シヨードインス式微壓計に就て

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吉田理學士が此頃改良せられた微壓計に就いて（前號を見よ）其倍率が外の氣壓の變化の種類（例へば長週期の變化との短週期の變化とか）に依りてどう變るか、又附屬してある毛管はどの様に働くか又ペンの記象が實際の變化に比してどの位の位相の變化があるか等の事を知りたいと思つて計算をして見た。

細かい計算は英文の方を御覽を願ひたい、大體の計算の方針は先づ太氣の種々なる變化に就きて一々吟味する事は不可能でもあり又不必要であるからして、外氣の壓力は簡單なる調和運動をする者とした次に氣鐘の入りてある石油の運動は小さいから省略し、又器械の摩擦も小さいとして省略した氣鐘の質量、其加速度其れに對する空氣及石油の抵抗、毛管の作用等を勘定に入れた、又器械内部の空氣は皆均一なる密度と氣壓とを持つものとし、之等は時間と共に變るものとした、器械外部からの熱の導入を省略し従て空氣は内部に於て斷熱變化をなすものとした。

右の様な考へからして計算した結果次の結果を得た、

$$P = \text{外氣ノ壓力} = P_0 + \phi \sin qz \quad (\text{但シ } \phi \text{ ハ } P_0 \text{ ニ對シテ小ナリ})$$

p = 内氣ノ壓力 器内ニテハ各部一様トス 時間ト共ニ變ズ

m = 氣鐘及附屬品ノ質量但シ全部一様ノ早サニテ動クモノニシテ常數ナリ

S = 氣鐘ノ水平切斷面積ノ常數

V = 内外氣壓ガ平均シ内外石油面ノ全ク一致シタル時ノ器内ノ空氣ノ總體積、之ハ

常數ナリ

r = 氣鐘ノ運動ニ對スル抵抗係數、但シ抵抗ハ速度ニ比例スルモノトス

$k = \frac{\pi}{8\eta'} \frac{\lambda^4}{l}$ 但シ π ハ圓周率 η' ハ空氣ノ粘度、 λ ハ毛管ノ半徑、 l ハ其長サ、

$\gamma = 1.41$ 即空氣ノ兩比熱ノ比ナリ、

と置かば

$$\xi = \Xi \sin(qt + \varphi)$$

とあらば但し

$$\Xi = \frac{V\phi q}{r} \frac{1}{\sqrt{-N}}$$

$$-N = q^4 \left(\frac{rV}{S_r} + \frac{KP_o m}{S} \right)^2 + \left\{ qP_o \left(\frac{N^r}{S} + S \right) - lq^2 \frac{mV}{S_r} \right\}^2$$

$$\tan \varphi = \frac{q^2 \left(\frac{rV}{S_r} + \frac{KP_o m}{S} \right)}{-qP_o \left(\frac{K^r}{S} - S \right) + q^2 \frac{mV}{S_r}}$$

である、此結果に就いて考へると、 m, r 及 k の小さい時には振幅は

$$\eta = \frac{V\phi}{rSP_0} \left\{ 1 - \frac{Kr}{S^2} + \frac{q^2mV}{S^2P_0r} - \frac{1}{2} \frac{q^2}{P_0S^2} \left(\frac{rV}{Sr} - \frac{KP_0m}{S} \right)^2 \right\}$$

となる、即主要項は $\frac{V\phi}{rSP_0}$ である、之れは全く別の簡單なる計算より出したものと一致する、若し變化が等溫變化であるなれば此項は $V\phi/SP_0$ となる、

もし K の自乗又は其積が m に對して非常に小なるならば振幅は近似的に

$$\eta = \frac{V\phi}{rSP_0} \left\{ 1 + \frac{q^2mV}{S^2P_0r} \right\}$$

となる、即振幅は刺撃たる太氣壓力の變化の振動數に比例する處の q なる常數を含む故に器械の倍率は上の第二項が一に比して全然小ならざる限りは外氣變動の週期に關して變ずる事になる、此影響を除くには m を小さくするより外はない、何となれば V を小さくし又は S を大きくすれば倍率が惡くなり、 P_0 及 r は一定の値を持ち q は時によりて變る者である、故に m を小さくするより外に方法はない、即氣鐘はアルミニウムか何かでなる可く輕くするが宜敷と思ふ、

