On the Dynamical Consideration of the Correlation Fields at 500 mb Level

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Abstract

Some problems relating to the extended-period forecasting of 500 mb heights by the correlational analysis are studied. In order to examine how the 10-day lag correlation field varies according to the various choices of data periods, an equation which governs the correlation field is derived from the vorticity equation. It is next shown that the simple linear regression method cannot seize the dynamical non-linear effect. And correlation coefficients between the terms of barotropic vorticity equation which was divided into the linear and non-linear parts, are examined.

1. Introduction

The statistical forecasting of the height of isobaric surface has been made by many authors under the stimulus of the development of the high speed electronic computer and the numerical weather forecasting. Some of them used the isobaric heights themselves as predictors (White and Palson, 1955; Magata and collaborators, 1958) and another adopted dynamical factors such as advections of vorticity and thickness as predictors (Cooley, 1958). White and collaborators (1957), and Malone (1956) used the coefficients of the orthogonal functions by which the height pattern was expanded and Lorenz (1956) adopted the coefficients of the empirical orthogonal function. Whatever we may use as predictors, there are many statistical problems with respect to these methods.

We will mention these statistical problems and explain our aims of this paper in the following.

i) It is difficult to obtain stable good correlation coefficients by using data in long period and the climatological forecasting does not give good results. In that case it is desirable to classify the data by a proper rule and to calculate the correlation coefficient by using the classified data. But it is difficult

to obtain a proper rule of classification. In this paper it will be examined how the correlation field changes with the period of data. And an equation governing the change of correlation coefficient will be derived from the vorticity equation and it will be verified how that equation can interpret the actual change.

ii) It is difficult to interpret variations of the complicated patterns of isobaric height by the linear regression method. In this paper it will be shown that the dynamical linear model and the similar one with it are the only ones which may exactly be interpreted by the simple linear regression method.

iii) What is the best predictor? There are many factors such as isobaric height, tendency of isobaric height and coefficient of the expansion by the orthogonal functions. Isobaric height is most convenient but it is not clarified which is better for forecasting, isobaric height or tendency. It is the best one as the statistical treatment to use the coefficients of the empirical orthogonal functions, but as forecasting method it is not necessarily so.

iv) Relation between the statistical method and dynamical method should be inquired. In the statistical method the correlation coefficient between some quantities has often been

calculated from the formal or empirical reasons. On the other hand Cooley (1958) used advections of vorticity and thickness as dynamical factors in the statistical forecasting, but they did not give so good results. In this paper correlation coefficients between various terms of the vorticity equation which was divided into the linear and nonlinear parts, are calculated and it is shown that the stable correlation cannot be obtained from the nonlinear term.

Generally speaking, from the above consideration it may be summarized that there are two critical problems in the statistical forecasting, one of them is stability of correlation coefficient and the other is nonlinearity. It is necessary to clarify physical and synoptic meanings of the nonlinear effect and to select some proper statistical factors.

2. Variation of correlation field

The statistical forecasting of 500 mb height at a point by using ones at several points in the vicinity of that point, will be considered. In that case correlation coefficients between 500 mb heights at those points are very important. There is pretty difference between the correlation coefficients according to the period of data. For instance 1~10 day lag coefficients between 500 mb height at 50°N, 140°E and ones at 0°E, 10°E,, 160°E along 50°N latitude circle, were calculated. Fig. 1(a) shows the correlation field calculated from the data of December 1950~February 1951 and Fig. 1(b) shows the same field calculated from the data of January 1951. As may be seen from these figures, the correlation field moves toward the west as lag τ increases. And in Fig. 1(a) the distribution of pretty large correlation coefficients moves toward the west without reducing these values but in Fig. 1(b) large values of the correlation coefficients disappear gradually as lag τ increases. In the case of $\tau=1$ both values are of the same order. Namely when the period of data is relatively long, large values of the correlation coefficients disappear as τ increases and the forecasting would be getting impossible. But when the period of data is relatively short, the large values of correlation coefficients remain large even though τ increases and the behaviour is rather systematic as may be seen from Fig.

