The Theory of Turbulent Diffusion in the Atmosphere* (III)

(The Theoretical Distribution of Airborne Pollution from Tall Stacks)

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— Abstract —

Applying his theory of isotropic turbulent diffusion to the practical problem of atmospheric pollution, the author studies theoretically on the distribution of airborne pollution from an elevated continuous point source, considering to some extent the non-isotropy of atmospheric turbulence. The expression obtained here is in its form similar to those of Sutton's and Lowry's but in detail different from them. The most striking difference is that the numerical parameters involved in the previous authors' results are not constant in the case of the present author and are represented as the functions of the length of the sampling period.

In the latter part of this paper, some discussions are made on the atmospheric diffusion from an instantaneous point source and furthermore some numerical examples are shown.

§ 1. Introduction

In these years several authors have devoted their effort to advance the theory of turbulent diffusion in the atmosphere by applying the similarity hypohese of turbulence set out first by Kolmogoroff, and remarkable progress has been made (Batchelor 1949, 1950, Inoue 1950, 1951, 1952). The present author treated also the turbulent diffusion of floating particles emitted from a continuous point source in the infinitely extending isotropic turbulent field, and the width of the diffusion zone was determined (1952 a, b). In the present paper, the author intends to apply his theory to the practical problem of atmospheric pollution and determine the distribution of airborne pollution from a single tall stack. For these purposes, it is needed to take two factors into consideration, which were neglected in previous studies. The one is the existence of the ground surface, at which a certain boundary condition of the concentration must be satisfied, and the other is the non-isotropy of the atmospheric turbulence, because the characteristic quantities concerning the atmospheric

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turbulence are much different from each other for vertical and horizontal directions, *e. g.* the scale of the largest turbulent element (turbulon) is of the order of 1,000 km for the latter direction and, on the other hand, is only of the order of 100 m for the former, and hence the magnitude of the maximum eddy diffusion coefficients are about 10^{10} and 10^5 c. g. s. for the horizontal and vertical directions respectively.

Now, the problem of the atmospheric pollution has been investigated by many authors from various points of view (Hewson 1951) and a number of theoretical studies have been also made. Although these theoretical results agree fairly well with the observed data, it seems that some points remain unclarified as to the effect of the length of sampling period on the concentration of pollution. One of the purposes of the present study is to discuss these points in detail in close connection with the author's previous studies on the relation between the length of the averaging time and the statistical quantities of turbulence (1952, c).

§ 2. Theoretical treatments

In the following treatments, we shall use the right angle coordinate system, in which x_1 is measured in the direction of the mean wind, x_2 across the wind, and x_3 vertically. The ground surface is expressed by $x_3=0$. The main features of the model of atmospheric turbulent field used here are as follows:

- (i) The mean flow has a constant velocity U in the whole atmosphere.
- (ii) The characteristic quantities of turbulence coincide with each other in the directions of x_1 - and x_2 -axes.
- (iii) The standard deviations of the turbulent wind velocities for each direction have equal values throughout the whole atmosphere, and their values for x_2 and x_3 components are $\langle v^2 \rangle_\infty$ and $\langle w^2 \rangle_\infty$, respectively. The symbol $\langle \rangle_\infty$ is used for denoting clearly that the quantities with this symbol are obtained, when observations are carried out during the time interval which is sufficiently larger than the life time of the largest turbulent element in that turbulent field. As will be stated later, the concentration of diffused gas or minute particles varies considerably with the length of the sampling period, even if gases or particles are emitted continuously at the same rate from the source. This fact is interpreted as follows: the atmospheric turbulence consists of turbulent elements of various dimensions superposed upon one another, the largest one may be of the order of 1,000 km and the smallest of only 1 cm. All these turbulent elements, however, can not be regarded as effective ones for all diffusion phenomena. For example, the diffusion of chimney smoke during 1 minute is chiefly controlled by small elements. Large elements, such as of the scale of 10 km or 100 km, have no influence on this diffusion phenomena and only contribute to carry these particles as the mean flow. But, as the length of sampling period increases, the scales of the elements which have effect on diffusion become larger, and ultimately cyclones and anticyclones are considered as effective elements for the global meteorological phenomena.