次に太氣の非常に徐々たる變動では q が小さいからして此場合に振幅は略

$$\eta = \frac{V\phi}{rSP_0} \frac{1}{1 + \frac{Kr}{S^2}}$$

となる、 K が大きくなればそれに從て振幅はいくらでも小さくなる、即毛管は長週期の變化を取り除く役

をして居る事が解る、

玆にKは r と積になりてあるからして、此毛管の影響は氣鐘に對して空氣や石油やの抵抗が多い程善く
きく。

次に非常に早い氣壓變化に對しては

$$\Omega = \frac{S\phi}{q^2m}$$

となる、之に依りて見れば或程度以上に早い變化は記録し難い事が解る、但し之れはどんな器にも起る
事で瞬間的の變動に對しては氣鐘の動く間がないからである、

次にmが左程小ならざる時には

$$\Omega = \frac{V\phi}{r} \frac{1}{P_0 S \sim \frac{q^2 m V}{S_r}} \left\{ 1 - \frac{P_0 K_r}{S^2 P_0 - q^2 \frac{m V}{r}} - \frac{1}{2} q^2 \left(\frac{r V + K P_0 m r}{P_0 S^2 r - q^2 m V} \right)^2 \right\}$$

となるよりて器械の構造に於て

$$P_0 S - \frac{q^2 m V}{S_r}$$

が零に近い様な事を避けなければならぬ、此れが零になる場合には r 及Kが小さい時には振幅は非常に
大きくなり其變化が急激であるからバログラフの用をしなくなる、

次に週期はK及 r の入りたる項が分子に入りある故に之等の小なる間は小である、但し q が全體に含ま

れてあるからして此方は倍率の様に略一定の價を取る事が不能で變動の種類を異にすれば變はる、故に器械を善くするには出来る丈此れを小さくすれば宜敷い此爲には r 及 m を小さくすればよい、

以上によりて構造上注意す可き點は出来る丈氣鐘及附屬品を軽くする事である、又 K は目的に應じて適當なる者を用ゆる事が出来る r も小さくしたがい宜敷い、

大正元年八月十七日夜の上田強震に就きて

理學博士 大 森 房 吉

緒言、八月十七日夜の上田地震は長野、松本にては軽く感じたるも、東京にては無感覺なりき、尤も微動計觀測に依るに東京發震時は午後十一時二十一分五十四秒にして震動の總繼續時間は約十五分に及べり然るに大なる破壊的地震の場合には微動計記象上、總繼續時間が一時間以上數時間に亘るを常とするものなるが、此の上田地震の繼續時間が比較的甚だ短なりしは其局部的性質の現象なるを示せるなり「長野は東京よりも震源に近きを以て發震時も稍々早くして同地方測候所の測定に依れば午後十一時二十一分三十二秒なりき。

震源の位置、震動は上田町に於て最も強かりしが、其れとても破壊的程度には達せず、微少なる土地の裂罅、器物の轉落、屋壁の破損等を生ぜるに過ぎず、而して石碑類が震動の爲めに移動せるは多くは北微

ON THE THEORY OF THE SHAW-DINES MICROBAROGRAPH.

BY

S. FUJIWHARA.

1. *Introduction.*—In the theory of their microbarograph, Dr. Shaw and Mr. Dines propounded the statical theory, assuming that in the closed vessel containing air the isothermal change of air takes place.⁽¹⁾

One of the most important conditions in constructing this microbarograph is that the vessel must be enclosed in non-conducting walls for heat, since the temperature variation of air within, due to introduction of heat from the outwards, seriously affects the air-pressure in the vessel, and makes the instrument almost useless. Another important point is that, it has a construction to record the comparatively rapid variation of the atmospheric pressure, which will cause an adiabatic change of the air in the vessel. This consideration leads me to develop a dynamical and adiabatic theory of the microbarograph.

2. *Construction.*—The construction of the instrument is shown diagrammatically in fig. 1. A is an air vessel with a comparatively large volume. The walls of A are protected from the heat invasion from the outwards with some nonconductive materials of heat. T_1 and T_2 are tubes of comparatively great diameter. c_1 c_2 is a capillary tube which serves to obliterate the effect of the slow, but not small variations of the atmospheric

(1) W. N. Shaw and W. H. Dines: The study of the Minor fluctuations of atmospheric pressure. Quarterly Journal of the Royal Met. Soci. January, 1905. p. 39—52.

then we have, as the equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,$$

or if we assume that the velocity potential exists, then the equation becomes

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \dots \dots \dots (2)$$

Suppose that the apparatus is initially in the state of equilibrium, so that there is no movement of the liquid as well as the belljar at $t=0$, and the internal and external surface of the liquid are in the same level at that moment *i. e.* we have as the initial conditions.

$$\xi = \eta = \zeta = 0, u = v = w = 0 \quad \text{at } t = 0, \dots \dots \dots (3)$$

where η and ζ denote the heights of the surface of the liquid out and in the bell.

