A Numerical Experiment of the Atmospheric Radiation

By K. Takahashi*, A. Katayama** and T. Asakura*

Japan Meteorological Agency, Meteorological Research Institute***

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Abstract

Net flux of long wave radiation and its divergence in the model atmospheres were calculated numerically and tabulated. Glancing over the tables, it was found that the change of the net flux and its divergence due to the change of water vapour amount was comparatively small while the change by cloudiness was large. The values of cooling rate of the atmosphere calculated by the present authors were compared with London's calculations. It was found that both values agreed roughly in middle troposphere, but that there were differences near the earth's surface and the upper stratosphere.

1. Introduction

The role of atmospheric radiation is very important in the atmospheric heat budget. The analytical study of the atmospheric radiation, however, is quite difficult owing to the complicated effects of various factors, such as temperature distribution, cloud and water vapour. In such cases, empirical rules derived from the results of numerical calculations for various models are useful as a method of research. Phillips¹⁾ has made a research on the general circulation of the atmosphere by numerical calculations under some simplified assumptions. He called such the method of the research a numerical experiment. Similar method may be applicable to the present problem.

Of course, many investigations on the atmospheric heat budget have been made already by many authors, such as, Ångström²⁾, Baur and Phillips³⁾, Simpson⁴⁾, Houghton⁵⁾, London⁶⁾ and others. Their purposes of the calculation, however, were to found the heat budget in the real atmosphere. As the state of the real atmosphere is complex, it was not easy to inspect the effects of various factors on the radiation amount. Their researches should be called the case studies. In the present paper, the long wave radiation flux and its convergence will be calculated numerically for simplified atmospheres. And the general characters of radiation amount will be discussed.

2. Method of calculations and the adopted model atmosphere

Scheme of the numerical calculations of radiation flux and its convergence has been planned by Katayama. Here only a rough sketch of the method of calculations will be shown. Details of the calculations will be referred to his paper. The principle of the present calculation is equivalent to the graphical calculation using Yamamoto's radiation chart including the correction of CO₂. The radiation is calculated numerically for the model atmosphere divided by 20 layers of equal pressure difference, assuming that the surface pressure is 1000 mb. The calculations were done by the I.B.M. 704 electronic computer prepared in Japan Meteorological Agency.

The model atmosphere adopted in the present calculation is determined by the following assumptions.

- 1) Lapse rate in the troposphere is constant.
- 2) The stratosphere is isothermal and its temperature is decided by the surface temperature. The relationship between them is obtained by the climatological data.
- 3) The difference between atmospheric temperature and dew point is uniform. This is roughly equivalent to the relation that the relative humidity is uniform.

4) The cloud layer is assumed to be a black body. Cloud height and cloud thickness are decided by the surface temperature. The numerical values of the model atmosphere determined by the above assumptions are shown in Table 1 and in Fig. 1.

Table 1. Model atmosphere

Surface temperature		-40	-30	-20	-10	0	10	20	30°C
Pressure									
1000 mb		-40.0	-30.0	-20.0	-10.0	0.0	10.0	20.0	30.0°C
900		-38.4	-31.6	-23.0	-14.3	-4.6	4.2	14.0	23.3
800		-37.0	-33.0	-26.4	-19.2	-9.6	-1.7	7.3	16.5
700		-35.1	-34.7	-30.0	-24.2	-14.8	-7.8	0.5	9.5
600		-34.0	-36.5	-34.0	-30.0	-20.5	-14.5	-6.6	1.3
500		-34.0	-38.0	-38.0	-36.0	-27.5	-22.3	-15.0	-8.0
400		-34.0	-38.0	-41.0	-43.0	-36.0	-32.0	-25.9	-19.2
300		-34.0	-38.0	-41.0	-45.0	-46.2	-44.2	-37.7	-31.7
200		-34.0	-38.0	-41.0	-45.0	-51.0	-56.5	-54.0	-49.8
100		-34.0	-38.0	-41.0	-45.0	-51.0	-58.0	-66.0	-75.0
0		-34.0	-38.0	-41.0	-45.0	-51.0	-58.0	-66.0	-75.0
Clouds									
Low	Base	960	950	940	930	920	910	900	900mb
	Top	920	900	870	850	850	810	800	800
Middle	Base	850	810	770	740	700	670	630	600
	Top	810	760	700	660	610	570	530	500
High	Base	670	580	480	380	290	210	170	150
	Top	650	550	450	350	250	170	120	100

Fig. 1. Model atmosphere, vertical distribution of air temperature and clouds

3. Effective radiation flux at the earth's surface

In the following, the results of the calculations will be explained. At first, the effective radiation flux at the surface will be shown in Table 2, for various surface temperatures, dew-point depressions (the difference between air temperature and dew point) and clouds. Glancing over the table, the following characters are found.

