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A Numerical Study of the Air-Mass Transformation over the Japan Sea in Winter

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Abstract

One of the most important causes for snowfall over the Japanese Islands in winter is associated with the transformation of continental polar air-masses during passage over the Japan Sea. The cP air-masses are rapidly modified over the relatively warm water owing to the addition of sensible and latent heat. The theory of air-mass transformation due to eddy diffusions is re-examined by taking account of the phase change of water vapor and the variable exchange coefficient dependent on the static stability. Numerical experiments are made to test the model.

It is shown that the present simple model can describe the average weather condition over the Japan Sea in winter, if the variation of exchange coefficients in the model is taken into consideration.

1. Introduction

The northwest monsoon passing over the Japan Sea in winter is one of the most typical examples of the transformation of continental polar air-mass. This process is known to play an important role in the snowfall of the north-west coastal regions of the Japanese Islands. During passage over the Japan Sea, the cold air from Siberia receives a great deal of sensible heat and water vapor in its lower atmospheric layer.

Recently, some quantitative estimations of the heat supply were made, based on a heat and moisture budget analysis. An extensive network of surface and radiosonde observations stations in Japan, Korea and Siberia surrounding the Japan Sea were used for this purpose. Manabe (1957, 1958), first to investigate this problem, found that the amount of sensible heat supplied from the sea to the atmosphere is often as much as 1000 ly day^{-1} during the period of a typical outburst (for example, late Dec. 1954). It is much larger than the heat supplied by radiation or condensation. The sensible heat supplied from the sea averaged approximately 500 ly day^{-1}

during the entire three months of winter (Dec. 1954—Feb. 1955). The amount of latent heat supplied through evaporation from the sea was estimated to reach about a half of the sensible heat supply. Manabe (1957) pointed out that the transport mechanism of heat may be different from that of water vapor in an unstable stratification as is observed over the Japan Sea in winter. The reason for this is that the Bowen ratio, which Manabe obtained, was remarkably larger than would be expected from the mixing length theory.

Recently, Ninomiya (1964) and Nakamura (1964)** evaluated these quantities along the line adopted by Manabe and obtained similar results. These results are summarized in Table 1. Ninomiya, in particular, performed

Table 1.

Author	Sensible heat (ly day^{-1})	Latent heat (ly day^{-1})	Remark
Manabe (1957)	1030	450	outburst
Manabe (1958)	555	340	mean
Ninomiya (1964)	1120	320	heavy snow
Nakamura (1964)	1000	500	outburst
Kondo (1964)		495	outburst

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the budget analysis over both the Japan Sea and the Japanese Islands during 10 days (January 16—25, 1963) of heavy snowfall in the northwest coastal region of the Japanese Islands. Data obtained from the special radiosonde observations, gathered by a research project during heavy snowfall in the Hokuriku District, were added to the usual radiosonde data. It was shown that sensible heat supply increased remarkably in the situation in which a cold vortex was located over the Japan Sea. Furthermore, Ninomiya found that the liquid and/or solid states of water should be taken into account since the types of clouds formed over the Japan Sea affect seriously the snowfall in the northwest coastal region of the Japanese Islands.

The above investigations were based on the heat budget all over the Japan Sea. Kondo (1964), however, estimated the amount of evaporation from the Japan Sea by using his evaporation diagram and obtained the value of 495 ± 55 ly day⁻¹. This estimation was made for the period from 21 to 31 January of 1962 during which no cyclone passed through over the Sea.

On the other hand, Saito (1938), Arakawa (1940), Kurooka (1957) and others investigated how an air-mass is modified during the passage over the Japan Sea and attempted to explain some of the typical observed phenomena. Their explanations were based on the heat conduction theory applied first by Taylor (1915) to the study of an air-mass transformation off Newfoundland.

Takahashi (1940) criticized that in the above treatments the coefficients of eddy diffusion were assumed to be constant despite the fact that the magnitude of the coefficients depends strongly on the static stability. Burke (1945) employed an approach similar to Takahashi's in the study of air-mass transformation off the east coast of the United States.

In the present treatment, the exchange coefficients are assumed to be functions of the static stability and the phase change of moisture is taken into consideration. Our approach, however, differs from both Takahashi's and Burke's mentioned above.