The relation between the length of sampling period (or averaging time) T_* and the

energy of turbulence effective for diffusion during T_* was obtained as follows (1952, b):

$$\begin{aligned} \langle u_i^2 \rangle_{T_*} &= \langle u_i^2 \rangle_{\infty} \frac{T_*}{\tau_{0i}}, & T_* &\leq \tau_{0i}, & i=1, 2 \text{ or } 3 \\ &= \langle u_i^2 \rangle_{\infty}, & T_* &\geq \tau_{0i}, \end{aligned} \quad (2.1)$$

provided Taylor-Karman's correlation function between the turbulent velocities at two points separated by the distance x is expressed by

$$\begin{aligned} R(x_i) &= 1 - \left(\frac{x_i}{L_{0i}} \right)^{2/3}, & x_i &\leq L_{0i}, \\ &= 0, & x_i &\geq L_{0i}, \end{aligned} \quad (2.2)$$

where L_{0i} and τ_{0i} are the size and life time of the largest element for x_i -direction in that turbulent field, respectively, and the relation between them is

$$L_{0i} = \langle u_i^2 \rangle_{\infty}^{1/2} \cdot \tau_{0i}. \quad (2.3)$$

Then, if we denote by t the time which elapses since a certain particle has been emitted from the source, the standard deviation of the diffused cloud during T_* is given as follows:

$$\begin{aligned} \langle X_i^2 \rangle &= \langle u_i^2 \rangle_{T_*} t^2 \left(1 - \frac{t}{3\tau_{0T_*i}} \right), & 0 &\leq t \leq \tau_{0T_*i}, \\ &= \langle u_i^2 \rangle_{T_*} \tau_{0T_*i} \left(t - \frac{1}{3}\tau_{0T_*i} \right), & t &\geq \tau_{0T_*i}, \end{aligned} \quad (2.4)$$

where the τ_{0T_*i} is the life time of the effective largest element for the length of sampling period T_* , and is approximately given by

$$\begin{aligned} \tau_{0T_*i} &= T_*, & T_* &\leq \tau_{0i}, \\ &= \tau_{0i}, & T_* &\geq \tau_{0i}, \end{aligned} \quad (2.5)$$

i. e. the life time of the effective largest element increases in proportion to T_* so much as T_* is smaller than the life time of the largest turbulent element in that turbulent field.

The Lagrangian correlation coefficient which is derived from (2.4) is

$$\begin{aligned} R(\xi) &= 1 - \left(\frac{\xi}{\tau_{0T_*i}} \right), & \xi &\leq \tau_{0T_*i}, \\ &= 0, & \xi &\geq \tau_{0T_*i}. \end{aligned}$$

Then, if the height of the point source is denoted by h and the distribution function of concentration of particles is assumed to be Gaussian, the mean concentration during the time interval T_* at (x, y, z) is obtained by considering the source and its image on the plane $z=0$, and is

$$\begin{aligned} \bar{X}(x, y, z) &= \frac{Q}{2\pi \langle Y^2 \rangle^{1/2} \langle Z^2 \rangle^{1/2} U} \exp \left(- \frac{y^2}{2 \langle Y^2 \rangle} \right) \\ &\quad \times \left[\exp \left\{ - \frac{(z-h)^2}{2 \langle Z^2 \rangle} \right\} + \exp \left\{ - \frac{(z+h)^2}{2 \langle Z^2 \rangle} \right\} \right], \end{aligned} \quad (2.6)$$

where

Q : the strength of the source, *i. e.* number of particles emitted from the source per unit time,

