On the Development of the Relatively Small Scale Cyclone due to the Release of Latent Heat by Condensation

By T. Nitta

Electronic Computation Center, Japan Meteorological Agency

(Manuscript received 1 May 1964)

Abstract

The problem of development of the relatively small scale cyclone (whose characteristic horizontal scale has the order of magnitude of hundred kilometers) is investigated from the standpoint of the effect of latent heat released by condensation through the process of convective action.

The appearance of region of hyperbolicity in the so-called ω -equation in balance system or predominancy of very small waves in the solution of primitive equations of motion is avoided by treating the contribution of non-adiabatic heating as a force from outside for ω -equation as proposed by Ooyama (1963).

The growth rate of disturbance is obtained analytically in the case of linear problem and also by means of numerical method as a non-linear problem. Comparisons between the growth rates in both cases and discussions on the results are given. Both results indicate that the most favorable wave lengths for development are found in several hundred kilometers.

The problem of supply of water vapor from outside is studied, although the dissipation at ground is not taken into account. Suggestion to the control of development of low is proposed and examined in connection with the supply of water vapor.

1. Introduction

The remarkable importance of the role the latent heat plays has been emphasized not only for the long term but also even for the short term evolutions of the atmosphere. (Manabe, 1956; Gambo, 1963; Arakawa, 1964 and etc.)

In the short range atmospheric evolutions, diabatic heating due to condensation contributes in various extents and influences in several manners. For example, this effect strengthens and intensifies the development of the large scale synoptic disturbance at mid-latitude, accelerating the speed of the movement of wave in lower troposphere, although the main source of baroclinic development is attributed to the eddy available potential energy produced by the horizontal temperature gradient of the field in-the-large. (Gambo, 1963) On the other hand, in the case of small scale cyclone at middle latitudes and typhoon or hurricane, the heat source due to the con-

densation plays the prime role for their formations and maintenances against the dissipation at ground surface, in relation to the activities of convections or cumulus clouds.

In order to incorporate the process of convective action to the large scale motion, many trials were examined to fail, only obtaining the predominancy of very small scale waves or noises in the integrated solutions.

Recently, Ooyama (1963) proposed the revised method to avoid this failure, dealing with the effect of condensation in convection as the forcing term from outside (strictly speaking, yet as inner force) for the thermal equation of the larger scale field and then for ω -equation.

In this paper, we will follow the improved models by Ooyama and show the results of analytical considerations and numerical experiments in which some trials are performed to see indirectly the work of self-controlling mechanism of water vapor supply on the developing system.

2. Basic assumptions

Although we investigate the effect of latent heat released through condensation process due to convective cumulus cloud, we shall deal with the motion in macro-scale and express the process by the quantities in macro-motion. Since Rossby number is considered to be almost less than unity for all time, we shall assume that this motion is described by the quasi-geostrophic equation system and then after the development of relatively small scale cyclone, the detailed representation would not be accurate.

For simplicity, 2-level model is used and integrated.

Modeling assumptions adopted here are as follows.

- (1) Motion is confined in x - p plane, where x denotes the horizontal coordinate, p the vertical.
- (2) To make clear the physical effect of condensation and also for simplicity, we shall ignore the advective terms and β -effect in the vorticity equation.
- (3) As the upper and lower boundary conditions for the vertical p -velocity, we will put

$$\omega = \omega_0 = 0 \quad \text{at } p=0,$$

and

$$\omega = \omega_4 = -\kappa \nabla^2 \phi_3 \quad \text{at } p=1000 \text{ mb}$$

where κ is the constant coefficient denoting the continuity of mass in boundary layer and connecting frictional convergence in boundary layer and vorticity in lower layer. At the lateral boundary, we assume that there is no time change.

- (4) Lateral diffusion is considered in Fickian type and a same constant coefficient is used for vorticity and thickness equations. Skin friction is neglected since the present work pays primary interest on the influence of the non-adiabatic heat source and its relation to the evolution of the atmosphere. (Syōno and Matsuno (1964) incorporated both effects in their numerical experiment of typhoon formation.)