2. Brief review

Taylor (1915) and many others studied air-

mass transformation by applying the theory of heat conduction in a semi-infinite rod to the atmosphere. They solved the equation of heat conduction,

$$\frac{\partial \theta}{\partial t} = K \frac{\partial^2 \theta}{\partial z^2}, \quad (2.1)$$

under the following type of boundary and initial conditions.

$$\theta = \theta_a + \gamma z \quad \text{for } z > 0, t = 0, \quad (2.2)$$

$$\theta = \theta_s \quad \text{for } z = 0, t \geq 0, \quad (2.3)$$

where θ is the potential temperature, K the coefficient of eddy conductivity and $\gamma \equiv \partial \theta / \partial z$ which is assumed to be constant. θ_s and θ_a , respectively, denote the potential temperatures of water and air at the sea surface. t and z are time and vertical coordinate. The solution obtained is

$$\theta = \theta_a + \gamma z + (\theta_s - \theta_a) \left\{ 1 - E \left(\frac{z}{\sqrt{4Kt}} \right) \right\}, \quad (2.4)$$

where E represents the error function, *i.e.*,

$$E(\xi) = \frac{2}{\sqrt{\pi}} \int_0^\xi e^{-x^2} dx. \quad (2.5)$$

Let us define z_m as the height at which the potential temperature, after a given time t , has increased by $(\theta_s - \theta_a)/2$ and t_m as the time after which θ has risen by $(\theta_s - \theta_a)/2$ at a given level. According to Haurwitz (1941), there exists an approximate relationship between z_m and t_m that

$$z_m = 0.477 \sqrt{4Kt_m}. \quad (2.6)$$

The heating effect spreads upwards rapidly within the lowest layer, and it proceeds slowly towards higher altitudes. When $K = 1 \sim 10$ m² sec⁻¹, for example, the height of mean heating z_m after 1 day is of the order of several hundred meters. It requires approximately 5 days for z_m to reach 0.7 km with $K = 1$ m² sec⁻¹ and 2 km with $K = 10$ m² sec⁻¹. In this case, the sensible heat supplied from the sea surface during time period, τ , can be approximated as follows

$$H_\tau = 2 C_p \rho (\theta_s - \theta_a) \sqrt{\frac{K\tau}{\pi}}, \quad (2.7)$$

where $C_p=0.24$ cal gr^{-1} , the specific heat of air at constant pressure, and $\rho=1.3\times 10^{-3}$ gr cm^{-3} , the density of air. If we adopt $K=10$ $\text{m}^2 \text{sec}^{-1}$, $\theta_s-\theta_a=10^\circ\text{C}$ and $\tau=1$ day, H_τ becomes 325 ly day^{-1} .

Priestley (1957) solved equation (2.1) under the following boundary condition to evaluate the heat flux at the surface. He assumed that the heat flux at the surface is proportional to the difference between air and surface temperatures and also to the surface wind speed *i.e.*,

$$H_0 = -\rho C_p C U (\theta_a - \theta_s) \quad \text{at } z=0, \quad (2.8)$$

and

$$\theta = \theta_i + \gamma z \quad \text{at } t=0. \quad (2.9)$$

θ_s is the potential temperature of water at the surface and θ_i the potential temperature of the air close to the surface at $t=0$. θ_a and U are the potential temperature of the air and the wind speed close to the surface respectively (approximately 10 meters altitude). C is a constant coefficient. When θ_s is assumed to be constant, the solution is

$$\theta = \left(\theta_i - \frac{K\gamma}{CU} \right) \psi(\xi, \tau) + \gamma z + \frac{K\gamma}{CU}, \quad (2.10)$$

$$\psi(\xi, \tau) = E(\xi) + e^{2\xi\tau + \tau^2} \{1 - E(\xi + \tau)\}, \quad (2.11)$$

where $\xi = z/\sqrt{4Kt}$ and $\tau = CU\sqrt{t/K}$. The heat supply for 1 day is estimated to be 235 ly for $C=0.0026$, $U=10$ m sec^{-1} , $\theta_s-\theta_i=10^\circ\text{C}$ and $K=10$ $\text{m}^2 \text{sec}^{-1}$. Even if K increases to 10^2 $\text{m}^2 \text{sec}^{-1}$, it would attain only 400 ly day^{-1} . Therefore, an intense upward transfer of heat through a fairly deep layer would be expected over the Japan Sea during a certain period in winter. On the other hand, the continental polar air is usually characterized by a remarkable inversion in the lower layer which may be liable to prevent the formation of severe, tall convection. Hence the behavior of an inversion is studied simultaneously with the process of upward transfer of sensible and latent heat.

