On the Relation between Solar Incidence and Cloud Coverage*

By K. Miyakoda

Geophysical Institute, Tokyo University

(Manuscript received 21 Jan. 1957)

Abstract

A computing formula of the intensity of insolation which reaches the horizontal area on the earth's surface is presented, where the shielding effect of cloud, the absorbing and scattering effect of water vapour and air molecules, and the variation of the sun's zenith distance are considered. In order to calculate the contribution of cloud coverage, two layer model of cloud is used, and 0.44 and 0.36 are adopted as mean albedos of the lower and the upper cloud layers, respectively. For the determination of the transmission coefficient due to water vapour and air molecules, Kimball's graph is used. As the result, it is proved that the introduction of this effect improves the result considerably.

1. Introduction

With the rapid development of the technique of numerical weather prediction, the knowledge on heat supplied to the atmosphere is urgently required from the point of view of the long-range forecasting. It is well known that there are several ways in heat supply to or removal from the atmosphere, *i.e.*, the absorption of insolation in water vapour and air molecules, the divergent flux of long wave radiation, the eddy transfer of sensible heat, and the release of latent energy by condensation.

Concerning the two former problems, much have been investigated, and a high accuracy of computation technique has been obtained. But these discussions are mostly confined to the case of cloudless sky. It is obvious that the cloud coverage plays an important role in trapping the beam intensity in either the long wave or the short wave radiation. In this meaning, it is desired to get informations how and in what degree the cloud shields the transmitted energy.

Concerning the third problem, *i.e.*, the eddy transfer of sensible heat, many authors investigated it in lieu of the diurnal variation of air temperature near the ground. According to the results obtained, the quantity of sensible heat over the land is closely related

to the intensity of solar incidence which has reached the earth's surface, though the quantity over the sea does not so largely depend on it. At any rate, for the forecasting of the sensible heat, it is necessary to determine definitely the intensity of insolation on the surface. In this case also, the cloud coverage is a predominant element for its determination.

Thus, in total, in order to get the quantity of heat supply to the atmosphere except the released heat by condensation, the knowledge on the shielding effect by cloud is ultimately required. One way of approaching to this purpose is to get a computing scheme of the intensity of insolation on the ground including the contribution of cloud coverage. Then, if this attempt is achieved, it would also become feasible to estimate the absorbed quantity of the short wave radiation in the upper atmosphere.

On this problem, Kimball [5] made a research in 1919, and proposed an experimental formula as follows

$$I = I_0 (1 - 0.78N), \quad (1.1)$$

where I is the intensity of solar incidence, I_0 the same quantity when the sky is clear and N the total cloudiness. Later on, Ångström [2] obtained 0.765 instead of 0.78 in (1.1) from the observation at Stockholm, and Haurwitz

* Division of Meteorology, Contribution No. 100.

[2] 0.78 or 0.70 at Bule Hill, Mosby [2] 0.56. for the Arctic and Fritz [2] 0.61 in the United States.

The present author also used this formula and examined its accuracy of computation. As will be shown later, the result was fairly good, but not sufficiently satisfactory. Furthermore, the coefficient in the above formula, for instance, 0.78 is too large as an average value of cloud albedo. This unnaturalness probably comes from its artificial computation scheme using the total cloudiness.

It is evident that there exist two or three cloud layers in the actual atmosphere. Then, the total cloudiness is, so to speak, the projected cloud amount of those on various levels to a plane, and accordingly, the coefficient in (1.1) represents an apparent value for cloud albedo.

In these circumstances, the present author intends to attack this problem by using two layer model of clouds, for the cloud amounts on the middle levels routinely reported are available. Further, an attempt is made to get a computing scheme of solar insolation, where not only the effect above mentioned, but also the contribution of the absorption and scattering by water vapour and air molecules as well as the variation of the sun's zenith distance are taken into account. With respect to the absorption and scattering by water vapor and air molecules, much have been investigated exclusively in the term of Trübungsfaktor [7] or from the theoretical point of view. Concerning the third effect also, many investigators made research, and examined the relation between the sun's zenith distance and the observed solar intensities when the sky is clear. Thus, what are intended in this paper are to summarize these various effects, to examine the contribution of each effect to the total intensity of insolation and to compare the results thus calculated with the observed data.