- (5) Frictional boundary layer which is assumed to be full of saturated water vapor and infinitesimally thin, is settled as is shown in Fig. 1. The vapor is forced to be transported

Fig. 1. Vertical resolution of 2 level model with infinitesimally thin frictional boundary layer.

upward corresponding to the macro-scale viscous convergence in the boundary layer to satisfy the law of mass conservation as is mentioned in assumption (3).

- (6) Water vapor transferred through frictional layer is assumed to condense immediately and warm the whole air column simultaneously, but space mean thickness is held to be constant for all time implicitly assuming the cooling due to radiation. Latent heat released by condensation due to the micro-scale motion is parameterized to be given in proportion to ω_4 in the place where is upward current ($\omega_4 < 0$), according to Ooyama's proposal. Proportional constant, η , may be expressed as

$$\eta = \frac{Lq}{c_p T} \frac{\theta}{\Delta\theta}$$

where L is the latent heat of condensation, q the specific humidity in the boundary layer, T and θ the mean temperature and potential temperature of the troposphere, and $\Delta\theta$ the difference between the average potential temperature of upper and lower layers. η may be interpreted as the constant relating the entropy obtained in the frictional boundary layer and the entropy required to transport the air parcel upward. In actual observed data, η seems to be 2~3, but in numerical experiment sometimes 6 is used for exaggeration of the effect.

- (7) All dependent variables are separated into the space mean parts and deviations from

the average such as, for instance,

$$\phi = \bar{\phi} + \phi'$$

where $\bar{\phi} = \int_0^D \phi dx / \int_0^D dx$ and $\phi' = \phi - \bar{\phi}$.

Hereafter all superscript prime will be omitted for the sake of simplicity.

3. Equations of motion

The equations of motion which satisfy assumptions in the previous section are the following.

(i) Vorticity equation

$$\frac{\partial}{\partial t} \nabla^2 \phi_1 = \frac{f^2}{\Delta p} \omega_2 + \lambda_1 \nabla^2 \cdot \nabla^2 \phi_1 \quad (3.1)$$

and

$$\frac{\partial}{\partial t} \nabla^2 \phi_3 = \frac{f^2}{\Delta p} (\omega_4 - \omega_2) + \lambda_3 \nabla^2 \cdot \nabla^2 \phi_3 \quad (3.2)$$

where ϕ is geopotential and λ the coefficient of lateral diffusion.

(ii) Thermal equation

$$\frac{\partial}{\partial t} h = S \Delta p (\omega_2 - (\eta \omega_4)) + \lambda_h \nabla^2 h \quad (3.3)$$

where $h = \phi_1 - \phi_3$, $\eta \neq 0$ for $\omega_4 < 0$ and $\eta = 0$ for $\omega_4 > 0$, and $\lambda_1 = \lambda_3 = \lambda_h = \lambda$. The second term in the parenthesis of right hand side of (3.3), *i.e.*, $(\eta \omega_4)$, is the non-adiabatic heating term due to condensation and η shows the proportional constant coefficient mentioned in section 2 as assumption (6).

Equations (3.1)~(3.3) suffice the complete set of equations of motion to determine three unknowns, *i.e.*, $\partial \phi_1 / \partial t$, $\partial \phi_3 / \partial t$ and ω_2 under the boundary conditions and initial conditions for ϕ and ω .

(iii) ω -equation

For diagnostic convenience, we shall derive the ω -equation at middle level, vanishing time derivative of ϕ among eqs. (3.1)~(3.3) and it may read

$$\nabla^2 \omega_2 - \frac{2f^2}{S \Delta p} \omega_2 = - \frac{f^2}{S \Delta p} \omega_4 + \nabla^2 (\eta \omega_4) \quad (3.4)$$