U : mean wind velocity

and

$$x=x_1, \quad y=x_2, \quad z=x_3.$$

It is easily seen that the equation (2.6) satisfies the boundary condition at $z=0$:

$$\partial\chi/\partial z|_{z=0}=0 \quad (2.7)$$

and the continuity equation

$$\int_0^z \int_{-\infty}^{\infty} \chi(x, y, z) U dy dz = Q. \quad (2.8)$$

When $t \ll \tau_{0T_{*y}}$ and $t \ll \tau_{0T_{*z}}$ (2.6) becomes

$$\begin{aligned} \chi(x, y, z) = & \frac{QU^2}{2\pi \langle v^2 \rangle_{T_*}^{1/2} \langle w^2 \rangle_{T_*}^{1/2} Ux^2} \exp\left(-\frac{y^2 U^2}{2 \langle v^2 \rangle_{T_*} x^2}\right) \\ & \times \left[\exp\left\{-\frac{(z-h)^2 U^2}{2 \langle w^2 \rangle_{T_*} x^2}\right\} + \exp\left\{-\frac{(z+h)^2 U^2}{2 \langle w^2 \rangle_{T_*} x^2}\right\} \right] \end{aligned} \quad (2.9)$$

With $z=0$, (2.9) gives the concentration at the surface as

$$\chi_0 = \frac{2QU^2}{2\pi \langle v^2 \rangle_{T_*}^{1/2} \langle w^2 \rangle_{T_*}^{1/2} Ux^2} \exp\left\{-\frac{U^2}{2x^2} \left(\frac{y^2}{\langle v^2 \rangle_{T_*}} + \frac{h^2}{\langle w^2 \rangle_{T_*}}\right)\right\} \quad (2.10)$$

and its maximum as

$$\chi_m = \frac{2Q}{e\pi U h^2} \left(\langle w^2 \rangle_{T_*}^{1/2} / \langle v^2 \rangle_{T_*}^{1/2} \right) \quad (2.11)$$

which occurs at a distance x_m from the source, given by

$$x_m = \frac{hU}{\sqrt{2} \langle w^2 \rangle_{T_*}^{1/2}}. \quad (2.12)$$

In order to illustrate the effect of the length of sampling period on the concentration, we shall consider the concentration at $z=y=0$ in the case of $h=0$, which is given by

$$\chi_0 = \frac{QU^2}{\pi \langle v^2 \rangle_{T_*}^{1/2} \langle w^2 \rangle_{T_*}^{1/2} Ux^2}. \quad (2.13)$$

The values of $\langle v^2 \rangle_{T_*}^{1/2}$ and $\langle w^2 \rangle_{T_*}^{1/2}$ in (2.13) vary as expressed by (2.1). Hence, the time-mean concentration considerably varies with T_* , for the fixed values of x , U and Q , according to the law:

$$\left. \begin{aligned} \chi_0 &\propto T_* & \text{for } \tau_{0y} > \tau_{0z} > T_*, \\ \chi_0 &\propto T_*^{1/2} & \text{for } \tau_{0z} < T_* < \tau_{0y}, \\ \chi_0 &\propto \text{const} & \text{for } \tau_{0z} < \tau_{0y} < T_*. \end{aligned} \right\} \quad (2.14)$$

For the diffusion of minute particles or gases from an infinite line source across the wind, we can easily obtain the following solution:

$$\chi(x, y, z) = \frac{Q}{(2\pi)^{1/2} \langle Z^2 \rangle^{1/2} U} \left[\exp \left\{ -\frac{(z-h)^2}{2 \langle Z^2 \rangle} \right\} + \exp \left\{ -\frac{(z+h)^2}{2 \langle Z^2 \rangle} \right\} \right] \quad (2.15)$$

When $t \leq \tau_{0T_*z}$, (2.15) is reduced to

$$\chi(x, y, z) = \frac{QU}{(2\pi)^{1/2} \langle w^2 \rangle_{T_*}^{1/2} Ux} \left[\exp \left\{ -\frac{(z-h)^2 U^2}{2 \langle w^2 \rangle_{T_*} x^2} \right\} + \exp \left\{ -\frac{(z+h)^2 U^2}{2 \langle w^2 \rangle_{T_*} x^2} \right\} \right] \quad (2.16)$$

§ 3. Comparison of theoretical results with observed data

Unfortunately, observed data of the concentration of smoke or gas emitted from a single source which are reliable have been scarcely obtained. Sutton (1947) derived the following conclusions from the experiments on Salisbury Plain:

(I) The concentration at any point in a continuously generated cloud is directly proportional to the strength of source.