2. Two Layer Model of Cloud

Cloud shows very complicated feature in form and thickness distribution. Therefore, the intensity of solar beam which comes down through the cloud layer, should also show very complicated distribution, although

the degree of complexities would be decreased a little because of the addition of diffused radiation. At any rate, it is almost impossible to treat such kind of fluctuating quantity without the detailed information of cloud form and thickness. But if we integrate even such quantity over a certain time interval, we could obtain a smooth distribution with respect to both time and space. In this meaning, the intensity of solar incidence treated herein will be confined to the averaged quantity with time.

Layer Model of Cloud

In the first place, we consider the transmission of a beam through a cloud layer of cloudiness n . Here n is taken in decatant and is defined such that, for instance, $n=1$ represents the overcast sky and $n=0$ the clear sky. Letting the beam intensity emitted from the sun be I_0 , the transmitted intensity through the broken clouds floating in the atmosphere is computed as follows. As shown in Fig. 1 the fraction of the beam which

Fig. 1. Beam transmission through cloud layer.

comes down directly through vacant space between clouds is $(1-n)I_0$, and the remaining fraction, *i.e.*, nI_0 is incident on cloud surface. Among the incident beam, some part is reflected back to space, while another penetrates into the cloud, the intensity of the latter is $(1-\alpha)nI_0$, where α is the albedo of cloud. Of the traversing portion, some fraction is absorbed in cloud droplets and the remainder continues to go downward. The intensity of the latter is $(1-\alpha)(1-\beta)nI_0$, where β is the coefficient of absorption. Therefore, in total the quantity which reaches the space beneath the cloud is

$$\begin{aligned} & (1-n)I_0 + (1-\alpha)(1-\beta)nI_0 \\ & = (1-\tau n)I_0, \end{aligned} \quad (2.1)$$

where

$$\tau \equiv \alpha + \beta - \alpha\beta \quad (2.2)$$

Two Layer Model of Cloud

We proceed our discussion to the case of two layer model, as shown in Fig. 2. In this

Fig. 2. Schematic illustration of two layer model of cloud, and transmission and reflection of beam.

figure, the quantities of the upper layer are attached by prime and those of the lower layer are not. Then the simple calculation leads to that the beam intensity transmitted through the two layers without being reflected by cloud is $(1 - \tau'\tau)(1 - \tau n)I_0$, which is indicated as the Direct Transmission in Fig. 2. If we further consider the fraction which has experienced multiple reflections, we get the following values

Direct Transmission:

$$(1 - \tau'\tau)(1 - \tau n) I_0$$

Primary Reflection:

$$(1 - \tau'\tau)(1 - \tau n) n n' \alpha \alpha' I_0$$

Secondary Reflection:

$$(1 - \tau'\tau)(1 - \tau n) (n n' \alpha \alpha')^2 I_0$$

where the quantity named as Primary Reflection is the beam which comes down to the lowermost space after being reflected twice, and the similar definitions hold for another quantities, such as Secondary Reflection, etc.

Then, we sum up them and get total intensity as follows

$$\text{Total} = I_0 \frac{(1 - \tau'\tau)(1 - \tau n)}{1 - n n' \alpha \alpha'} \quad (2.3)$$

3. Albedo of Cloud

Determination of τ and τ'

It is necessary to determine the values of τ and τ' in (2.3). But the functional form of