3. Basic model for numerical experiment

The classical theory of air-mass transformation due to the heat conduction will be improved by taking account of the effects of moisture and the variable eddy exchange coef-

ficients which depend on static stability in the model.

In order to increase space resolution in the lower layer of the atmosphere, we introduce a new vertical coordinate, Z , which is similar to the special coordinate system proposed by Murakami (1964):

$$Z = \left(\frac{z}{H} \right)^{1/2}. \quad (3.1)$$

Here H is assumed to be 8 km for convenience. The atmospheric layer with a depth of 8 km is divided into 20 sublayers, all being equal in Z . The geometrical height for each level and its corresponding pressure in the standard atmosphere are listed in Table 2.

Table 2. Conversion of coordinates.

j	Z	$z(\text{m})$	$p(\text{mb})$
1	0	0	1,013
2	0.05	20	1,011
3	0.1	80	1,004
4	0.15	180	993
5	0.2	320	975
6	0.25	500	955
7	0.3	720	930
8	0.35	980	901
9	0.4	1,280	869
10	0.45	1,620	833
11	0.5	2,000	795
12	0.55	2,420	754
13	0.6	2,880	712
14	0.65	3,380	667
15	0.7	3,920	622
16	0.75	4,500	577
17	0.8	5,120	532
18	0.85	5,780	486
19	0.9	6,480	441
20	0.95	7,220	397
21	1	8,000	356

If we assume a steady state of air flowing with a constant speed U along the x -axis and neglect the horizontal diffusion, the following equations may be obtained,

$$U \frac{\partial \theta}{\partial x} = \frac{1}{2HZ} \frac{\partial}{\partial Z} \left(\frac{K_\theta}{2HZ} \frac{\partial \theta}{\partial Z} \right) + Q, \quad (3.2)$$

$$U \frac{\partial q}{\partial x} = \frac{1}{2HZ} \frac{\partial}{\partial Z} \left(\frac{K_q}{2HZ} \frac{\partial q}{\partial Z} \right) - M, \quad (3.3)$$

$$U \frac{\partial m}{\partial x} = \frac{1}{2HZ} \frac{\partial}{\partial Z} \left(\frac{K_m}{2HZ} \frac{\partial m}{\partial Z} \right) + M. \quad (3.4)$$

These equations govern the distribution of the potential temperature, θ , the specific humidity q and the liquid (solid) water content, m , respectively. Q is the rate of heating due to condensation (sublimation) of water vapor. The negative value of Q represents the rate of cooling due to the evaporation (sublimation) of liquid (solid) water. M is the rate of condensation (sublimation). K_θ , K_q and K_m denote the eddy exchange coefficients for sensible heat, water vapor and liquid (solid) water. We assume here, however, that $K_\theta = K_q = K$ and $K_m = 0$.

The change, δq_s , of the saturation specific humidity q_s following an air parcel moving horizontally can be expressed by that of θ , $\delta\theta$, assuming an isobaric process, *i.e.*,

$$\delta q_s = \frac{L}{R_v} \left(\frac{p_0}{p} \right)^\kappa \frac{q_s}{\theta^2} \delta\theta, \quad (3.5)$$

where L is the latent heat of water vapor, R_v the gas constant of water vapor, p_0 a reference pressure (1000 mb) and $\kappa = (c_p - c_v)/c_p$ in which c_p and c_v denote the specific heats of air at constant pressure and volume, respectively. Here the saturation specific humidity q_s is evaluated using the Tetens' empirical formula (1930), *i.e.*,

$$q_s = 3.8 \times 10^{\frac{aT}{T+b}} / p, \quad (3.6)$$

where T is the temperature in centigrade and p the pressure in millibar. $a=7.5$ and $b=237.3$ over water, while $a=9.5$ and $b=265.5$ over ice.