(II) The concentration at any point from a continuous point or line source is approximately inversely proportional to the wind velocity for a given strength of source.

(III) The time-mean width of the cloud from a continuous point source is 35 m at 100 m from the source, and shows only very small variations with wind velocity.

(IV) The time-mean height of the cloud from a continuous line source is 10 m at 100 m from the source, and shows only very small variations with wind velocity.

(V) The central or peak concentration from a continuous point source decreases with distance downward (x) according to the law:

$$\text{concentration} \propto x^{-1.76}.$$

(VI) The peak concentration from an infinite across-the-wind continuous line source decreases with distance downward according to the law:

$$\text{concentration} \propto x^{-0.9}.$$

As to the case of diffusion from a top of a chimney, C. H. Bosanquet and J. L. Pearson (1936) made a theoretical investigation. These authors used a type of semi-empirical analysis to obtain expressions for the concentration, and obtained, among others, the following results:—

(VII) The concentration at ground level rises rapidly to a maximum at about 10 m of chimney height downward and then falls off more slowly to zero with increasing distance from the chimney base.

(VIII) The maximum concentration at ground level is inversely proportional to the square of the chimney height.

(IX) The maximum concentration at ground level is almost independent of the mode

of variation of the diffusion coefficient with height, and completely independent of the numerical value of the same coefficient.

We shall now show that these observational results can be reasonably explained by our theoretical results. It is first to be noted that many experimental results of the fluctuations of wind velocities show that the intensity of turbulence, *i. e.* $\langle v^2 \rangle_{T_*}^{1/2} / U$, is almost independent of the mean wind velocity U , provided the value of T_* is fixed. Then it can be seen from (2.4) or

$$\langle Y^2 \rangle = \langle v^2 \rangle_{T_*} t^2 \left(1 - \frac{t}{3\tau_{0T_*y}} \right) = \frac{\langle v^2 \rangle_{T_*}}{U^2} x^2 \left(1 - \frac{x}{3U\tau_{0T_*y}} \right),$$

that the width of the cloud from a continuous point source shows only very small variations with wind velocity. This conclusion agrees well with the experimental result (III). When $\langle Y^2 \rangle^{1/2}$ is 35 m at 100 m from the source downward, as it was obtained actually by experiment, $\langle v^2 \rangle_{T_*}^{1/2} / U$ is reasonably obtained as 0.35.

On the other hand, the relation between $\langle w^2 \rangle_{T_*}^{1/2}$ and U have been scarcely established by observations, owing to the difficulty of precise measurements of the vertical turbulent wind velocity. It may be reasonable, however, to assume, from the hypothesis of the local isotropy of the turbulence, that the same relation exists between $\langle w^2 \rangle_{T_*}^{1/2}$ and U as that between $\langle v^2 \rangle_{T_*}^{1/2}$ and U , *i. e.* that $\langle w^2 \rangle_{T_*}^{1/2} / U$ is almost independent of U . Then it is also derived from (2.4) that the time-mean height of the cloud shows also only very small variation with wind velocity, corresponding to (IV). This fact was recently confirmed also by Record (1951) experimentally and by Inoue (1952) theoretically.