4. Growth rate in linear case

If we assume that η is irrelevant to the sign of ω and may be constant everywhere in the whole domain concerned with, the fundamental equations (3.1)~(3.3) become linear system

and we can solve the problem from analytical view-point. Inserting wave solution of the form $\phi \sim e^{ikx} e^{\gamma t}$, where $k = 2\pi m/D$, D is the horizontal length of domain, m the wave number and γ the growth rate, into (3.1)~(3.3) under upper and lower boundary conditions, we obtain the growth rate determining equation

$$A\gamma^2 + B\gamma + C = 0 \quad (4.1)$$

where

$$A = 2 + \frac{S \Delta p^2}{f^2} k^2$$

$$B = \kappa \frac{f^2}{\Delta p} - S \Delta p \kappa (\eta - 1) k^2 + \frac{S \Delta p^2}{f^2} (\lambda_1 + \lambda_3) k^4 + (\lambda_1 + \lambda_3 + 2\lambda_h) k^2$$

and

$$C = k^2 \left(\lambda_h \kappa \frac{f^2}{\Delta p} + \lambda_h (\lambda_1 + \lambda_3) k^2 - S \Delta p \kappa \lambda_1 (\eta - 1) k^2 + \frac{S \Delta p^2}{f^2} \lambda_1 \lambda_3 k^4 \right)$$

Solutions of (4.1) are

$$\gamma = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \quad (4.2)$$

(4.2) shows that the inequality $\eta > 1$ is the necessary condition for $\gamma > 0$, *i.e.*, the development of disturbance. We will hereafter consider the growth rate $\gamma = (-B + \sqrt{B^2 - 4AC})/2A$. The quantity in square root is almost positive for parameters adopted here.

The assumption on η being constant over the whole area results the overestimation of the effect of latent heat, because $(\eta \omega_4)$ in down-ward region contributes to the cooling and intensifies the contrast in non-adiabatic heating, thus we will obtain the stronger contrast in the field of vertical p -velocity than the case of holding η zero over the domain where ω has positive sign.

In Fig. 2, the diagram of the growth rate γ versus wave length obtained from (4.2) is shown for various η . λ is taken to be $5 \times 10^3 \text{ m}^2 \cdot \text{sec}^{-1}$. We can easily see that the most unstable wave has the wave length in order of magnitude of hundred kilometers (as is shown by an arrow) and that the more the value of η , the shorter the most unstable wave length

Fig. 2. The growth rate γ versus wave length L for various η in linear case.

Fig. 3. Same as Fig. 2 but for various λ in linear case. Single and double circles denote the results of numerical calculation.

and the larger the growth rate. For $\eta=2\sim3$, we have 5~600 km as the most favorable wave length for development. e -folding time in this case lies between 1 and 2 days or a little more. These values would seem to be reasonable for explaining the evolution of relatively small-scale cyclone at middle latitude and/or the development of tropical low. Thus, we may say that, although the present description of cooperative process between cumulus

convection and macro-field is extremely simple and rough, the result gives one of the solution for this type of instability in the atmosphere.

Fig. 3 denotes the distribution of γ in wave length space in the case of $\eta=6$ changing the value of λ . (Single and double circles are the results of numerical calculation. See next section.). The wave length of the most unstable disturbance becomes longer as the value of λ tends to be large and vice versa. Al-

though we incorporate the action of the lateral diffusion alone, the dynamical effect of micro-motion influences and controls the horizontal size of macro-motion.

5. Numerical tests in non-linear case

When we pose the value of η more accurately, *i.e.*, to be zero over downward region and positive constant value more than unity over upward area, the equations of motion (3.1)~(3.3) become non-linear in the sense that η takes selectively constant value or zero according to the sign of ω .

Numerical time integration is performed for the following three cases changing physical assumption concerning to the form of supply of water vapor from boundary layer. The present model has no self-controlling mechanism of energy income, but these trials give the suggestions on the work of control of water vapor supply upon development of motion.

Case 1: Supply of saturated vapor from frictional layer is endless, *i.e.*, the layer is always fulfilled with water vapor.

Case 2: Total amount of water content in gaseous phase within layer is limited and no supply from outside of boundary layer is expected.

Case 3: Water vapor is transported from outside with constant rate into the frictional layer. In the present case, we chose the smaller incoming rate than the outgoing one.

In actual computations of cases 2 and 3, we predict the value of η by solving the equation for moisture flux under assumption of $\eta \propto q$ ($=$ mixing ratio). As the surface dissipation is not taken into account, the motion which would appear in case 1 could not approach toward equilibrium.

In order to exaggerate the effect of latent heat clearly, $\eta=6$ is used in the numerical computation, although the value is too large compared with observation as mentioned in section 2. Space interval d used in numerical calculation equals to 30 km and time increment $\Delta t=10$ min.