The procedure for solving the set of the equations (3.2)–(3.5) is outlined below. The boundary conditions will be described in Section 4.

(i) First, the terms of Q and M are omitted from the equations (3.2)–(3.4), neglecting phase change of water substance. Then, θ , q , m and q_s are computed tentatively, which are denoted by θ^* , q^* , m^* and q_s^* .

(ii) If $q^* - q_s^* = \delta M > 0$ (that is, if the air is over-saturated), δM is divided into two parts. One is the condensed water δM_1 , and the other is the increase of water vapor to be stored, δM_2 . The warming due to condensation, $\delta\theta_1$, is

$$\delta\theta_1 = \frac{L}{c_p} \left(\frac{p_0}{p} \right)^\kappa \delta M_1. \quad (3.7)$$

Then, δM_2 is obtained using Clapayron's law, as follows,

$$\begin{aligned} \delta M_2 &= \frac{L}{R_v} \left(\frac{p_0}{p} \right)^\kappa \frac{q_s}{\theta^2} \delta\theta_1 \\ &= \frac{L^2}{c_p R_v} \left(\frac{p_0}{p} \right)^{2\kappa} \frac{q_s}{\theta^2} \delta M_1. \end{aligned} \quad (3.8)$$

Thus, the ratio r_1 , of δM_1 to $\delta M (= \delta M_1 + \delta M_2)$ is obtained as

$$r_1 = 1 / \left\{ 1 + \frac{L^2}{c_p R_v} \left(\frac{p_0}{p} \right)^{2\kappa} \frac{q_s}{\theta^2} \right\}. \quad (3.9)$$

Fig. 1. The vertical distributions of the ratio, $r_1(\delta M_1/\delta M)$, for the surface temperature of 0°C and 30°C, respectively, with the lapse rate of $6.5^\circ\text{C km}^{-1}$.

The vertical distributions of r_1 in some cases are shown in Fig. 1 as a reference. r_1 increases with decreasing temperature and δM_1 becomes larger than δM_2 in the situation concerned here.

We then correct the tentative values, θ^* , q^* , m^* and q_s^* to those at a just-saturated state, as follows:

$$\left. \begin{aligned} \theta &= \theta^* + \frac{L}{c_p} \left(\frac{p_0}{p} \right)^\kappa r_1 \delta M, \\ q &= q^* - r_1 \delta M, \\ m &= m^* + r_1 \delta M, \\ q_s &= q. \end{aligned} \right\} \quad (3.10)$$

(iii) If $q^* - q_s^* \equiv \delta M < 0$ and $m^* > 0$, evaporation should be expected. The maximum possible evaporation would be $-r_1 \delta M$.

a) If $m^* + r_1 \delta M \geq 0$, the evaporation attains the maximum possible value $-r_1 \delta M$. Thus, the adjusted values of θ , q , m and q_s can be computed from equations (3.10).

b) If $m^* + r_1 \delta M < 0$, the total amount of the existing liquid (solid) water m^* should evaporate, and hence,

$$\left. \begin{aligned} \theta &= \theta^* - \frac{L}{c_p} \left(\frac{p_0}{p} \right)^\kappa m^*, \\ q &= q^* + m^*, \\ m &= 0, \\ q_s &= q_s^* - \frac{L^2}{c_p R_v} \left(\frac{p_0}{p} \right)^{2\kappa} \frac{q_s^*}{\theta^{*2}} m^*. \end{aligned} \right\} \quad (3.11)$$

Fig. 3(a). Mean soundings at Vladivostok and Wajima during January 1963. Solid lines denote temperature and broken lines dew-point temperature. Thin slant lines are dry and moist adiabats of 270°K , respectively.

(iv) In the other cases, a correction to the tentative values obtained in (i) is not required.

4. Boundary conditions

We will adopt the atmospheric layer of 8 km in depth and 800 km in horizontal distance to approximate the fetch of the Japan Sea from Vladivostok in Siberia to Wajima in Japan as shown in Fig. 2.

The mean sounding at Vladivostok P_1 , dur-

Fig. 2. The region to be considered for numerical experiment.