When both $\langle v^2 \rangle_{T_*}^{1/2} / U$ and $\langle w^2 \rangle_{T_*}^{1/2} / U$ are independent of U , (2.9) shows that the concentration at any point is inversely proportional to U , corresponding to (II). The experimental results (I) and (VIII) agree well with the theoretical ones (2.9) and (2.11). Furthermore, theoretical results (2.9) and (2.15) show that the central concentration from a continuous point- or line-source decreases with distance downward according to the law:

$$\begin{aligned} \text{concentration} &\propto x^{-2} \\ \text{concentration} &\propto x^{-1} \end{aligned}$$

respectively. These results disagree slightly with (V) and (IV) and it is desirous to study this point more precisely, and also the results (VIII) both theoretically and experimentally.

§ 4. Comparison of the present theoretical results with other authors' ones

As is mentioned at the beginning, the problem of turbulent diffusion of suspended particles emitted from a chimney top has been treated theoretically by several authors. In this section, we shall compare our results with those of other authors' and make clear the different points between them.

In 1932, Sutton developed an equation of diffusion for a point source at the earth's surface by assigning the following expression to the Lagrangian correlation coefficient introduced by Taylor in his theory of diffusion by continuous movements:

$$R(\xi) = \left(\frac{\nu}{-\nu + \langle u^2 \rangle \xi} \right)^n. \quad (4.1)$$

He extended in 1947 his treatment to cover an elevated point source. The general expression derived by him is

$$\chi = \frac{Q \exp\left(-\frac{y^2}{C_y^2 x^{2-n}}\right)}{\pi C_y C_z U x^{2-n}} \left[\exp\left\{-\frac{(z-h)^2}{C_z^2 x^{2-n}}\right\} + \exp\left\{-\frac{(z+h)^2}{C_z^2 x^{2-n}}\right\} \right], \quad (4.2)$$

where z is distance upward; C_y and C_z are virtual diffusion coefficients for the across the wind and vertical directions, respectively, given by

$$\left. \begin{aligned} C_y^2 &= \frac{4\nu^n}{(1-n)(2-n)U^n} \left(\frac{\langle v^2 \rangle}{U^2} \right)^{1-n}, \\ C_z^2 &= \frac{4\nu^n}{(1-n)(2-n)U^n} \left(\frac{\langle w^2 \rangle}{U^2} \right)^{1-n}, \end{aligned} \right\} \quad (4.3)$$

where n is a numerical parameter obtained by fitting the theoretical profile

$$u = u_1 z^{n/(2-n)} \quad (4.4)$$

to the observed change of wind speed with height.

With $z=0$, (4.2) gives the concentration at the surface as

$$\chi_0 = \frac{2Q}{\pi C_y C_z U x^{2-n}} \exp\left\{-\frac{1}{x^{2-n}} \left(\frac{y^2}{C_y^2} + \frac{z^2}{C_z^2} \right)\right\} \quad (4.5)$$

and its maximum as

$$\chi_m = \frac{2Q}{e\pi U h^2} \left(\frac{C_z}{C_y} \right), \quad (4.6)$$

which occurs at a distance x from the source, given by

$$x_m = (h^2 / C_z^2)^{1/(2-n)}. \quad (4.7)$$

On the basis of meteorological data, Sutton gives values of C_y varying from 0.21 for a source at the surface to 0.07 for the source at 100 m and values of C_z from 0.12 to 0.07 for the same range of heights of source; for the sources of the height 25 m and higher the values of C_y and C_z are the same. These values have been checked of their validity through measurements of average concentrations at fixed points for a period not less than three minutes.

In 1951, Lowry has given an equation for maximum surface concentrations as follows:

$$\bar{\chi}_m = \frac{2Q}{e\pi h^2} \left(\frac{a_m}{U} \right), \quad (4.8)$$

where $\bar{X}_m$ is the time-mean concentration at a fixed point for a period of one hour, a_m is the frequency during an hour of the most frequent wind direction at the top of the stack.