Numerical tests are planned to start at the moment when latent heat is released after sufficient containment of saturated vapor and preexisting weak disturbance is assumed at

Fig. 4. Time evolutions of vorticity and vertical p -velocity (a) in case 1 and (b) in case 2. Abscissa is x -coordinate.

initial time.

Now we shall look at the results of numerical experiments. Firstly, in Fig. 4, we can see the warming of the air column above the cyclone region as time advances and also the increasing of the vertical p -velocity w over low area. Cyclone continues to develop for all time in case 1 (Fig. 4(a)) since supply of energy always goes on into disturbance, while in case 2 (Fig. 4(b)) slow development is observed and at last the increase of central vorticity begins to cease after 70 hours according to the decrease of water vapor supply (for reference, see Fig. 8). In these cases wave length of initial disturbance, L_0 , is taken to be 360 km and a very weak amplitude is given. Needless to say, the control and reduction of the supply of water vapor for developing system is too much artificial, but the difference of evolution between two cases clearly shows the vital role of water vapor for deepening the depression and importance of self-controlling mechanism of the supply. In Fig. 5,

Fig. 5. Time evolution of the central vorticity of cyclone (ordinate) for various η in case 1. Abscissa is time in hour.

Fig. 6. Same as Fig. 5 but for various λ in case 1.

Fig. 7. Same as Fig. 5 but for various L_0 in case 1.

the difference of development with various η is shown under the same initial condition. Ordinate is the central vorticity of low and abscissa time in hour. As η increases more and more, the rate of intensification of low becomes larger as is easily imagined from the larger value of growth rate for larger η in the linear case. Adoption of $\lambda=0$ might seem to be curious, for in the result of linear analysis vanishing of lateral diffusion means excitation of the smallest wave length, but as we give a finite value for L_0 at $t=0$ and the calculation is simple and runs smoothly, we do not find any other modes of disturbance at least during 30 hours. In all numerical examples, initial weak depressions alone developed. The effect of lateral diffusion is observed in Fig. 6, fixing other parameters. As the value of coefficient λ becomes larger and larger, the curve of central vorticity versus time inclines to flat. Too large value of λ suppresses the development of the cyclone in the domain of short wave and shifts the

most unstable wave length for development towards longer. This agrees well with the result of linear case as is shown in Fig. 3. The growth rates obtained from Fig. 6 are plotted in Fig. 3 as marked with circles together with other results of $L_0=720$ and 1080 km, which are shown in Fig. 7. The difference of growth rate between the numerical solution in the non-linear case and the analytical one in the linear case may come from the overestimation of condensation heating in the latter.

Comparisons of time sequences of the vorticity of cyclone center among cases 1, 2 and 3 are made in Fig. 8 together with time variation of η . At $t=0$, we pose $\eta=6$ and $L_0=360$ km and λ is taken to be zero for all time. Remarkable effect of variation of η on time evolution of low in case 2 is interesting and suggesting, as is already seen in Fig. 4. The situation of case 3 in the present calculation is a special one among many combinations of receipt and payment of energy. As the rate

Fig. 8. Same as Fig. 5 but for case 1, 2 and 3. Changes of η with respect to time are also plotted in case 1 and 2.

of income is taken to be less than that of outgo, the disturbance ceases its development and at last tends to decay just like as is seen in case 2. Generally speaking, results of numerical experiments illustrated in these figures show good agreement with those obtained by analytical method.

6. Discussions

Interpretation of convective activities by means of language of large scale atmospheric motion has now become one of the most important problem in the field of theoretical meteorology, not only for the explanation of the formation and maintenance of typhoon, but also for the construction of general circulation theory.

The assumptions and models used in the present test are, of course, too simple and elementary to attack the problem substantially. We have to incorporate terms ignored in the present examination. Besides, at middle latitude, we must take into consideration the advection of water vapor by southerly wet tongue and also the mechanism of dissipation at ground surface.

The remarkable ability of latent heat to intensify and excite the cyclone is, however, clearly shown in the preceding section. Yet, the model has no self-controlling mechanism of heating due to condensation by micromotion and no feedback system of water vapor supply towards convection, but indirect discernment on some aspects of control is performed. Gross stabilization of system as the result of warming of cyclonic air column might propose other point of constraint on the evolution.