Table 2. Radiation flux at the earth's surface
No cloud

Dew point depression	0	5	10	15	20°C
	ly/day				
30°C	167	190	218	238	264
20	177	199	222	246	271
10	186	206	228	250	273
0	183	202	222	242	263
-10	183	209	219	238	258
-20	168	184	201	219	238
-30	144	158	175	191	207
-40	102	114	129	144	158

Low cloud

Dew point depression	0	5	10	15	20°C
	ly/day				
30°C	39	41	42	45	46
20	35	36	38	39	41
10	30	32	33	34	35
0	21	22	23	24	24
-10	17	18	18	19	19
-20	10	10	11	11	11
-30	4	4	4	5	5
-40	-3	-3	-3	-3	-3

Middle cloud

Dew point depression	0	5	10	15	20°C
					ly/day
30°C	86	94	102	111	120
20	80	87	94	101	108
10	72	78	83	88	93
0	57	61	64	68	72
-10	49	52	54	57	60
-20	31	32	34	35	37
-30	12	13	13	14	14
-40	-10	-10	-11	-11	-11

High cloud

Dew point depression	0	5	10	15	20°C
					ly/day
30°C	150	169	188	209	231
20	151	168	186	204	222
10	148	163	177	192	207
0	123	134	144	155	166
-10	101	108	116	123	131
-20	62	66	70	74	78
-30	25	26	27	29	30
-40	-20	-21	-22	-23	-28

- 1) The change of the amount of the effective radiation is comparatively small for the change of the surface temperature. Weak maximum is found in the region between 10°C and -10°C for the cloudless sky. As the radiation from the earth's surface decreases with the decrease of the surface temperature, these results seem to be curious. But it is explained by the increase of the radiation from the atmosphere due to the increase of vapour.

The effective radiation from the surface has been studied already by many authors. Brunt⁷⁾ derived an empirical formula as follows:

$$R = \sigma T^4 (0.474 - 0.065 \sqrt{e}) \quad (1)$$

where R is the effective radiation for clear sky, σ the Stefan-Boltzmann constant, T the temperature and e the water vapour pressure in the air.

Two values of the radiation flux, one calculated from the above empirical formula and the other obtained by the present calculation, will be shown in Table 3.

Table 3. Effective radiation from the surface for clear sky

Dew point depress.	0		10		20°C	
Surface temp.	Emp.	Num.	Emp.	Num.	Emp.	Num.
						ly/day
30°C	50	167	157	218	246	264
10	189	186	238	228	278	273
-10	207	183	233	219	246	258
-30	180	144	187	175	191	207

Here Emp. expresses the values calculated by Eq. (1) and Num. that obtained by the present calculation. Both values agree comparatively well except at high temperature.

The agreement shows that the present calculations are reasonable. Disagreement at high temperature seems to be due to the inadequacy of the Brunt's formula at high temperature.

- 2) The effective radiation increases with the decrease of dew point, though its increase is small.
- 3) The effect of clouds is large. For example, when low clouds cover the sky, the effective radiation becomes 16 percent of the amount for clear sky, provided the surface temperature is 10°C. Of course, such a percentage depends on the surface temperature, but its effect is not large.

The calculated values of the radiation flux are also checked in the following way. The decrease of the effective radiation is estimated by the rate of nocturnal cooling. In Table 4, such a decrease of the effective radiation is shown for various clouds estimated by the present calculation as well as estimated from the nocturnal cooling by Asklöf, Takahashi and Liutertshtein⁸⁾.

Table 4. Percentage of the effective radiation for various sky conditions

Cloud condition	No cloud	High cloud	Middle cloud	Low cloud
				%
Numerical calculation	100	78	36	15
Asklöf	100	80	25	14
Takahashi	100	80	30	15
Liutertshtein & Chudnovskii	100	80	40~30	20~10

The percentage of the decrease of the effective radiation obtained by both methods agrees at least in the first approximation. Thus, our calculations are justified.

4. Effective radiation at the top of the atmosphere

The effective radiation flux at the top of the atmosphere, that is, the outgoing infrared radiation from the earth's atmosphere is shown in Table 5, for various surface tem-

Table 5. Radiation flux at the top of the atmosphere (Long wave).