As the boundary conditions we have

$$u = v = w = 0 \quad \text{at } r = R, \dots \dots \dots (4)$$

that is on the wall of the vessel ; (r denotes the radial distance from the center of the Belljar) ;

$$u = v = w = 0 \quad \text{at } z = d \dots \dots \dots (5)$$

i. e. at the bottom of the vessel ;

$$u = v = w = 0 \quad \text{at } r = c \dots \dots \dots (6)$$

i. e. at the wall of the pipe T_2

$$u = v = w = 0 \quad \text{at } r = a, \frac{\eta}{\zeta} > z > \xi + b \dots \dots \dots (7)$$

i. e. on the wall of the belljar ;

$$w = \frac{d\eta}{dt}, \quad \frac{P}{\rho'} = -g\eta - \frac{\partial \phi}{\partial t} - \frac{1}{2}w^2 \dots \dots \dots (8)$$

at $a < r < R, z = \eta$ *i. e.* at the external free surface of the liquid (where ρ' denotes the density of the liquid) and

$$w = \frac{d\zeta}{dt}, \quad \frac{p}{\rho'} = -g\zeta - \frac{\partial\phi}{\partial t} - \frac{1}{2}w^2 \dots\dots\dots (9)$$

at $c \leq r \leq a$, $z = \zeta$ i. e. at the inner free surface of the liquid.

The equation of motion for the liquid is

$$\frac{p'}{\rho'} = -gz - \frac{\partial\phi}{\partial t} - \frac{1}{2}U^2, \dots\dots\dots (12)$$

where p' and U denote the pressure in and the velocity of the liquid.

The total mass M of the air enclosed in the instrument i. e. in the jar, tubes and in the air vessel may be put as

$$M = \rho \{V - S(\xi + \zeta)\}$$

where ρ is the density of air within, which may be approximately assumed to be uniform and V denotes the whole volume of air within, in the initial state. We shall have, by differentiating the above formula,

$$dM = d\rho \{V - S(\xi + \zeta)\} - \rho S(d\xi + d\zeta) \dots\dots\dots (13)$$

The mass of air coming into the instrument through the capillary tube c_1 c_2 in a time interval dt is, according to O. E. Meyer

$$k(P - p)\rho dt, \dots\dots\dots (14)$$

where
$$k = \frac{\pi}{8\eta^4} \frac{\lambda^4}{l},$$

in which η^4 is the viscosity of air, λ the radius of the capillary tube and l is its length.

Hence we shall have by equating (13) and (14)

$$d\rho \{V - S(\xi + \zeta)\} - \rho S(d\xi + d\zeta) = \kappa(P - p)\rho dt \dots (15)$$

If the air undergo the adiabatic change then

$$\frac{dp}{p} = \gamma \frac{d\rho}{\rho}, \dots\dots\dots (16)$$

where γ is the ratio of the two specific heats of air.

But if it be the isothermal change, then

$$\frac{dp}{p} = \frac{d\rho}{\rho} \dots\dots\dots (17)$$

I shall improve the equation (15) in near future,
applying the theory of resonators.

4. *Statical theory.*—Since the acceleration, resistance and friction of the belljar are very small we may assume, as the first approximation,

$$P=p \text{ or } dP=dp \dots\dots\dots (18)$$

In this case we have to neglect the effect of the capillary tube and to put $\kappa=0$. Then from (8), (9) and (18), we get $\eta=\zeta=0$, and hence there is no sensible motion in the liquid. Thus equation (15) becomes

$$d\rho\{V-\xi S\} - \rho S d\xi = 0$$

When the change goes on adiabatically then we have, by introducing (16) into the above equation,

$$\frac{1}{r} \frac{dp}{p} = \frac{S d\xi}{V - \xi S}.$$

If we neglect ξS comparing with V , then

$$d\xi = \frac{1}{r} \frac{V}{S} \frac{dp}{p} \dots\dots\dots (19)$$

On the contrary, if the change be isothermal, we shall have

$$\frac{dp}{p} = \frac{S d\xi}{V - \xi S}$$

or as in the former case,

$$d\xi = \frac{V}{S} \frac{dp}{p} \dots\dots\dots (20)$$

This result agrees with that obtained by⁽¹⁾ Dr. Shaw and

Dines.