1(a). The author examined what behaviour the correlation field shows when the period of data is relatively short and it is displaced in order of data. In that case lag was fixed 10 days. The correlation coefficients of 500 mb height at 50°N, 140°E and ones at 10°E, 20°E,, 160°E, were calculated by using the data which were displaced by five days over the range of December 1950~February 1951 and each correlation coefficient was calculated by using 20 data. Fig. 2 shows that correlation fields. From this figure the behaviour of the correlation fields may be clarified to some extent. It is necessary to know a rule of variation of the correlation field in order to be able to forecast the heights. As may be seen from Fig. 2, the east-west displacement of the peaks of the correlation coefficients appears relatively regularly during January 1951. In the next paragraph this point will be discussed correlating with the variation of the zonal current.

3. Variation equation of correlation field

The author showed that east-west variation of the correlation field correlates with the rate of propagation of the disturbance (Magata and collaborators, 1958). Thus the east-west variation of correlation field will be expected to have a relation with the general current. In this place a variation equation of correlation field will be derived from the barotropic vorticity equation. Rectangular coordinates x and y are taken, x pointing to east-west and y to south-north, and 500 mb height pattern is assumed to be expressed in the following form :

$$Z = c - \frac{f}{g} U(t)y + z(x, y, t), \quad (1)$$

where c is a constant, $z(x, y, t)$ the deviation from uniform flow pattern, f the Coriolis parameter, g the acceleration of gravity and $U(t)$ a uniform general current which is a function of time only. Substituting (1) into the barotropic vorticity equation

$$f^2 \frac{\partial Z}{\partial t} = J \left(\frac{g}{f} f^2 Z + f, Z \right), \quad (2)$$

the following equation may be obtained :

$$f^2 \frac{\partial z}{\partial t} + U(t) \frac{\partial}{\partial x} f^2 z + \beta \frac{\partial z}{\partial x} - J \left(\frac{g}{f} f^2 z, z \right) = 0, \quad (3)$$

Fig. 1. Lag correlation coefficients between 500 mb heights at 140°E and 0°E, 10°E, 20°E, ..., 160°E along the latitude 50°N. Lower diagrams correspond to the correlation coefficients in upper tables. (a) and (b) are cases calculated from the data of December 1950~February 1951 and January 1951 respectively.

Fig. 2. 10-day lag correlation coefficients between 500 mb heights at 140°E and 0°, 10°E, ..., 160°E along the latitude 50°N, calculated by using data which were displaced by five days over the range of December 1950~February 1951. Black spots show the locations of the peaks of the correlation coefficients obtained by 10-day forecasting.

where $\nabla^2 z = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$ and $J(A, B) = \frac{\partial A}{\partial x} \frac{\partial B}{\partial y} - \frac{\partial A}{\partial y} \frac{\partial B}{\partial x}$.

Now z_1 and z_2 are defined as follows:

$$z_1 = z(X - \xi, Y - \eta, t), \quad (4)$$

$$z_2 = z(X, Y, t - \tau), \quad (5)$$

where X, Y and τ are constants. If z_1 is assumed to be a function of ξ, η and t , the equation of z_1 corresponding to (3) may be written as follows:

$$\nabla^2 \frac{\partial z_1}{\partial t} - U(t) \frac{\partial}{\partial \xi} \nabla^2 z_1 - \beta \frac{\partial z_1}{\partial \xi} - J\left(\frac{g}{f} \nabla^2 z_1, z_1\right) = 0, \quad (6)$$

where ∇^2 and J are executed with regard to ξ and η . If both sides of (6) are multiplied by z_2 and averaged with t

$$\overline{\nabla^2 \left(z_2 \frac{\partial z_1}{\partial t} \right)} - \overline{U(t) \frac{\partial}{\partial \xi} (\nabla^2 (z_1 z_2))} - \beta \frac{\partial}{\partial \xi} (\overline{z_1 z_2}) - \overline{z_2 J\left(\frac{g}{f} \nabla^2 z_1, z_1\right)} = 0, \quad (7)$$

where $\bar{A} = \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} A dt$.