Fig. 3(b). Temperature and dew-point temperature profiles assumed at the point P_1 for experiment.

ing January 1963 is illustrated in Fig. 3(a). The remarkable stable layer (inversion) ranging from 750 to 900 mb level which may be the result of an extreme cooling in its low layer should be mentioned. The mean sounding at Wajima close to the terminal point P_2 during the same period is shown for comparison in Fig. 3(a). The inversion layer, which separates the lower moist unstable layer from the upper dry stable layer, is usually observed around the 800 mb level at Wajima in winter. The inversion is not, however, as stationary nor noticeable as the one at Vladivostok, so that it may become indistinct on the average over a period of a month. The relative humidity around the 800 mb level is almost always over 90% and a cloud layer exists there constantly. The condition assumed at the starting point, $x=0$, is shown in Fig. 3(b). It is characterized by an isothermal layer (about -20°C) below around the 700 mb level ($j=13$) and by a constant lapse rate of $6.5^\circ\text{C km}^{-1}$ above. The relative humidity is assumed to be 80% throughout the whole layer. It should be noted that the air above around the 700 mb level at Wajima was traced back westwards. Hence the temperature profile above the 700 mb level at the starting point was simulated using temperature soundings at Osan in the middle part of Korea.

Fig. 4 shows the distributions of the mean surface water temperature T_1 and the air temperature at the anemometer level T_2 from Vladivostok to Wajima. These temperature distributions are shown by broken lines for the period of 30 years and circles for the month of January 1963*.

The solid line shows the surface water temperature distribution used in the present experiment. The lower boundary conditions assumed here are prescribed by the fluxes of sensible heat and water vapor in the following forms,

$$\left. \begin{aligned} F_s &= \rho c_p C U (\theta_1 - \theta_2), \\ F_v &= \rho C U (q_1 - q_2), \end{aligned} \right\} \quad (4.1)$$

where θ_1 and q_1 denote the potential tempera-

Fig. 4. Mean surface water temperature (T_1) and air temperature at the anemometer level (T_2) from Vladivostok to Wajima over the Japan Sea during 30 years are shown in broken line and those during January 1963 in circles. Solid line is the surface water temperature assumed for experiment.

ture and the saturation specific humidity at $Z=0$, while θ_2 and q_2 represent the potential temperature and the specific humidity at $Z=0.05$; C is the transfer coefficient and is analogous to the drag coefficient. Deacon *et al.*

Table 3. The cases used for numerical experiment $K=K^*/[1+\alpha(\partial\theta/\partial z)]$. K^* in $\text{m}^2 \text{sec}^{-1}$ and α in $\text{m } ^\circ\text{C}^{-1}$.

Case	C	K^*	α	Remarks
1	0.0018	10	0	dry model without moisture supply
2	"	"	"	
3	"	1	"	
4	"	50	"	
5	0.0026	10	"	variable K below this case
6	0.0036	"	"	
7	0.0018	"	10^2	
8	"	"	5×10^2	
9	0.0026	"	10^2	
10	0.0018	"	"	relative humidity of 20%
11	"	"	"	top of inversion at $j=10$
12	"	"	"	sublimation occurred

* The data used have been provided by T. Fujita, Meteorological Research Institute in Tokyo. Fujita summarized the observational data by ships over the Japan Sea during January 1963.

(1956) obtained the drag coefficient ranging from 0.001 to 0.003 under the condition of $U=10 \text{ m sec}^{-1}$ at 10 m above the sea surface in the neutral stratification. Jacobs (1942) found $C=0.0022$ from the calculation of energy balance over the major oceans using climatological data. At the upper boundary, $Z=1$, no fluxes of sensible heat and water vapor are assumed.

The following 12 cases were studied for various values of C and K as listed in Table 3.

5. A check of computation

We adopt the forward difference method (along the direction of flow) for equations (3.2)

Fig. 5. Solid line shows the exact solution which represents the vertical profile of temperature increase at the terminal point. Crosses and circles denote the corresponding solutions obtained by numerical integration in z and Z coordinates, respectively.

—(3.4). The space difference Δx necessary to satisfy the computational stability criterion should be taken as follows (Richtmyer, 1957),

$$\Delta x < \frac{U_{\min}(2HAZ \cdot Z_{\min})^2}{2K_{\max}} \quad (5.1)$$

If we assume the minimum value of U , $U_{\min}=10 \text{ m sec}^{-1}$ and the maximum value of K , $K_{\max}=10 \text{ m}^2 \text{ sec}^{-1}$, then

$$\Delta x < 800 \text{ m}, \quad (5.2)$$

for $H=8 \text{ km}$, $\Delta Z=0.05$ and $Z_{\min}=0.05$.