(2.3) is too complicated to treat it. Therefore, neglecting the contributions multiple reflections, we practically use the simple form as

$$I \propto I_0 (1 - \tau' n') (1 - \tau n). \quad (3.1)$$

As a method of determination of τ and τ' , we adopt the following technique. Bearing in mind that these values are less than unity, we divide them into ten classes, *i.e.*, 0.1, 0.2, ... 1.0. Then we take an arbitrary combination of τ and τ' , and calculate the right hand side of (3.1) for the observed cloud amounts n and n' . The quantity thus calculated is compared with the observed intensity of horizontal insolation. The comparisons are made with respect to daily series of data over a month, and then the correlation coefficient is computed. In this way, among every possible combination, we seek an optimum one, which provides the highest correlation coefficient. Speaking in more detail, there possibly exist 10^{10} combinations; for instance, (0.1, 0.1), (0.1, 0.2), (0.1, 0.3) ... (0.9, 1.0), (1.0, 1.0). It seems too labourious to compute correlation coefficients for all cases. But, in practice, we start this calculation with an arbitrary one and give a small perturbation to one of the values τ and τ' . If then a higher correlation is obtained than before, we continue to give further perturbation based on this combination newly obtained and if otherwise, we stop there.

The data used here are the hourly observations of solar incidence measured by Robitzsch pyrliometer and of cloud amounts at the lower and the middle levels for June, 1955, at Akita in Japan. As the value of n or n' in (3.1), we use the averaged value of those at the beginning and at the end of the period concerned. The cloudiness observation is not so reliable, especially that of the middle level. But we could not find more effective way than to use such routine surface observation.

The optimum values of τ and τ' thus obtained for each one hour interval during the daytime in June are shown in Table 1.

From this table, it may be concluded that: 1) the values τ or τ' are not so largely deviated from each other with respect to time, 2) the systematic variation of the values with the sun's altitude is not recognized, and

Table 1. Optimum values for γ and γ' .

Hour	γ	γ'
6~7 h	0.48	0.32
8~9	0.32	0.56
10~11	0.40	0.32
12~13	0.48	0.32
14~15	0.32	0.32
16~17	0.56	0.32
mean	0.44	0.36

3) the mean values of γ and γ' are 0.44 and 0.36, respectively. The reason that γ is larger than γ' may be that the albedo and the absorptivity of the lower cloud layer are larger compared with those of the upper one, because the former is generally thicker than the latter.

Comparison of this Result with the One obtained from Conventional Method

The correlation coefficients between the observed and the calculated intensities which are obtained from (3.1) with use of the values of γ and γ' in Table 1, are given in Table 2, where the results obtained from (1.1) are also shown for comparison.

Table 2. Correlation coefficients between the observed and the calculated intensities of incidence obtained from eq. (1.1), from eq. (3.1) (Optimum values of γ and γ' are used.) and from eq. (3.1) (Mean values of γ and γ' are used.)

Hour	From eq. (1.1) coeff. 0.78	From eq. (3.1) optimum γ and γ' used	From eq. (3.1) mean γ and γ' used
6~7 h	0.69	0.68	0.67
8~9	0.61	0.78	0.69
10~11	0.65	0.73	0.68
12~13	0.67	0.91	0.88
14~15	0.65	0.79	0.77
16~17	0.66	0.70	0.69
mean	0.66	0.77	0.73

This table reveals that eq. (3.1) gives better result than eq. (1.1) except one case for 6~

Fig. 3. Observed surface albedo of cloud vs cloud thickness (after Neiburger)

7 h, the correlation coefficient being increased, on the average, by 0.07.

It is noted here that the coefficient 0.78 in eq. (1.1) might be compared with the sum of γ and γ' , i.e., 0.44 and 0.36, in eq. (3.1).

Albedo of Cloud

According to the results theoretically obtained by Hewson [4] and others, the amount of absorption of solar radiation in cloud-droplets is rather small compared with those of reflection and transmission. From this, it follows that γ defined in (2.2) is approximately expressed as

$$\gamma = \alpha + \beta - \alpha\beta \approx \alpha \quad (3.2)$$

Therefore, the values obtained above, i.e., 0.44 and 0.36 may represent the mean albedos of cloud at the lower and the middle levels.

The cloud albedos were measured by several investigators. Namely, Hauwitz [3], Fritz [1], Luckiesh [2] and Aldrich [2] made research on the relation between albedo and cloud type, and Neiburger [8] measured the variation of cloud albedo with cloud thickness (Fig. 3). Fig. 3 shows the albedo has very wide range of values from 0 to 0.80.