Now,

$$\overline{U(t) \frac{\partial}{\partial \xi} (\nabla^2 (z_1 z_2))} = \frac{\partial}{\partial \xi} \overline{U \cdot z_1 z_2 + U' (z_1 z_2')}, \quad (8)$$

where

$$A' = A - \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} A dt.$$

Neglecting the fourth term of (7) and the second term of the right hand side of (8), the following equation may be obtained:

$$\nabla^2 \left(z_2 \frac{\partial z_1}{\partial t} \right) - \overline{U(t) \frac{\partial}{\partial \xi} \nabla^2 (z_1 z_2)} - \beta \frac{\partial}{\partial \xi} (\overline{z_1 z_2}) = 0. \quad (9)$$

In the same manner, if z_1 and z_2 are put as follows,

$$z_1 = z(X, Y, t), \quad (10)$$

$$z_2 = z(X + \xi, Y + \eta, t - \tau), \quad (11)$$

the following equation may be obtained:

$$\nabla^2 \left(z_1 \frac{\partial z_2}{\partial t} \right) + \overline{U(t - \tau) \frac{\partial}{\partial \xi} \nabla^2 (z_1 z_2)} + \beta \frac{\partial}{\partial \xi} (\overline{z_1 z_2}) = 0. \quad (12)$$

If the correlation coefficient of $z(X - \xi, Y - \eta, t)$ and $z(X, Y, t - \tau)$ is assumed to be equal to one of $z(X, Y, t)$ and $z(X + \xi, Y + \eta, t - \tau)$, the following equation may be obtained by

adding both (9) and (12):

$$\frac{\partial}{\partial t} \nabla^2 (\overline{z_1 z_2}) + \{ \overline{U(t - \tau)} - \overline{U(t)} \} \frac{\partial}{\partial \xi} \nabla^2 (\overline{z_1 z_2}) = 0, \quad (13)$$

where z_1 and z_2 are defined by (10) and (11) respectively. The solution of (13) becomes as follows:

$$\overline{z_1 z_2} = F\left(\xi - \int_0^t \{ \overline{U(t - \tau)} - \overline{U(t)} \} dt, \eta\right), \quad (14)$$

where $F(x, y)$ is an arbitrary function. (14) shows that the correlation field at $t=0$ moves to x -direction after t hours by the following distance:

$$\int_0^t \{ \overline{U(t - \tau)} - \overline{U(t)} \} dt. \quad (15)$$

In Fig. (2) black spots show the locations of the peaks of the correlation coefficients obtained by 10-day forecasting by (14). These locations correspond to the actual locations, especially when the variations of the correlation fields are systematic.

In the analogous way, it may be shown that the correlation field moves westwards with lag τ . Namely if z_1 and z_2 are put as follows,

$$z_1 = z(X, Y, t),$$

$$z_2 = z(X + \xi, Y + \eta, t - \tau),$$

where X, Y and t are first assumed to be constants, the equation which is a function of ξ, η and τ may be written as follows:

$$-\frac{\partial}{\partial \tau} \nabla^2 z_2 + U(t - \tau) \frac{\partial}{\partial \xi} \nabla^2 z_2 + \beta \frac{\partial z_2}{\partial \xi} - J\left(\frac{g}{f} \nabla^2 z_2, z_2\right) = 0.$$

Multiplying the above by z_1 and averaging with t , the following equation may be obtained (neglecting the same terms as the former):

$$-\frac{\partial}{\partial \tau} \nabla^2 (\overline{z_1 z_2}) + \overline{U(t - \tau) \frac{\partial}{\partial \xi} \nabla^2 (z_1 z_2)} + \beta \frac{\partial}{\partial \xi} (\overline{z_1 z_2}) = 0,$$

where $\bar{A} = \frac{1}{T} \int_0^T A(t) dt$.