In our treatment, we assumed that the mean wind velocity is independent of the height, similarly with the case of Sutton. The value of n in (4.4) is therefore zero. Then, the comparison of our result (2.9) with that of Sutton's (4.2) brings out the fact that expressions are exactly similar, provided

$$C_y = \frac{\sqrt{2} \langle v^2 \rangle^{1/2} T_*}{U}, \quad C_z = \frac{\sqrt{2} \langle w^2 \rangle^{1/2} T_*}{U}. \quad (4.9)$$

Furthermore, comparison of (2.11) and (4.6) with (4.8) yields that a_m introduced by Lowry is

$$a_m = \frac{C_z}{C_y} = \frac{\langle w^2 \rangle^{1/2} T_*}{\langle v^2 \rangle^{1/2} T_*}. \quad (4.10)$$

It seems to the present author that one of the most important points in these authors' researches exists in the fact that Lowry finds from his experiments that $a_m < 1$, although Sutton assumed $C_y = C_z$ for the source higher than 25 meters. This circumstance may be reasonably explained by (4.10), *i. e.* Sutton derived his conclusions from the measurement of average concentration for a period of several minutes. For the diffusion during such a small time interval T_* and at a considerable height, the assumption of isotropy of turbulence may be reasonable. If T_* becomes as long as an hour, as in the case of Lowry, $\langle w^2 \rangle^{1/2} T_*$ reaches its ultimate value of $\langle w^2 \rangle_\infty^{1/2}$. The value of $\langle v^2 \rangle^{1/2} T_*$, however, increases according to the law of (2.1) and becomes larger than $\langle w^2 \rangle_\infty^{1/2}$. For such a value of T_* , it holds that $a_m < 1$.

Furthermore, it is to be remarked that Lowry stated that, in the light of measured concentrations at Brookhaven National Laboratory, Sutton's equation gives concentrations which are too high if a time mean of an hour is used. Summarizing these facts, Hewson (1951) stated as follows: "When, under average conditions, a general estimate of the mean concentration for several minutes is sufficient, either Bosanquet and Pearson's equation or Sutton's expressions will be adequate.—When mean concentrations at a point for longer period are required, Lowry's expression will be more reliable." Further he stated, emphasizing the important role of the length of sampling period in the measurement of concentration, "The work of Lowry and Barad at Brookhaven indicates that the magnitude of time-mean concentrations is a function of the length of the sampling period, and that such time mean concentration have limited significance unless the periods are specified." According to our theoretical results, this relation between the length of sampling period and the concentration is expressed by (2.13), or in words, the magnitude of the concentration decreases with the length of sampling to the law (2.13), even if the particles are emitted continuously with the equal rate and the concentration is measured at a fixed point. This fact is closely connected with another one that the diffusion angle at the source increases

with T_* , as it was discussed previously in the other paper (1952, b) *i. e.* since particles are diffused into larger and larger regions, as T_* increases, the concentration measured at a fixed point decreases. It seems to the author very desirous to check the validity of the relation (2.13) by precise measurements of concentrations.

§ 5. Numerical example

In order to illustrate more clearly the feature of the diffusion described above, we shall now show a numerical example in the case of the diffusion from a continuous point source on the ground surface, using the following values of quantities characterizing the turbulent field:

$$\begin{aligned} \langle w^2 \rangle_{\infty}^{1/2} &= 5 \text{ cm/sec}, & \tau_{0z} &= 2 \times 10^3 \text{ sec} = 33 \text{ min}, \\ \langle v^2 \rangle_{\infty}^{1/2} &= 106 \text{ cm/sec}, & \tau_{0y} &= 10^6 \text{ sec} = 11.5 \text{ day}, \\ U &= 5 \text{ m/sec}, & Q &= 1/\text{sec}. \end{aligned}$$