5. *Dynamical Theory*.—As the second approximation we shall retain m , r and κ , but shall neglect the friction and the motion of the liquid in the vessel. The equations becomes in this case

$$m \frac{d^2 \xi}{dt^2} = (P - p)S - r \frac{d\xi}{dt},$$

$$d\rho(V - \xi S) - \rho S d\xi = k\rho(P - p)dt.$$

If the change be adiabatic, then

$$\frac{d\rho}{\rho} = \frac{1}{r} \frac{dp}{p}.$$

Eliminating p and ρ between these equations we have

$$\frac{1}{r} \left(\frac{dP}{dt} - \frac{r}{S} \frac{d^2 \xi}{dt^2} - \frac{m}{S} \frac{d^3 \xi}{dt^3} \right) (V - \xi S) - S \frac{d\xi}{dt} \left(P - \frac{r}{S} \frac{d\xi}{dt} - \frac{m}{S} \frac{d^2 \xi}{dt^2} \right) =$$

$$= \kappa \left(\frac{m}{S} \frac{d^2 \xi}{dt^2} + \frac{r}{S} \frac{d\xi}{dt} \right) \left(P - \frac{r}{S} \frac{d\xi}{dt} - \frac{m}{S} \frac{d^2 \xi}{dt^2} \right)$$

Since $\xi, \frac{d\xi}{dt}, \frac{d^2 \xi}{dt^2}, \frac{d^3 \xi}{dt^3}$ are small, we may safely neglect the squares and products of these terms, and then we get

$$\frac{V}{r} \left(\frac{dP}{dt} - \frac{r}{S} \frac{d^2 \xi}{dt^2} - \frac{m}{S} \frac{d^3 \xi}{dt^3} \right) - SP \frac{d\xi}{dt} =$$

$$= \kappa P \left(\frac{r}{S} \frac{d\xi}{dt} + \frac{m}{S} \frac{d^2 \xi}{dt^2} \right) \dots \dots \dots (21)$$

Suppose that the barometric change which takes place in the outside of the apparatus is of a simple harmonic type, then we have

$$P = P_0 + \phi \sin qt, \dots \dots \dots (22)$$

in which ϕ is very small compared with P_0 .

Again neglecting small quantities of the second order we have

$$\frac{mV}{S_r} \frac{d^3 \xi}{dt^3} + \left(\frac{rV}{S_r} + \frac{kP_0 m}{S} \right) \frac{d^2 \xi}{dt^2} + \left(\frac{kr}{S} + S \right) P_0 \frac{d\xi}{dt} = \frac{V}{r} q \phi \cos qt.$$

Assume

$$\xi = A \sin qt + B \cos qt,$$

where A and B are constants to be determined.

Introduce this into the above equation, and equate coefficients of $\sin qt$ and $\cos qt$, then we have

$$\begin{aligned} Bq^3 \frac{mV}{S\gamma} - Aq^2 \left(\frac{rV}{S\gamma} + \frac{kP_0 m}{S} \right) - Bq \left(\frac{kr}{S} + S \right) P_0 &= 0; \\ -Aq^3 \frac{mV}{S\gamma} - Bq^2 \left(\frac{rV}{S\gamma} + \frac{kP_0 m}{S} \right) + Aq \left(\frac{kr}{S} + S \right) P_0 &= \frac{V\phi}{\gamma} q. \end{aligned}$$

From these we get

$$A = \frac{M_1}{N} \frac{V\phi q}{\gamma},$$

$$B = \frac{M_2}{N} \frac{V\phi q}{\gamma},$$

where

$$M_1 = -qP_0 \left(\frac{kr}{S} + S \right) + q^3 \frac{mV}{S\gamma},$$

$$M_2 = q^2 \left(\frac{rV}{S\gamma} + \frac{kP_0 m}{S} \right),$$

$$N = -q^4 \left(\frac{rV}{S\gamma} + \frac{kP_0 m}{S} \right)^2 - \left\{ qP_0 \left(\frac{kr}{S} + S \right) - q^3 \frac{mV}{S\gamma} \right\}^2.$$

If $m=r=k=0$, then we have

$$A = \frac{V\phi}{S\gamma P_0}, \quad B = 0,$$

Hence

$$\xi = \frac{V}{\gamma S P_0} \phi \sin qt,$$

$$\text{or} \quad d\xi = \frac{V}{\gamma S P_0} q \phi \cos qt \, dt = \frac{V}{\gamma S P_0} dP,$$

which coincides with the results obtained in the last paragraph.

