On the Contradiction and Modification of the Equation of Diabatic Wind Profile

—Its Relations to the Scale of Turbulence—

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The equation of diabatic wind profile $\bar{u}(z)$ in the surface layer was given as follows, by Ellison (1957), Yamamoto (1959) and Panofsky *et al.* (1960), from further investigations on the similarity theory that was firstly introduced to the atmospheric turbulence by Monin-Obukhov (1954):

$$\varphi^4 - \gamma \frac{z}{L} \varphi^3 = 1 \quad (1)$$

where φ is Monin-Obukhov's universal function, $\varphi = (kz/u_*) (\partial \bar{u} / \partial z)$ and L is stability length defined by

$$L = - \frac{u_*^3}{k \frac{g}{T_0} \frac{q}{C_p \rho}}$$

and

z ; height
 $\bar{u}$; wind speed
 u_* ; friction velocity
 k ; von Kármán's constant
 ρ ; density
 T_0 ; temperature
 g ; acceleration due to gravity
 C_p ; specific heat at constant pressure
 q ; vertical heat flux
 γ ; empirical constant.

On the other hand, the scale of turbulence should be uniquely derived from the rate of turbulent energy dissipation E and kinematic viscosity or turbulent diffusion coefficient K , based on Kolmogoroff's similarity hypothesis. When the divergence of energy can be neglected, the rate of turbulent energy dissipation is given by the following equation under steady state:

$$E = u_*^2 \frac{\partial \bar{u}}{\partial z} + \gamma \frac{g}{T_0} \frac{q}{C_p \rho} \quad (2)$$

While the turbulent diffusion coefficient is given by

$$K = u_*^2 \left(\frac{\partial \bar{u}}{\partial z} \right)^{-1} \quad (3)$$

Then the scale of turbulence l can be determined dimensionally as follows:

$$l = kz \varphi^{-1} \left(1 - \gamma \frac{z}{L} \varphi^{-1} \right)^{-\frac{1}{4}} \quad (4)$$

Substituting eq. (1) into eq. (4), it can be shown that the scale of turbulence is independent of the atmospheric stability:

$$l = kz \quad (5)$$

The present scale l must be closely connected with the other scale, l_1 , say, the integral of correlation coefficient $R(z)$ of fluctuating wind speed u' at two different heights:

$$l_1 = \int_0^\infty R(z) dz \quad (6)$$

where

$$R(z) = \overline{u'(0)u'(z)} / \overline{u'^2}$$

Therefore the correlation coefficient should be independent of the atmospheric stability, such as the scale of turbulence. However it is clear that the correlation coefficient varies with stability in various experiments (ex. Haugen, 1959).

It can be concluded that eq. (1) of diabatic wind profile cannot take into account of the dependency of the scale of turbulence on the atmospheric stability. On the other hand, could the variation of the scale also be derived uniquely from z/L , the following relation should be rational:

$$l = kz \phi \left(\frac{z}{L} \right) \quad (7)$$

where ϕ is a universal function of z/L . Then eq. (1) must be modified as follows, to take into account of the scale variation according

to the stability:

$$\varphi^4 - \gamma \frac{z}{L} \varphi^3 = \varphi^{-4} \quad (8)$$

The function φ should be determined from observational data.

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非断熱大気の風速分布の方程式の矛盾と修正について

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