This shows that the correlation field moves at a speed

$$-\overline{U(t - \tau)} + \frac{\beta}{k^2}$$

as lag τ increases, where k is the wave number of correlation field. Assuming $T=75 \sim 90$ days, $U(t - \tau)$ at the latitude $50^\circ N$ calculated from the data of December 1950~February

1951 are shown in Table 1. As may be seen from this table, the correlation pattern moves at almost constant speed. Fig. 1(a) shows this character.

Table 1.

τ	0 (day)	5	10	15	20	25
$U(t-\tau)$	8.6 (m/s)	8.5	8.4	8.3	8.4	8.5

4. Linear regression method

In the statistical forecasting, a linear combination between a predictant and predictors has been assumed. For instance, when 500 mb height $z(t, x, y)$ at a point (x, y) is predicted by using heights $z(t-\tau, x_1, y_1)$, $z(t-\tau, x_2, y_2)$, ..., $z(t-\tau, x_M, y_M)$ at points in the vicinity of that point, the following linear combination would be assumed:

$$z(t, x, y) = A_0 + \sum_{i=1}^M A_i z(t-\tau, x_i, y_i), \quad (16)$$

where A_i may be determined by the least square method with the aid of past data. But it is generally difficult to illustrate the complicated height changes by the linear combination. Then, what variations may be illustrated by the linear combination? For the sake of simplicity, a simple model of one dimension will be considered. The isobaric height $z(x, t)$ is assumed to be given as follows:

$$z(x, t) = \sum_{n=0}^{\infty} \{a_n(t) \cos nx + b_n(t) \sin nx\}. \quad (17)$$

Types of function $a_n(t)$ and $b_n(t)$ will be decided so as to be able to apply the linear combination such as (16) without error term. Assuming the mean height to be zero, the following relation will be considered to hold good:

$$z(x, t) = \sum_{i=1}^M A_i z(x + x_i, t - \tau). \quad (18)$$

If the right hand side of (18) is rearranged with the aid of (17),

$$\begin{aligned} z(x, t) &= \sum_{i=1}^M \sum_{n=0}^{\infty} A_i \{a_n(t-\tau) \cos n(x+x_i) \\ &\quad + b_n(t-\tau) \sin n(x+x_i)\} \\ &= \sum_{n=0}^{\infty} \sum_{i=1}^M [A_i \{a_n(t-\tau) \cos nx_i \\ &\quad + b_n(t-\tau) \sin nx_i\} \cos nx \\ &\quad + A_i \{-a_n(t-\tau) \sin nx_i \\ &\quad + b_n(t-\tau) \cos nx_i\} \sin nx]. \end{aligned}$$

From this relation and (17), the following relations may be obtained:

$$\begin{aligned} a_n(t) &= \left(\sum_{i=1}^M A_i \cos nx_i \right) a_n(t-\tau) \\ &\quad + \left(\sum_{i=1}^M A_i \sin nx_i \right) b_n(t-\tau), \\ b_n(t) &= - \left(\sum_{i=1}^M A_i \sin nx_i \right) a_n(t-\tau) \\ &\quad + \left(\sum_{i=1}^M A_i \cos nx_i \right) b_n(t-\tau). \end{aligned}$$

If p_n and q_n are put as follows

$$\begin{cases} p_n = \sum_{i=1}^M A_i \cos nx_i \\ q_n = \sum_{i=1}^M A_i \sin nx_i \end{cases} \quad (19)$$

the above relations may be written as follows:

$$\begin{cases} a_n(t) = p_n a_n(t-\tau) + q_n b_n(t-\tau) \\ b_n(t) = -q_n a_n(t-\tau) + p_n b_n(t-\tau) \end{cases} \quad (20)$$