Then the scales of the largest turbulent element defined by $L_{0y} = \langle v^2 \rangle_{\infty}^{1/2} \tau_{0y}$ and $L_{0z} = \langle w^2 \rangle_{\infty}^{1/2} \tau_{0z}$ are

$$L_{0y} = 10^8 \text{ cm} = 1,000 \text{ km}, \quad L_{0z} = 10^4 \text{ cm} = 100 \text{ m},$$

and the magnitude of the eddy diffusion coefficient defined by $K_{0y} = L_{0y} \langle v^2 \rangle_{\infty}^{1/2}$ and $K_{0z} = L_{0z} \langle w^2 \rangle_{\infty}^{1/2}$ are

$$K_{0y} = 10^{10} \text{ c. g. s.}, \quad K_{0z} = 10^4 \text{ c. g. s.}$$

Figures 1~3 show the relation between t and $\langle Y^2 \rangle^{1/2}$ or $\langle Z^2 \rangle^{1/2}$ for the cases of $T_* = 5 \text{ min}$, 1 hour and 10 hours, respectively, calculated by means of (2.4). In the case of $T_* = 5 \text{ min}$, it holds approximately that $\langle Y^2 \rangle^{1/2} = \langle Z^2 \rangle^{1/2}$, hence this turbulent field can be regarded as isotropic for the diffusion phenomena during

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

such a short time interval as 5 min. As T_* increases, the difference between $\langle Y^2 \rangle^{1/2}$ and $\langle Z^2 \rangle^{1/2}$ becomes more and more striking. Figures 5~7 show the concentration versus height at $y=0$ and a certain fixed point of x , calculated from (2.9). Figure 4 shows the relation between T_* and concentration at $y=0$ and $x=900$ m (or $t=3$ min), which illustrates more clearly the relation between the length of sampling period and the concentration.

§ 6. The case of instantaneous point source

We have hitherto confined ourselves to the case of diffusion from a continuous point source. We shall now consider the case of instantaneous point source, *i. e.* diffusion of the clouds of particles which were concentrated at the origin of diffusion at the initial time. It is also assumed, after Frenkiel (1949), that these particles will be distributed in the cloud according to the Gaussian distribution function:

$$\chi = \frac{Q}{(2\pi)^{3/2} \langle X^2 \rangle^{1/2} \langle Y^2 \rangle^{1/2} \langle Z^2 \rangle^{1/2}} \exp \left\{ -\frac{1}{2} \left(\frac{x^2}{\langle X^2 \rangle} + \frac{y^2}{\langle Y^2 \rangle} + \frac{z^2}{\langle Z^2 \rangle} \right) \right\}, \quad (6.1)$$

where Q is the number of diffused particles, and x, y, z are measured from the centre of the cloud. The standard deviations $\langle X^2 \rangle$, $\langle Y^2 \rangle$ and $\langle Z^2 \rangle$ are obtained by replacing T_* in (2.4) by t , *i. e.*

$$\left. \begin{aligned} \langle X_i^2 \rangle &= \frac{2}{3} \langle u_i^2 \rangle_{\infty} \frac{t^3}{\tau_{0i}}, & t \leq \tau_{0i}, \\ &= \langle u_i^2 \rangle_{\infty} \tau_{0i} \left(t - \frac{1}{3} \tau_{0i} \right), & t \geq \tau_{0i}. \end{aligned} \right\} \quad (6.2)$$

Figure 8 shows the calculated results of time change of the standard deviations, using the values of $\langle u_i^2 \rangle_{\infty}$ and τ_{0i} adopted in § 5.

§ 7. Conclusion

The author investigated in this paper theoretically the turbulent diffusion of smoke or gases emitted continuously from an elevated point source and obtained the mathematical expression for the concentration which agrees fairly well with the observed data. The effect of the length of sampling period on this concentration was also made clear. But it is to be remarked that this treatment is carried out under a number of simplifications or assumptions, *e. g.* it has not taken into consideration the change of the mean wind velocity U with height, and therefore the effect of the thermal stability of the atmosphere on the concentration appears to be only an implicit one as the determining factor of the magnitude of vertical turbulent velocity $\langle w^2 \rangle^{1/2}$. Furthermore, the author assumed that the non-isotropy of the atmospheric turbulence can be taken into consideration only by using the different values for the characteristic quantities of horizontal

Fig. 8.

and vertical turbulence. It is obvious that this assumption is very unsatisfactory and that more complete treatments should be taken, and these points will be investigated in the future.

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