As mentioned in Section 2, equation (2.1) with a constant exchange coefficient has been solved analytically under the boundary condition (2.2) and the initial condition (2.3). Hence, the exact solution (2.4) is used to check the accuracy of the present method of numerical integration. Fig. 5 illustrates the vertical distribution of the temperature increase ΔT at 800 km downwind from that at the starting point. The exact solution is illustrated by the solid line. Crosses and circles show the approximate solutions obtained by the numerical integration on the basis of the z coordinate, in which 400 meters is used for Δz , and the Z coordinate, respectively. Here we assumed that $U=10 \text{ m sec}^{-1}$, $K=10 \text{ m}^2 \text{ sec}^{-1}$ and the difference between water and air temperatures at the sea surface is 10°C . The agreement between the numerical solutions obtained in both coordinate systems and the exact solution is very good.

6. Results of calculations

We will discuss the major results obtained from the 12 cases listed in Table 3. For convenience, the results are shown by converting potential temperature θ into temperature T .

(1) Inclusion of moisture

Fig. 6 shows the vertical distributions of temperature increases at the terminal point for cases 1 and 2, and of water vapor increase, liquid water content and relative humidity for case 2. The cloud layer may be observed between the altitudes from 300 to 1000 meters, and the total amount of liquid water in the cloud layer was 0.44 mm. On the other hand, the amounts of water vapor and sensible heat supplied from the sea were 4.3 mm (250 ly) and 340 ly, respectively. This indicates that only

Fig. 6. Vertical profiles of temperature increases (solid line in case 2, circles in case 1), water vapor increase (broken line), liquid water content (dotted line) and relative humidity (solid line on the left side) at the terminal point, respectively.

10% of the water vapor supplied from the sea is consumed by the condensation and the remaining 90% is stored in the form of vapor, since the remarkable warming concurrently occurs owing to a great amount of sensible heat supply. Inclusion of water vapor in case 2 contributed to increase the temperature of the lower part of the atmosphere by only 0.5°C compared with case 1. Although the amount of condensed water increases much more in other cases as shown below, the direct influence of condensation through eddy exchange mechanism considered here may be rather slight upon the temperature field.

(2) Constant exchange coefficient

The dependence of air-mass transformation on the magnitude of exchange coefficient K is shown in Fig. 7(a). As is expected, the rate of transformation becomes more effective as the coefficient increases. In other words, the initial inversion layer was destroyed only below 800 m in case 3. However, the inversion has almost disappeared for $K=50\text{ m}^2\text{ sec}^{-1}$.

The sensible heat supply, however, does not necessarily increase with increasing K . It is shown in Fig. 7(c) that the sensible heat supply in case 4 only attains a value of about 300 ly. This value is a little smaller (by 30 ly) than that in case 2. The increase of K above a certain value did not contribute to increase the sensible heat supply unless the other conditions, such as the magnitude of transfer coefficient at the sea surface, were changed. The reason for this is due to: (i) less upward heat transport in the low layer resulting from rapid formation of neutral stratification and, (ii) the increase in the downward transport in the stable upper layer with an increase of K . Hence, the surface air temperature in the lowest level tends to approach the surface water temperature and the sensible heat supply decreases (Fig. 7(b)).

We should notice that no cloud layer is found in case 4 with the exception of the saturation in the uppermost layers due to the fictitious boundary condition, that is, no flux of entities is assumed at the upper boundary. Thus, the assumption of a constant exchange coefficient seems to bring on an unrealistic result.

(3) Transfer coefficient at the lower boundary

Here we took three different constants for the transfer coefficient at the lower boundary, C , for the same value of exchange coefficient, $K=10\text{ m}^2\text{ sec}^{-1}$. The results of those are illustrated in Fig. 8(a), (b) and (c). The amounts of sensible and latent heat supply and the profiles of temperature and moisture in case 2, are not so different from those for cases 5 and 6 in which $C=0.0026$ and $C=0.0036$, respectively, were used as coefficients. It is indicated in Fig. 8(c) that the supply of both sensible and latent heat increases only by about 20% as the magnitude of the coefficient doubles. This is easily understood from Fig. 8(b) which shows both the temperature and the specific humidity in the lowest level approaching those of the sea surface as the transfer coefficient increases. Therefore the increase of transfer coefficient at the lower boundary is not so effective in increasing heat supply from the sea, unless it is associated with an appropriate increase of upward heat transport in the upper layer adjacent to the boundary layer. This is indicated clearly below.