If $n=0, 1, 2, \dots, K$ and $M=2K+1$ are assumed, $A_i (i=1, 2, \dots, M)$ can be decided as solutions of the simultaneous linear equations (19) for arbitrarily given p_n and $q_n (q_0=0)$. The solutions $a_n(t)$ and $b_n(t)$ of the difference equations (20) will satisfy (18) with the above decided A_i . Thus the difference equations (20) should be solved. Eliminating $b_n(t)$ from (20),

$$a_n(t) - 2p_n a_n(t-\tau) + (p_n^2 + q_n^2) a_n(t-2\tau) = 0.$$

If $a_n(t) = \lambda^t$, (21) is substituted into the above equation, the following equation for λ may be obtained:

$$\lambda^{2\tau} - 2p_n \lambda^\tau + p_n^2 + q_n^2 = 0.$$

The roots of this equation are

$$\lambda^\tau = \sqrt{p_n^2 + q_n^2} e^{\pm i\theta},$$

where

$$\theta = \tan^{-1} \frac{p_n}{q_n}.$$

If this λ is substituted into (21) and its real part is taken, $a_n(t)$ can be decided. Similarly $b_n(t)$ can be obtained. Thus the general solutions of (20) may be written as follows:

$$\begin{cases} a_n(t) = (p_n^2 + q_n^2)^{\frac{t}{2\tau}} (A_n \cos nc_n t - B_n \sin nc_n t) \\ b_n(t) = (p_n^2 + q_n^2)^{\frac{t}{2\tau}} (A_n \sin nc_n t + B_n \cos nc_n t) \end{cases} \quad (22)$$

where A_n and B_n are arbitrary constants and c_n are decided from

$$\cos nc_n\tau = \frac{p_n}{\sqrt{p_n^2 + q_n^2}},$$

$$\sin nc_n\tau = -\frac{q_n}{\sqrt{p_n^2 + q_n^2}}.$$

Substituting (22) into (17) and rearranging, z may be written as follows:

$$z = \sum_{n=0}^{\infty} (p_n^2 + q_n^2)^{\frac{t}{2\tau}} \{A_n \cos n(x - c_n t) + B_n \sin n(x - c_n t)\}.$$

Therefore if the amplitudes of the waves increase or decrease at rates $(p_n^2 + q_n^2)^{t/2\tau}$ and the wave moves with the constant speeds c_n , the linear combination such as (18) will be very effective. As a special case, if $p_n^2 + q_n^2 = 1$, it corresponds to the dynamically linear model.

5. Selection of predictors

In this place the correlation field in the case when the isobaric height z is used as predictor, will be compared with that when $(z)_{t+\tau} - (z)_t$ is used as predictor. For instance the height z will be assumed to be given as follows:

$$z = \sum_{n=0}^{\infty} \{A_n \cos n(x - c_n t) + B_n \sin n(x - c_n t)\}, \quad (23)$$

where A_n , B_n and c_n are constants.

Generally a correlation coefficient $\rho(f, g)$ of two quantities $f(t)$ and $g(t)$ in $[0, -\infty]$ may be defined as follows

$$\rho(f, g) = \frac{\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^0 \{f(t) - \bar{f}\} \{g(t) - \bar{g}\} dt}{\sqrt{\left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^0 \{f(t) - \bar{f}\}^2 dt \right]} \sqrt{\left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^0 \{g(t) - \bar{g}\}^2 dt \right]}} \quad (24)$$

By this definition, the correlation coefficient with lag τ of the heights at two points $x = x_1$ and $x = x_2$ becomes as follows:

$$\rho(z(t, x_1), z(t - \tau, x_2)) = \frac{\sum_{n=1}^{\infty} (A_n^2 + B_n^2) \cos n(x_2 - x_1 + c_n \tau)}{\sum_{n=1}^{\infty} (A_n^2 + B_n^2)}. \quad (25)$$

Fixing x_1 and assuming x_2 and τ to be variables, (25) shows that the correlation field moves toward the inverse direction with the actual disturbances. Fig. 1 illustrates this respect.