But unfortunately it is impossible to get information of cloud thickness in routine observations. Therefore, we adopt the mean values, i.e., 0.44 and 0.36. In Fig. 4, calculat-

Fig. 4. An example of relation between the observed intensity of incidence I and the calculated quantity, $(1-\gamma n)(1-\gamma' n')$ for 12-13 hr June 1955 at Akita ($\gamma=0.44$ and $\gamma'=0.36$ are used).

ed quantity from (3.1) with the mean values for γ and γ' is taken in abscissa and the observed one in ordinate. In this graph, it will be seen that when the value is large the calculation gives the underestimated value, and when small it gives the overestimated one. This implies that when the intensity of incidence is large, higher value of albedo than the mean would give better result, and when small, lower value is desirable. These tendencies are, to some extent, acceptable, for in overcast sky condition the cloud would frequently be thick, and when cloudiness is small, it would be thin. In this meaning there exists some relationship between cloudiness and albedo, at least, in the extreme cases as mentioned above.

4. Depletion due to Absorption and Scattering by Water Vapour and Air Molecules

So far we have discussed only on the shielding effect by cloud layer. But there exist another diminishing factors for incoming radiation, *i.e.*, absorption and scattering due to water vapour, air molecules, dust and ozone, which have been much studied in the term of "Trückungs-faktor". Among them, what are immediately utilized using the routine observations are those by water vapour and air molecules.

Concerning the depletion due to these

factors, using the absorption coefficient obtained by Fowle, Kimball [2] [10] designed a graph which gives the transmission percentage as a function of air mass and water vapour contents. Fig. 5 in Kimball's graph, where the air mass ($=p/p_0 \sec z$, p_0 : pressure at the ground surface, p : pressure at the position concerned, z : the sun's zenith distance) is taken in abscissa, the transmission percentage (a) in ordinate, and the curves are drawn for various amounts of precipitable water (w). (Precipitable water is defined as an integration of water vapour contents from the bottom to the top of the atmosphere.) Fritz [2] further made a very convenient graph which gives directly the intensity of solar incidence, making use of Kimball's graph. For the present research, however,

Fig. 5. Kimball's graph, which gives the transmission percentage as a function of air mass and precipitable water content.

we use Kimball's graph. Concerning the informations of water vapour, we obtain them from the radiosonde data, calculating the precipitable water from Premigram designed by

Fig. 6. An example showing transmission percentage due to the absorption and scattering by water vapour and air molecules and the observed intensities of incidence on the ground surface, for 12~13 hr June 1955 at Akita.

Wadati. Since the upper air soundings are made twice a day, the precipitable water at the mid-time of the observation is computed by interpolation. The determination of the sun's altitude will be mentioned later.

In Fig. 6, the calculated results of this contribution are shown as reference. Then, we see that this contribution is not small in order of magnitude, but it is small in its daily variation compared with that of cloud coverage, and therefore, the correlation between the observed and the transmitted quantities calculated only from the information of air mass and water vapour content is not so good. But these effects should, of course, be considered in the following form as

$$I \propto I_0 a (1 - \gamma' n') (1 - \gamma n) \quad (4.1)$$

where a is the depletion coefficient due to these effects.

The results of introduction of this effect is given in Table 3 by the correlation coefficients.

Table 3. Correlation coefficients between the observed and the calculated intensities of incidence obtained from eq. (4.1).

Hour	From eq. (4.1)
6~7 h	0.72
8~9	0.75
10~11	0.70
12~13	0.90
14~15	0.82
16~17	0.81
mean	0.78

This table proves that the coefficients are increased without exception by 0.05 on the average.

5. Results

Sun's Zenith Distance

Eq. (4.1) is still in the proportional form. In order to get the complete equation, we should consider the variation of incident angle of sun's beam with time and season. On the relation between the sun's altitude and intensity of incidence, Stagg [9] reported very detailed observation. In his paper, the envelope of the plotted intensities of solar incidence against the sun's altitude represents the form of $\cos z$, where z is the sun's zenith distance. $\cos z$ is the obliqueness of the incidence to the horizontal surface. Thus, we have the total intensity which reaches the earth's surface as follows

$$I = I_0 a' \cos z (1 - \gamma' n') (1 - \gamma n) \quad (5.1)$$

where a' is the depletion coefficient due to other reasons than cloud coverage, water vapour absorption and air molecule scattering.