No cloud					
Dew point depres.	0	5	10	15	20°C
Surface temp.	ly/day				
30° C	553	576	600	623	646
20	518	540	559	579	598
10	484	501	517	533	549
0	451	463	476	489	501
-10	419	427	436	445	454
-20	389	394	400	405	410
-30	362	364	366	368	370
-40	346	344	343	342	340

Low cloud					
Dew point depres.	0	5	10	15	20°C
Surface temp.	ly/day				
30° C	520	539	558	576	594
20	484	500	516	531	546
10	455	468	480	492	504
0	434	444	454	463	472
-10	397	404	411	418	425
-20	379	383	387	391	395
-30	358	359	361	362	364
-40	348	347	346	345	344

Middle cloud					
Dew point depres.	0	5	10	15	20°C
Surface temp.	ly/day				
30° C	438	448	459	470	479
20	427	436	446	454	462
10	402	410	418	426	433
0	387	394	401	407	414
-10	366	370	374	378	382
-20	357	360	362	365	367
-30	352	353	354	355	356
-40	353	352	351	351	350

High cloud					
Dew point depres.	0	5	10	15	20°C
Surface temp.	ly/day				
30° C	171	171	171	171	171
20	216	216	216	216	216
10	242	242	242	242	242
0	279	280	280	281	281
-10	304	304	304	304	304
-20	327	327	327	328	328
-30	341	341	341	342	342
-40	360	360	360	360	360

peratures, dew-points and cloud conditions. Glancing over the table, the following characters are summarized.

- 1) The outgoing radiation decreases slightly with the decrease of dew point.
- 2) The outgoing radiation decreases in most cases with the fall of surface temperature, but in case of high-cloud cover, the outgoing radiation increases with the fall of surface temperature.
- 3) The effect of cloud conditions on the amount of outgoing radiation is less than that on the radiation flux at the surface.

The present table shows that the earth can not be considered as black body for long wave radiation, if we take a surface temperature as an index of the earth's temperature. The amount of outgoing radiation R (ly/day) is expressed roughly by the following empirical formula.

$$\begin{aligned}
 R &= 468 + 3.5\theta \quad \text{no cloud} \\
 R &= 447 + 2.9\theta \quad \text{low cloud} \\
 R &= 403 + 1.6\theta \quad \text{middle cloud}
 \end{aligned} \quad (2)$$

Here θ is the surface temperature expressed in centigrade.

If we assume that the earth is black body, the outgoing radiation must be calculated by Stefan-Boltzmann equation $R = \sigma T^4$

or approximately by

$$R = \sigma T_0^4 + 4\sigma T_0^3\theta = 660 + 10.0\theta \quad (3)$$

Thus, the absolute value as well as the change of the amount of outgoing radiation due to the change of surface temperature is much larger than a real atmosphere as shown in Fig. 2.

Fig. 2. Radiation flux at the top of the atmosphere

5. Cooling rate of the whole atmosphere

The difference of the radiation flux between the top and the bottom of the atmosphere is equal to the mean cooling rate of the whole atmosphere. In Table 6, this cooling rate of the atmosphere is shown for various surface temperatures, dew-points and clouds.

Table 6. Cooling amount of the whole atmosphere

No cloud					
Dew point depres.	0	5	10	15	20°C
Surface temp.	ly/day				
30° C	386	386	382	385	380
20	341	341	337	333	327
10	298	295	289	283	276
0	268	262	254	247	238
-10	236	218	217	207	196
-20	221	210	199	186	172
-30	218	206	191	177	163
-40	244	230	214	198	182

Low cloud					
Dew point depres.	0	5	10	15	20°C
Surface temp.	ly/day				
30° C	481	498	516	531	548
20	449	464	478	492	505
10	425	436	447	458	469
0	413	422	431	439	448
-10	380	386	393	399	406
-20	369	373	376	380	384
-30	354	355	357	357	359
-40	351	350	349	348	347

6. Cooling rate of the atmosphere

Next, the cooling rate of each layer of the atmosphere will be examined. In Fig. 3 the cooling rate of every 100 mb thickness of the atmosphere is shown where the ordinate expresses the pressure level and the abscissa the surface temperature.