Next $\eta(t, x)$ is put as follows:

$$\eta(t, x) = z(t, x) - z(t - \mu, x). \quad (26)$$

And $\eta(t, x)$ will be considered instead of $z(t, x)$. The correlation coefficient with lag τ of $\eta(t, x_1)$ and $\eta(t - \tau, x_2)$ can be written as follows:

$$\rho(\eta(t, x_1), \eta(t - \tau, x_2)) = \frac{\sum_{n=1}^{\infty} (1 - \cos nc_n \mu) (A_n^2 + B_n^2) \cos n(x_2 - x_1 + c_n \tau)}{\sum_{n=1}^{\infty} (1 - \cos nc_n \mu) (A_n^2 + B_n^2)} \quad (27)$$

Also in this case, the same character as the former may be seen with respect to the movement. But the amplitudes of (27) contain $(1 - \cos nc_n \mu)$ and they vary with c_n even when μ is fixed. Therefore concerning actual case, it would give an instabilizing effect to the correlation coefficient.

6. Statistical features of the barotropic vorticity equation

The barotropic vorticity equation in spherical coordinates is

$$\frac{\partial \zeta}{\partial t} + \frac{v_\varphi}{a} \frac{\partial \zeta}{\partial \varphi} + \frac{v_\lambda}{a \cos \varphi} \frac{\partial \zeta}{\partial \lambda} + 2 \omega \cos \varphi \frac{v_\varphi}{a} = 0, \quad (28)$$

where

$$\begin{cases} v_\lambda = -\frac{g}{af} \frac{\partial z}{\partial \varphi} \\ v_\varphi = \frac{g}{af \cos \varphi} \frac{\partial z}{\partial \lambda} \end{cases} \quad (29)$$

$$\zeta = \frac{g}{fa^2 \cos \varphi} \left\{ \frac{\partial}{\partial \varphi} \left(\cos \varphi \frac{\partial z}{\partial \varphi} \right) + \frac{1}{\cos \varphi} \frac{\partial^2 z}{\partial \lambda^2} \right\}. \quad (30)$$

λ : longitude

φ : latitude

a : radius of the earth

g : acceleration of gravity

f : Coriolis parameter

ω : rotating velocity of the earth

ζ : vorticity

v_λ : east-west component of wind velocity

v_φ : south-north component of wind velocity

For the sake of numerical calculation, η is put as follows:

$$\eta = \log(\tan \varphi + \sec \varphi) + \text{const.} \quad (31)$$

If η is used instead of φ , v_λ , v_φ and ζ may be expressed as follows:

$$\begin{aligned} v_\lambda &= -\frac{g}{fa \cos \varphi} \frac{\partial z}{\partial \eta}, \\ v_\varphi &= \frac{g}{fa \cos \varphi} \frac{\partial z}{\partial \lambda}, \end{aligned}$$

Fig. 3(a)

Fig. 3. Variations of α, β, γ and δ at 50°N , 50°E during January 1951. (a) is a case of $\tau=1$, (b) $\tau=5$ and (c) $\tau=10$ days.

Fig. 3(b)

Fig. 3(c)

$$\zeta = \frac{g}{fa^2 \cos^2 \varphi} \left(\frac{\partial^2 z}{\partial \lambda^2} + \frac{\partial^2 z}{\partial \eta^2} \right).$$

And the vorticity equation (28) may be written as follows:

$$\frac{\partial}{\partial t} \nabla^2 z + J \left(z, \frac{g}{fa^2 \cos^2 \varphi} \nabla^2 z + \text{const.} \right) = 0, \quad (32)$$

$$\text{where } \nabla^2 z = \frac{\partial^2 z}{\partial \lambda^2} + \frac{\partial^2 z}{\partial \eta^2},$$

$$J(A, B) = \frac{\partial A}{\partial \lambda} \frac{\partial B}{\partial \eta} - \frac{\partial A}{\partial \eta} \frac{\partial B}{\partial \lambda}.$$