6. *The amplitude* Ξ of the variation of ξ becomes

$$\Xi = \sqrt{A^2 + B^2} = \frac{V\phi q}{r} \frac{1}{\sqrt{-N}}.$$

If m, r and κ be small then

$$\Xi = \frac{V\phi}{rSP_0} \left\{ 1 - \frac{\kappa r}{S^2} + \frac{q^2 m V}{S^2 P_0 r} - \frac{1}{2} \frac{q^2}{P_0^2 S^2} \left(\frac{rV}{S r} - \frac{\kappa P_0 m}{S} \right)^2 \right\}.$$

If κr , r^2 and κ^2 are small compared with m , then we shall have

$$\Xi = \frac{V\phi}{rSP_0} \left\{ 1 + \frac{q^2 m V}{S^2 P_0 r} \right\}$$

Since the correction term for Ξ contains q , the multiplication of the instrument depends on the exciting disturbance in such a manner that, for the larger value of q *i.e.* for the more rapid barometric change, we shall have the greater multiplication and *vice versa*.

For a very slow barometric change, *i.e.* for a very small value of q , $-N$ becomes

$$-N = \left\{ q P_0 \left(\frac{\kappa r}{S} + S \right) \right\}^2.$$

And hence

$$\Xi = \frac{V\phi q}{r} \frac{1}{q P_0 \left(S + \frac{\kappa r}{S} \right)} = \frac{1}{r} \frac{V\phi}{S P_0} \frac{1}{\left(1 + \frac{\kappa r}{S^2} \right)}$$

This indicates clearly the part played by the capillary tube, that is, for the slow barometric change, the corresponding movement of the recording pen becomes small as κ increases, or in the other words, the less multiplication for the greater flow of air through the capillary tube. It is also seen from the above formula that the action of the capillary tube is more efficient for the greater value of r , *i.e.* damping action of the liquid to the belljar gives an assistance to the *fainting action* of the capillary tube.

On the contrary, when q is very great that is when the baro-

metric change is very rapid, then $-N$ becomes

$$\sqrt{-N} = q^2 \frac{mV}{Sr}$$

and hence

$$\Xi = \frac{V\phi q}{r} \frac{Sr}{q^2 mV} = \frac{S\phi}{q^2 m}.$$

This shows that a very rapid change of the barometric pressure can not be recorded by the instrument, which may be easily supposed by common sense.

If κ and r be small, but m not so, then

$$\Xi = \frac{V\phi}{r} \frac{1}{P_0 S \sim \frac{q^2 mV}{Sr}} \left(1 - \frac{\frac{P_0 \kappa r}{s}}{P_0 S - \frac{q^2 mV}{Sr}} - \frac{1}{2} \frac{q^2 \left(\frac{rV}{Sr} + \frac{\kappa P_0 m}{S} \right)^2}{\left(P_0 S - \frac{q^2 mV}{Sr} \right)^2} \right)$$

If

$$P_0 S \sim \frac{q^2 mV}{Sr}$$

be very near to zero, then the multiplication becomes very great, and therefore for such a barometric disturbance, whose q is very near to

$$\sqrt{\frac{PS^2 r}{mV}},$$

the barograph will record something next to nonsense. Thus in designing the instrument a proper caution must be taken to avoid common occurrence of such a case.

For $r=0$, the above equation reduces to

$$\Xi = \frac{V\phi}{r} \frac{1}{P_0 S \sim \frac{q^2 mV}{Sr}} \left(1 - \frac{1}{2} \frac{q^2 \kappa^2 m^2}{\left(S^2 - \frac{q^2 mV}{P_0 \kappa} \right)^2} \right).$$

7. The phase difference φ may be calculated by putting

$$\tan\varphi = \frac{B}{A} = \frac{q^2\left(\frac{rV}{S_r} + \frac{\kappa P_0 m}{S}\right)}{-qP_0\left(\frac{\kappa r}{S} - S\right) + q^2\frac{mV}{S_r}}$$

Since κ and r are very small, the retardation of the phase may not be striking.

If $r=0$, then

$$\tan\varphi = \frac{q\kappa P_0 m}{SP_0 + q^2\frac{mV}{S_r}},$$

and therefore the effect of the capillary tube on the phase retardation may be diminished by alleviating the belljar.

[*Remark.* If we put 1 in the places of r in all equations given above, we shall have the corresponding equations for isothermal change.],

In concluding this note, I must express my best thanks to Dr. Yoshida and to Prof. Dr. Okada. Dr. Yoshida, who has made some improvements to the Shaw-Dines microbarograph, called my attention to the work of [the theoretical side, and I took the work in hand according to the suggestion of Prof. Dr. Okada.

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