It is also noted that $\cos z$ is obtained from the following relation

$$\cos z = \sin \varphi \cdot \sin \delta + \cos \varphi \cdot \cos \delta \cdot \cos t, \quad (5.2)$$

where φ is the latitude, δ the sun's declination and t the sun's hour angle.

Concerning $I_0 a'$

As the value of $I_0 a'$, 114 cal/cm² hr is adopted. This means that a' is assumed constant. According to the results obtained

Fig. 7. Variation of $\cos z$ at Akita ($\phi=40^\circ\text{N}$).

and December, 1955 at Akita are shown by the correlation coefficients in Table 4. From this table, it is concluded that the coefficients are comparatively low when the sun's altitude is low. This may be ascribed to the adoption of inadequate values for cloud amounts, when the sun's altitude is low. Namely the reported cloudiness is for the cloud just above the station, while the cloud which affects the transmitted beam when the sun's altitude is low is different from it. Therefore the reported data does not always provide the information required for the present use.

so far, however, the contribution by dust shows considerable amount of variation with time, especially in the tropics [10]. Over Japan also [6], the effect of the yellow loam floating from Chinese Continent is not negligible. But since the knowledge on this effect is so meagre and besides the information on the amount of dust in the atmosphere cannot be obtained, we assume its contribution is constant.

Results

The results obtained from eq. (5.1) with respect to solar incidence for June, September

6. Averaging Time Interval

As stated at the beginning of § 2, the intensity of solar incidence we are treating with, is the averaged quantity over a certain time interval. This time scale would correspond to the period necessary for attaining the realization of the assumed model. Speaking in more detail, it is required that a part of cloud which passes over just above the observer during the time interval should stand for the characteristic sample of those of the whole sky. Accordingly, long averaging time corresponds to large sampling. In this meaning if we integrate the intensity over some time interval, we can get higher values of correlation coefficient with increase of the averaging time interval. But this tendency would cease when the realization of the assumed model is attained.

Table 4. Correlation coefficients of the observed and the calculated intensities of solar incidence for June, September and December 1955 at Akita.

Hour	June	September	December
6 ~ 7 h	0.72		
7 ~ 8	0.77	0.71	
8 ~ 9	0.75	0.69	0.62
9 ~ 10	0.85	0.65	0.76
10 ~ 11	0.70	0.71	0.72
11 ~ 12	0.86	0.77	0.65
12 ~ 13	0.90	0.84	0.69
13 ~ 14	0.89	0.81	0.72
14 ~ 15	0.82	0.82	0.73
15 ~ 16	0.79	0.70	
16 ~ 17	0.81		
17 ~ 18	0.78		
mean	0.80	0.74	0.70

Fig. 8. A veraging time interval vs the correlation coefficients between the observed and the calculated intensities of incidence.

There exists, however, another reason which increases the coefficient value. As mentioned in § 5, the correlation coefficient is high when the sun's altitude is large. Therefore, if we integrate the intensities with time, the coefficient would become larger than that of smaller sun's altitude.

In this way, there are two reasons which raise up the coefficient. These will be recognized in the curves in Fig. 8, where in abscissa the averaging time interval is taken and in ordinate the correlation coefficient. From this, we conclude some 3 hours of the time interval is suitable for calculation of

Fig. 9. Examples of comparison of the calculated intensities with the observed data for 3 hour intervals, June 1955 at Akita (a) 9~12 h, (b) 12~15 h and (c) 15~18 h.

Fig. 10. Examples of comparison of the calculated intensities with the observed data for one day, (a) June, (b) September and (c) December 1955 at Akita.

intensity based on eq. (5.1).

Thus the results of calculated intensity for 3 hours are shown in Fig. 9, and the totals for one day in Fig. 10. In the latter figure,

we see the calculated values are, in general, a little underestimated and the amplitude of variation is smaller in the calculated result than in that of observation.

7. Conclusions and Remarks

Conclusive summary and remarks are described below.