Now, if Λ is assumed to be a function of t only and put,

$$z_0 = c - \frac{f}{g} \Lambda a^2 \sin \varphi, \quad (33)$$

then

$$-\frac{g}{fa^2 \cos \varphi} \frac{\partial z_0}{\partial \varphi} = \Lambda,$$

z_0 representing the height pattern corresponding to the uniform zonal current with the angular velocity Λ . If the deviation of z from z_0 is again called z , the equation of new z may be written as follows:

$$\begin{aligned} \frac{\partial}{\partial t} \nabla^2 z + \Lambda \frac{\partial}{\partial \lambda} \nabla^2 z + 2(\Lambda + \omega) \cos^2 \varphi \frac{\partial z}{\partial \lambda} \\ + \frac{g}{fa^2 \cos^2 \varphi} J(z, \nabla^2 z) = 0. \end{aligned}$$

Since $\Lambda \ll \omega$ holds good, the above equation becomes as follows:

$$\begin{aligned} \frac{\partial}{\partial t} \nabla^2 z = -\Lambda \frac{\partial}{\partial \lambda} \nabla^2 z - 2\omega \cos^2 \varphi \frac{\partial z}{\partial \lambda} \\ + \frac{g}{fa^2 \cos^2 \varphi} J(\nabla^2 z, z). \end{aligned} \quad (34)$$

Integrating both sides of (34) from t to $t+\tau$,

$$\begin{aligned} (\nabla^2 z)_{t+\tau} - (\nabla^2 z)_t \\ = - \int_t^{t+\tau} \Lambda \frac{\partial}{\partial \lambda} \nabla^2 z dt - 2\omega \cos^2 \varphi \\ \times \int_t^{t+\tau} \frac{\partial z}{\partial \lambda} dt - \frac{g}{fa^2 \cos^2 \varphi} \int_t^{t+\tau} J(z, \nabla^2 z) dt. \end{aligned} \quad (35)$$

If α, β, γ and δ are defined as follows,

$$\alpha = (\nabla^2 z)_t - (\nabla^2 z)_{t+\tau}, \quad (36)$$

$$\beta = \int_t^{t+\tau} \Lambda \frac{\partial}{\partial \lambda} \nabla^2 z dt, \quad (37)$$

$$\gamma = 2\omega \cos^2 \varphi \int_t^{t+\tau} \frac{\partial z}{\partial \lambda} dt, \quad (38)$$

$$\delta = \frac{g}{fa^2 \cos^2 \varphi} \int_t^{t+\tau} J(z, \nabla^2 z) dt, \quad (39)$$

(35) may be written as follows:

$$\alpha = \beta + \gamma + \delta. \quad (40)$$

Each term of (40) was numerically calculated by the difference method with the aid of daily 500 mb charts of January 1951. And the correlation coefficients of α and the other quantities in (40) were calculated. These correlation coefficients were obtained at fifteen points

Table 2.

τ	1(day)			5			10		
Long.	$\rho(\alpha, \beta)$	$\rho(\alpha, \gamma)$	$\rho(\alpha, \delta)$	$\rho(\alpha, \beta)$	$\rho(\alpha, \gamma)$	$\rho(\alpha, \delta)$	$\rho(\alpha, \beta)$	$\rho(\alpha, \gamma)$	$\rho(\alpha, \delta)$
30°E	0.47	-0.29	-0.22	0.56	-0.53	-0.64	0.67	-0.50	-0.75
40	0.38	-0.32	-0.05	0.57	-0.71	-0.34	0.51	-0.77	-0.27
50	0.15	-0.26	-0.07	0.46	-0.67	0.21	0.33	-0.61	0.17
60	0.20	-0.34	-0.26	0.55	-0.76	-0.52	0.56	-0.64	-0.61
70	0.40	-0.49	-0.13	0.68	-0.76	-0.30	0.80	-0.56	-0.28
80	0.29	-0.22	0.05	0.48	-0.29	0.22	0.42	-0.05	0.09
90	0.35	-0.37	0.22	0.49	-0.49	0.07	0.03	-0.08	0.30
100	0.23	-0.24	-0.05	0.55	-0.59	-0.45	0.40	-0.33	-0.50
110	0.21	-0.19	0.00	0.51	-0.71	-0.04	0.38	-0.47	0.19
120	0.38	-0.23	-0.04	0.59	-0.37	0.09	0.57	0.04	0.24
130	0.26	-0.24	0.14	0.44	-0.35	-0.26	0.34	0.06	0.08
140	0.20	-0.17	0.10	0.34	-0.24	-0.15	0.37	-0.17	-0.27
150	0.29	-0.26	0.12	0.32	-0.28	0.00	0.36	-0.30	0.08
160	0.18	-0.25	-0.06	0.27	-0.55	0.16	0.37	-0.50	-0.22
170	0.24	-0.18	0.10	0.12	-0.14	-0.12	0.05	0.04	-0.14