Fig. 8(a), (b), (c). Same as Fig. 7 except for circles in case 5 and crosses in case 6.

Fig. 9(a), (b), (c). Same as Fig. 7 except for circles in case 7 and crosses in case 8. The vertical profiles of K at the terminal point are added in (a) and case 9 denoted by triangles in (b) and (c). Broken lines are the same as in Fig. 4.

The assumption that C and K are independent of each other may possibly become one weakness in the present model.

(4) Variable exchange coefficient

The coefficient of eddy exchange K is assumed to depend only upon the static stability in the following form,

$$K = \frac{K^*}{1 + \alpha \frac{\partial \theta}{\partial z}} = \frac{K^*}{1 + \frac{\alpha}{2HZ} \frac{\partial \theta}{\partial Z}} \quad (6.1)$$

The relation (6.1) was derived empirically by Takahashi (1963)**. It implies only that K increases with decreasing stability and decreases with increasing stability. This is similar to the expression, $K = K^* \exp(-\alpha(\partial\theta/\partial z))$ adopted by Fisher and Caplan (1963) in the prediction model of fog. K^* is the exchange coefficient which might be expected under adiabatic conditions and is prescribed to be $10 \text{ m}^2 \text{ sec}^{-1}$ here. α is assumed to be $10^2 \text{ m}^\circ \text{C}^{-1}$ except that in case 8, it is assumed to be $5 \times 10^2 \text{ m}^\circ \text{C}^{-1}$.

The vertical profiles of temperature and moisture at the terminal point P_2 are shown in Fig. 9(a) for cases 2, 7 and 8, respectively. Cases 7 and 8 are compared with case 2. In the former cases, the exchange coefficients depend to a variable degree on the static stability as contrasted with case 2 in which there is a constant exchange coefficient. The increases in temperature and moisture in cases 7 and 8 are less in the layer below the 200 meter level and greater in the higher layer when compared with case 2.

Cases 7 and 8 are characterized by the presence of deeper cloud layers with higher bases. These cloud layers have the liquid water content amounting to 1.0 and 1.9 mm, respectively. They acquire 20 and 30% of the water vapor supplied, respectively. If the liquid water content is reduced by one half as the clouds move through the distance of 100 km downwind from the point P_2 , the amount of precipitation occurring would be about 4 and 8 mm per day in cases 7 and 8, respectively. These amounts of cloud and precipitation expected may be fairly realistic during the period of moderate cold air outbursts.

In addition to the increase of sensible and

latent heat supplies in the cases with variable exchange coefficients, we should notice a remarkable increase of them in case 9. This is shown by triangles in Fig. 9(c). The sensible and latent heat supplies in case 5 increase only 45 and 25 ly day^{-1} , more than those in case 2, respectively. These cases are compared under conditions of the same constant exchange coefficient. Their increases attain 120 and 150 ly day^{-1} , respectively under conditions of a variable exchange coefficient (compared case 9 with case 7). It is thus suggested that the increase of transfer coefficient C is very effective in increasing the heat supplied from the sea when associated with an appropriate increase of exchange coefficient in the layer adjacent to the boundary layer.

In each case of experiment, the difference between the surface water temperature and the lowest level air temperature is assumed to be about 20°C at the starting point. This temperature difference decreases rapidly to a half of its original value before arriving at a point 100 km downwind. The difference then becomes constant after the air reaches a point 200 km downwind. As for the water vapor, the corresponding difference in the values of specific humidities tends to increase with the distance from 100 km downwind (Fig. 9(b)).

The stratification which might be expected at the terminal point in case 8 is shown in Fig. 10. The inversion layer in the model did not disappear in each case except for the

Fig. 10. Sounding to be expected at the terminal point in case 8.