As the surface temperature is high in low

Middle cloud					
Dew point depres.	0	5	10	15	20°C
Surface temp.	ly/day				
30° C	352	354	357	359	359
20	347	359	352	353	354
10	330	332	335	338	340
0	330	333	337	339	342
-10	317	318	320	321	322
-20	326	328	328	330	330
-30	340	340	341	341	342
-40	363	362	362	362	361

High cloud					
Dew point depres.	0	5	10	15	20°C
Surface temp.	ly/day				
30° C	21	2	-16	-38	-60
20	65	48	30	12	-6
10	94	79	65	50	35
0	156	146	136	123	123
-10	203	196	188	181	163
-20	265	261	257	254	250
-30	336	315	314	313	312
-40	380	381	382	383	388

Fig. 3a. Cooling rate of the atmosphere for clear sky (1y/day 100 mb)

Fig. 3b. Cooling rate of the atmosphere with middle clouds (1y/day 100 mb)

latitude and low in high latitude, Fig. 3 is equivalent to the vertical south-north cross section of the cooling rate of the earth's atmosphere as shown in London's paper. Comparing both Figures, we find many resemblances, for instance, the cooling rate of about $1\sim 2^{\circ}\text{C}$ per day and the excess of cooling in middle troposphere. The present figures, however, show large cooling near the surface and in the upper stratosphere and the minimum cooling near the tropopause. These points can not be found in London's Figure.

Of course, the present results would not be applicable to an upper stratosphere, because the state of that region is too simplified in the present calculation and the present method based on radiation chart is inadequate for an precise calculation in the stratosphere. Our qualitative results, however, may be considered to be better in a finer structure of radiative cooling than London's results which were calculated graphically, hence his results may be not enough to show such details.

It will be worth to notice that the maxi-

Fig. 4. Cooling rate of the atmosphere composed of dry and wet layers (1y/day 100 mb)

mum cooling rate in the middle troposphere is much distinct in London's Figure. This discrepancy is explained by the difference of the vertical humidity distribution between our model atmosphere and the atmosphere adopted by London. To confirm this point, the following trial is made. Assuming that the atmosphere is composed of two layers, viz., one is saturated air and the other is dry air whose dew point depression is 10°C , the cooling rate is calculated. The results of the calculation are shown in Figure 4, where the ordinate expresses the height in pressure level of the boundary between wet and dry airs.

When upper layer is dry and lower layer is wet, the cooling rate is large near the boundary between the both layers. While, when upper layer is wet and lower layer is dry, the cooling rate decreases near the boundary.

As the relative humidity decreases generally in upper layer of the atmosphere, one of the reason of the distinct cooling in middle atmosphere may be attributable to the nature of humidity distribution in the atmosphere.

Lastly, the distribution of the model atmosphere, assuming the following amount of cloudiness after London, is calculated as shown in Fig. 5.

Cloud \ Temp.	Temp.						
	-30	-20	-10	0	10	20	30°C
Low cloud	30	32	33	30	30	23	27%
Middle cloud	12	13	13	12	12	11	11
High cloud	7	8	9	11	11	11	11

Fig. 5. Cooling rate of the average atmosphere

This is the better approximation to the climatological state of the real atmosphere. By comparing Fig. 3 and 5, we find that clouds shield the outgoing radiation near the earth's surface and the maximum cooling rate near the surface for the clear sky disappears. On the other hand, the maximum cooling rate in the middle troposphere becomes more distinct than in case of clear sky. The height of maximum cooling layer is nearly equal to the average height of middle and low clouds. These results are in good accordance with those obtained by London.

7. Conclusion

Here only the numerical values of radiation flux and its divergence are shown briefly. These tables may be helpful for the research of heat budget in the atmosphere. For instance, it is derived that the variation of the surface temperature due to the change of solar radiation is magnified owing to the existence of vapour amount, if we assume radiation equilibrium.

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大気熱放射の数値実験

高橋浩一郎*・片山 昭**・朝倉 正*

(気象庁*, 気象研究所**)

大気の熱放射についてはすでに多くの人が研究しているが、ここではモデル的な気温、露点温度、雲の鉛直分布を仮定し、山本の放射図を基にし、大気を 50 mb の層にわけ、電子計算機により数値計算を行つた。

その結果は地表の気温、気温と露点温度の差、雲をパラメーターとして表示した。

この表をみると、露点温度が多少変化しても放射の流れや収れんはあまり大きく変わらないが、雲の状況は非常に大きな影響があり、大気熱放射を論ずる場合には雲の模様を正しく推定することが一義的に重要なことがわかる。また、熱放射による大気の冷却の模様を調べてみると、ロンドンが計算したように大気の中層に冷却の大きいところが出るが、それ以外にも成層圏の上部、地表付近にも冷却の大きいところが出る。また、地表の気温を大気の気温の目安とするとき、地表の気温がかなり変化しても大気外への放射量はあまり変化しないことがわかる。