Conclusions

a) As a computing formula of solar incidence, we have

$$I = I_0 a' a \cos z (1 - \gamma' n')(1 - \gamma n),$$

where I is the intensity of insolation, I_0 the intensity of solar beam at the outer boundary of the earth's atmosphere, a the depletion coefficient due to the absorption and scattering by water vapour and air molecules, a' the depletion coefficient due to other reasons than those mentioned above, z the sun's zenith distance, $\cos z$ the effect of obliqueness of the incidence to the horizontal surface, n the cloudiness of the lower level, n' that of the middle level, γ the albedo of cloud at the lower level and γ' that at the middle level. Practically the value of $I_0 a'$ is adopted as $114 \text{ cal/cm}^2 \text{ hr}$, the value of a is obtained from Kimball's graph, and the values of γ and γ' are taken as 0.44 and 0.36, respectively.

b) The results obtained from the conventional method $I = I_0 (1 - 0.78 N)$ are compared with those from the formula mentioned above, where N is total cloudiness. Then, it is proved that the latter is superior to the former.

c) Consideration of a gives a better result than the neglect of it, and the correlation coefficients between the observed and the calculated intensities is increased by 0.05.

d) The formula mentioned above is suitable for more than 3 hour interval calculation.

e) The calculated absolute values of intensity are, in general, a little smaller than the actual ones.

Remarks

a) In order to get higher accuracy of calculation, we need the information of cloud thickness. Concerning this problem, the recently developed technique of radar observation would provide some possibility.

b) According to our experience, it is desired that the variation of the depletion coefficient with water vapour content is greater

than that given by Kimball's graph. Speaking in more detail, it is preferable that the amount of depletion due to the scattering by air molecules is smaller and the variation of amplitude of the absorption by water vapour is larger.

c) In order to forecast the intensity of solar incidence, it is necessary to predict cloudiness. But this problem is very difficult.

d) Another problem accompanying to the estimation of insolation is to make clear how and in what degree the incident energy on the earth's surface serves to increase the air temperature.

Acknowledgements

The writer wishes to express his sincere thanks to Prof. S. Syōno who initially suggested this problem to him and who has encouraged him throughout this study. Thanks are due to Messrs. S. Manabe and M. Komabayashi, in Tokyo University, who gave him many kind advices and suggestions. The data of solar intensities by Robitzsch pyrheliometer were obtained by the courtesy of Mr. Suzuki, in the section of measurement in Japan Meteorological Agency. The present author would also like to express his thanks to Prof. G. Yamamoto in Tōhoku University and to the members of Colloquium on Sun's Energy in Tokyo, who gave him many effective suggestions.

References

1. Fritz, S. (1950): Measurements of the Albedo of Clouds. *Bull. Amer. Meteor. Soc.*, **31**, 25-27.
2. Fritz, S. (1951): Solar Radiant Energy and its Modification by the Earth and its Atmosphere. *Compendium of Meteor.* 13-33.
3. Haurwitz, B. (1948): Insolation in Relation to Cloud Type. *J. Meteor.* **5**, 110-113.
4. Hewson, E. W. (1943): The Reflection, Absorption and Transmission of Solar Radiation by Fog and Cloud. *Quart. J. Roy. Meteor. Soc.* **69**, 47-62.
5. Kimball, H. H. (1919): Variations in the Total and Luminous Solar Radiation with Geophysical Position in the United States. *Mon. Wea. Rev. Wash.* **47**, 769-793.
6. Kitaoka, T. and Matsuoka, Y. (1948): *Geophys. Mag.* **15**, 5.
7. Linke, F. (1943): *Handbuch der Geophysik.*

- Berlin, Gebr. Borntraeger.
8. Neiburger, M. (1949): Reflection, Absorption and Transmission of Insolation by Stratus Clouds. *J. Meteor.* **6**, 98-104.
 9. Stagg, J. M. (1956): Solar Radiation at Kew Observatory. *Geophys. Memoirs* No. 86.
 10. Yamamoto, G. (1955): Radiation (in Japanese). Chijin Shobo, Tokyo.
-