurs civils de France. Bul. de. Jan 1910.

reaserches of Eiffer ¹⁾, Maxim and others. For small and smooth surface without end such as that of steel wire as the kite string, the relation will become more simple than for finite plate, because the vortex, the main source of irregularity, finds less occasions to take place for the former than for the latter. We can suppose the effect of wind on an inclined long cylindrical surface of small thickness as a function of the inclination and really some of the formes $\mu v^2 \sin \theta$ or $\mu v^2 \sin^2 \theta$ etc. If the calculation show that the curves which are get from these suppositions differ in small amount, we are able to say that the true form of the string will be nearly the same to these curves.

For simplification we neglect the skin effect, so we put $\nu=0$. After the first supposition, the equation become

$$dT = \rho g \sin \theta \, ds; \quad T d\theta = (\rho g \cos \theta + \mu v^2 \sin^2 \theta) ds, \dots \dots (I)$$

and after the second,

$$dT = \rho g \sin \theta \, ds; \quad T d\theta = (\rho g \cos \theta + \mu v^2 \sin \theta) ds. \dots \dots (II)$$

As the first integral of (I), we have

$$T = T_0 \left(\frac{\sqrt{4a^2 + 1} + 1 - 2a \cos \theta}{\sqrt{4a^2 + 1} - 1 + 2a \cos \theta} \right)^{\frac{1}{\sqrt{1+4a^2}}} \dots \dots (I_a)$$

where T_0 is a constant. And from the (II) we get

$$T = T_0 \frac{\frac{a\theta}{1+a^2}}{(\cos \theta + a \sin \theta)^{\frac{1}{1+a^2}}} \dots \dots \dots (II_a)$$

where T_0 is the tension when $\theta=0$, and a means $\mu v^2 / \rho g$.

The second integral of (I) is

$$S = \frac{T_0}{\rho g} \int \left(\frac{\sqrt{1+4a^2} + 1 - 2a \cos \theta}{\sqrt{1+4a^2} - 1 + 2a \cos \theta} \right)^{\frac{1}{\sqrt{1+a^2}}} \frac{d\theta}{\cos \theta + a \sin \theta} + C,$$

where C is a constant.

The approximate form of S in the case in which a is very small is given in my previous paper. But a can varie nearly from zero to infinity, the most common value of it existing between $1/10$ to 5. If a is equal to zero these equations becomes that of catenary i. e.

$$T = \frac{T_0}{\cos \theta}; S = \frac{T_0}{\rho g} \operatorname{tng} \theta + C \dots \dots \dots (Ia')$$

The appearent difficulties to reduce these formula from the above are easily surmounted by the consideration that T_0/a in the equation (Ia) is finite for the limit of $a=\text{zero}$, and becomes T_0 in the equations (Ia').

For very great value of a it becoms

$$T = T_0 + \frac{1}{a} \log \operatorname{tng} \frac{\theta}{2}; S = \frac{T_0}{\rho g} \cotg \theta + S_0$$

In this case the curve becomes also a catenary but whose axis is horizontal.

Its equation is

$$x - \frac{T_0}{a} = \frac{1}{2} \left(e^{\frac{a}{T_0} y} + e^{-\frac{a}{T_0} y} \right)$$

where x is the horizontal distance and y is the height.

The form of the curve due to (Ib) in general is gained by the graphical method.

The second solution of (II) is

$$S = \frac{T_0}{\rho g} (\cos \chi)^n \frac{e^{m\theta}}{(2-n)(1-n)} \left[-m \cos^{2-n} \varphi + \right. \\ \left. - (2-n) \cos^{1-n} \varphi \sin \varphi + \frac{1}{(2-n)(1-n)} \frac{\Gamma(1+n')}{2^{n'}} \times \right. \\ \left. \sum_{S=0}^{\infty} \frac{\varepsilon_s (m \cos s\varphi + s \sin s\varphi)}{\Gamma\left(\frac{2+n'+S}{2}\right) \Gamma\left(\frac{2+n'-S}{2}\right) (m^2 + s^2)} \right] + C_2.$$

where

$$\text{tag } \chi = a \quad ; \quad m = \frac{a}{1+a^2} \quad ; \quad n = 1 + \frac{1}{1+a^2} \quad ; \quad \varphi = \theta - \chi \\ 0 \leq a \leq \infty \quad ; \quad -1 \leq m \leq 1 \quad ; \quad 1 \leq n \leq 2 \quad ; \quad 0 \leq n' = 2 - n \leq 1 \\ \varepsilon_0 = 1 \quad \& \quad \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \dots = 2 \quad C_2 = a \text{ const. and } \Gamma(x) \\ \text{is the second Eulerian function. The height } y \text{ and the horizontal distance } x \text{ are given by similar series as } S.$$

For the very small values of a these equations are reduced into those represent catenary.

For the greatest limit of a they become

$$T = T_0 e^{\frac{1}{a}}, \quad S = \frac{T_0}{\mu v^2} \log \tng \frac{\theta}{2} + S_0$$

and the equation of the curve in x and y is

$$e^{\frac{\mu v^2}{T_0} (x-x_0)} = \sin \frac{\mu v^2 (y-y_0)}{T_0}$$

where x_0 & y_0 are constants. This curve has a parabolic form.

I have calculated the values of S , y and x for the both

cases and find that they are slightly different in the corresponding state.

The detailed calculation of the above formulae will be published in the proceedings of the Tokyo Physico-mathematical society.

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