In principle, we must trace the behavior of water vapor itself and the more real and reasonable verisimilitude of cooperating system of large scale motion and convective activities

or clouds is necessary.

(After completion of the present paper, the author had the opportunity to read the paper written by Charney and Eliassen (1964) concerning the same problem as treated here. Although they showed only the analytical results in linearized version, the essence of the idea seems almost the same with Ooyama's model used here.)

Acknowledgement

The author wishes to express his heartily thanks to Prof. S. Syōno, Tokyo University, who gave him encouragement through this work. He also expresses his deep thanks to Dr. K. Gambo, Electronic Computation Center of Japan Meteorological Agency, who suggested the present problem, to Dr. A. Arakawa of our Center for his useful discussions and advises and staff members of our Center for their kind encouragement. Thanks are due to Misses K. Endo and Y. Atsuki for drafting figures and to Miss M. Yoshino for typewriting the manuscript.

References

1. Arakawa, A., 1964: to be published.
2. Charney, J.G., and A. Eliassen, 1964: On the growth of the hurricane depression. *Jour. Atm. Sci.*, **21**, 68-75.
3. Gambo, K., 1963: The role of sensible and latent heats in the baroclinic atmosphere. *Jour. Met. Soc. Japan, Ser. II*, **41**, 233-246.
4. Manabe, S., 1956: On the contribution of heat released by condensation to the change in pressure pattern. *Jour. Met. Soc. Japan, Ser. II*, **34**, 308-320.
5. Ooyama, K., 1963: A dynamical model for the study of tropical cyclone development (Unpublished manuscript).
6. Syōno, S., and T. Matsuno, 1964: to be published.

凝結の潜熱による小規模低気圧の発達について

新 田 尚

(気象庁電計室)

対流性積雲の活動による凝結の潜熱の解放にもとづいた小規模な低気圧の発生、発達について極端に単純化したモデルを用いて調べた。いわゆる ω -方程式における双曲型領域の出現と、まともに対流現象を対象に数値積分する場合の困難と失敗を避けるため、大山 (1963) のモデルを採用した。

このモデルでは、最下層に考えた摩擦層が飽和した水蒸気で満たされ、摩擦収束による上方への水蒸気輸送の結果、凝結の潜熱が解放され、低気圧発達のエネルギー源をなす。水蒸気の上方への輸送と潜熱解釈の機構は、対流性積雲の活動と密接に関係しているが、その作用が、より巨視的な場の運動にどう風統的に解放されるか、いいかえれば、どのようにパラメタライズしてより大規模な場の量で表現できるかについては、沢山の未知の問題を含んでいる。ここでは最も簡単に考えて、個々の積雲の作用の総和として、熱力学的には、バランスした巨視的運動場の摩擦収束による摩擦層上限の境界面での上昇流に比例した形で潜熱が解放され全気柱を高効率的に暖めると仮定した。他方力学的には、横方向の拡散作用をもたらすとした。

じょう乱の発達率を線型の場合について解析的に調べた結果、数百キロの波長の所に最大発達率があらわれ、その波長は、横方向の拡散係数が小さい程短波域へ、大きい程長波域に移動することがわかったが、これは、すでに、大山も示している所である。

更に非線型の場合について数値積分を行ない、水蒸気の補給の仕方による気圧等の発達の違いを調べ、上の線型の結果の確認を行ない、一方的な発達を制限する機構をさぐった。もちろん、これは単純化しすぎたモデルを用いているため、水蒸気補給の自己調節能力はもっていない。従って水蒸気補給の間接的な制御をテストするにすぎない。しかしこのテストによって、じょう乱が発達するにつれて水蒸気補給を減少させれば、発達がとり次第に衰えてゆくことが示唆される。

なお実際の気象で考えると、このモデルに用いた取扱いは低緯度におけるじょう乱の発達に対してよい近似をもつが、中緯度の場合は、摩擦境界層からの補給よりむしろ下層における飽和水蒸気の補給 (湿舌) が重要となり、その作用をも考慮にいれるべきであろう。またエネルギーの供給に重点をおいたため、地面の dissipation の問題にはふれなかった。