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Fig. 11. Cross sections in the vertical plane along the wind from P_1 to P_2 . Solid lines show liquid water content in g kg^{-1} , broken lines cloud layers and thick solid lines the top of unstable layer. (a) is for case 7, (b) for case 8, (c) for case 10 and (d) for case 11.

use of an abnormally large magnitude for the exchange coefficient. Usually, an inversion layer is observed around the 800 mb level over the Japanese Islands, although it often disappears during the period of heavy snowfall.

Fig. 11 shows the distributions of cloud layer (bounded by a broken line), liquid water content (solid line, g/kg) and the top of unstable layer (or the base of inversion). This diagram is made in the vertical plane along the wind from P_1 to P_2 . In cases 7 and 8 (illustrated in Figs. 11(a) and (b), respectively), clouds form in the restricted low layer immediately after leaving the point P_1 and then the top and the thickness of the cloud layer gradually increase downwind. The cloud top reaches the 1800 meter level at the terminal point where the cloud extends into the stable layer by several hundred meters. It may be seen from Figs. 11(c) and (d) that the origin of the cloud with respect to the starting point depends on the humidity of the initial airmass and the temperature difference between the surface water and air. In case 10, with extremely

dry air having a relative humidity of 20%, clouds form around 400 km downwind from the point P_1 . Since an air layer with relative humidity of 50~60% is most often observed around Vladivostok, we can expect a cloud formation to be observed as far as central parts of the Japan Sea. In case 11, with a rather small difference between the surface water and air temperatures, clouds are found at the distance of 500 km from the point P_1 . This condition would correspond to a situation over a continent where the polar air undergoes less cooling. In both cases 10 and 11, however, only small amounts of liquid water are contained in the thin cloud layers below approximately 1000 meters. These quantities would be 0.37 and 0.16 mm, respectively. Hence, the precipitation to be expected is too little.

Case 12, in which sublimation is assumed to occur, is little different from case 7 except for a small increase of water content in the present model.

7. Concluding remarks

It was shown in this paper that the diffusion model with a constant exchange coefficient fails to explain consistently phenomena of air mass transformation over the Japan Sea in winter. As far as moderate weather conditions are concerned, the revised diffusion model can simulate the characteristics of the thermal and moisture patterns and of energy exchange over the Sea. On the other hand, the present model will encounter serious problems which will remain unsolved when this model is applied for severe weather conditions. During this period, a number of cumulus and cumulonimbus, sometimes with thunder, have already formed over the Japan Sea and are often reported to reach the 6 km level or so. As suggested in Section 6, we may have to take into account the intense upward heat transport due to cumulus convection in the free atmosphere together with smaller scale eddy transport processes in the boundary and subcloud layers in the air-mass transformation.

In addition, a cumulus convection, especially tall cumulonimbus, will generally not develop in the cold season without the heat and moisture from outside of the system (for example, Asai (1964)). In the present case, however, we can expect a great deal of heat supply from the lower boundary and subcloud layers which also depends on the activity of cumulus convection in the upper layer. Another more important matter to be considered is the influence of larger scale atmospheric situations on the cumulus convection. Based on the synoptic analysis during the heavy snowfall period in January 1963, the cold cyclonic vortex observed typically in the middle level of the atmosphere over the Japan Sea was indicated to be associated with a heavy snowfall in the Japanese Islands (Matsumoto *et al* (1964) and Ninomiya (1964)). This situation may provide less stable stratification through deep layers and general upward motion associated with the horizontal convergence in the low layer.

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冬期日本海上における気団変質についての数値実験

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大陸極気団が暖かい日本海上を吹送する間に海面から顕熱・潜熱の補給を受けて変質する過程は冬期裏日本における降雪の重要な原因の一つである。

ここでは、渦拡散論に基づく既存の気団変質論に、水蒸気の相変化と、静的安定度に依存する渦拡散係数の可変性を考慮に入れて拡張した若干のモデルについて数値解を求め、その結果を検討した。

冬期日本海上での気団変質——海面から熱・水蒸気の補給、それらの大気中での分布、水蒸気の一部凝結等——を説明することは、一定係数の渦拡散論では困難であるが、安定度に伴って変化する渦拡散係数を用いれば少くとも冬期の平均状態に関する限り可能であることが示される。

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