30°E, 40°E, ……., 170°E along the latitude 50°N circle. Fig. 3 (a)~(c) show variations of α , β , γ and δ at 50°E, 50°N during the month. As may be seen from these figures, in the cases of $\tau=5$ and 10 days, positive correlation $\rho(\alpha, \beta)$ between α and β , and negative correlation $\rho(\alpha, \gamma)$ between α and γ , are predominant. The positive correlation of α and β originates from the movement of the disturbances by the general current A . If negative correlation of $\nabla^2 z$ and z is taken into account, the negative correlation of α and γ would be expected. There is no definite character on the correlation $\rho(\alpha, \delta)$ of α and δ which represents nonlinear effect. In the case of $\tau=1$, it is difficult to introduce some correlational relations from the figure.

Numerical values of the correlation coefficients are shown in Table 2. The various characters derived from the figure can be introduced from this table. After all it is relatively easy to see the effect of the movement by the general current with the aid of the correlation method but it is difficult to examine the nonlinear effect.

7. Conclusion

The limit of application of the correlation method was clarified from the various view points. There are two difficult problems relating to the correlation method, one of them is instability of the correlation coefficient and the other is nonlinearity. These problems should further be inquired from the view

point of statistical and physical mechanism of the phenomena.

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References

- Cooley, D. S., 1958: Statistical forecasting operators based on dynamical equations. *Tellus*, **10**, No. 3.
- Lorenz, E. N., 1956: Empirical orthogonal functions and statistical weather prediction. Scientific Report No. 1, Statistical Forecasting Project, M.I.T.
- Magata, M., T. Ozawa, E. Suzuki, T. Fujita and K. Tomatsu, 1958: On the correlation fields and the predicting possibilities by statistical methods at 500 mb level. *Pap. Met. Ceophys.*, **8**, 245-270.
- Malone, T. F., 1956: Application of synoptic climatology to weather prediction. Chapter 28 in Petterssen, Weather analysis and forecasting, 238-255.
- White, R. M., R. C. Derby, D. S. Cooley and A. F. Seaver, 1957: Hemispherical prediction by statistical techniques. *J. Meteor.*, **14**.
- White, R. M. and W. C. Palson, 1955: On the forecasting possibilities of empirical influence functions. *J. Meteor.*, **12**, 478-485.

500 mb 高度の相関場について

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同一の2量間の相関係数にしても、資料の期間の採り方によつて違う値が得られるので、比較的短期間の資料を用いて計算した相関係数の値が、資料期間を順次に変えて行つた場合に、どのように変動するかを調べ、力学的考慮によつて、その変動がある程度説明できることを示す。また、渦度方程式を線型部と非線型部に分けて、1日、5日、10日の移動平均を施し、各項間の相関係数を調べ、時間変化の項と、線型部の間には、一定の相関々係が見られるが、非線型項との相関は、不明瞭であることが分つた。いわゆる相関法によつて予報を行う場合に、安定した相関係数が必要なわけであるが、そうでない時にもその内部的性質を明らかにすることによつて、相関係数の変動が予報できるわけである。