NOTES AND CORRESPONDENCE (要報)

Comments on the Preferred Size of Steady Cumulus Convection Cell

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In a recent article, Asai and Kasahara (1967) developed an interesting model of cumulus convection cell. The compensating downward motion associated with the updraft was taken into account in the model. Cooperated with the hypothesis of maximum upward transport of sensible heat, this model was able to determine a preferred ratio between areas of ascending and descending motions in terms of relevant physical parameters such as the size of cell, the dissipation factor and the ratio between the static stabilities for the ascending and descending motions. A preferred size of cell remained to be determined however.

Asai (1967) then presented an extended model which included the effect of horizontal motion associated with the updraft and downward motions. The paper concluded that "the present model can determine a preferred size of steady cumulus convection cell....." However, the authors found some difficulties in following his mathematical development of Eq. (3. 13), a term representing the vertical exchange of horizontal momentum. The purpose of this note is to point out rather that, when slightly different boundary conditions are applied, the term for the vertical exchange of horizontal momentum vanishes identically and the conclusion cited above does not follow.

The boundary conditions used by Asai are

$$\begin{aligned} w &= 0 \quad \text{at } z=0 \quad \text{and } z=d, \\ u &= 0 \quad \text{at } r=0 \quad \text{and } r=b. \end{aligned} \quad (1)$$

Here the notations are the same as those used in Asai's paper. The velocity components are then divided into two parts; tangential averages (denoted by the tildes) and deviations from them (denoted by the primes). Asai assumed no eddy transport of momentum and heat through the

boundaries, *i. e.*

$$\begin{aligned} \widetilde{Aw'} &= 0 \quad \text{at } z=0 \quad \text{and } z=d, \\ \widetilde{Au'} &= 0 \quad \text{at } r=b. \end{aligned} \quad (2)$$

Instead of (2), let us now apply the following boundary conditions and see what follows:

$$\begin{aligned} w' &= 0 \quad \text{at } z=0 \quad \text{and } z=d, \\ u' &= 0 \quad \text{at } r=0 \quad \text{and } r=b. \end{aligned} \quad (3)$$

The term concerned here is the left hand-side of Eq. (3. 13) of Asai's paper. By definition, it is given by

$$\begin{aligned} \int_0^b [F_r]_{z=0}^{z=d} dr \\ = \int_0^b \left[-\frac{u'u'}{r} - \frac{\partial u'u'}{\partial r} - \frac{\partial u'w'}{\partial z} \right]_{z=0}^{z=d} dr \end{aligned} \quad (4)$$

The second term of the right-hand side of (4) vanishes, corresponding to the boundary conditions (3). With the aid of the continuity equation:

$$\frac{u'}{r} = -\frac{\partial u'}{\partial r} - \frac{\partial w'}{\partial z},$$

Eq. (4) is transformed to

$$\begin{aligned} \int_0^b [F_r]_{z=0}^{z=d} dr &= \int_0^b \left[\frac{\partial u'w'}{\partial z} \right]_{z=0}^{z=d} dr \\ &\quad - \int_0^b \left[w' \frac{\partial u'}{\partial z} \right]_{z=0}^{z=d} dr - \int_0^b \left[\frac{\partial u'w'}{\partial z} \right]_{z=0}^{z=d} dr \end{aligned} \quad (5)$$

The second term of the right-hand side of (5) again vanishes. It then follows

$$\int_0^b [F_r]_{z=0}^{z=d} dr = 0 \quad (6)$$

This is the counterpart of (3. 13).

With (6) instead of (3. 13), k_2 in (3. 17) of Asai's paper is given by

$$k_2 = \frac{2l\tau^2}{a^3(1-\sigma^2)^2} \left\{ 1 - \frac{\ln\sigma}{\mu(1-\mu)} \left(\frac{a}{d} \right)^2 \right\}^{-1},$$

and k_2 in (3. 22) becomes

$$k_2 = \frac{2\alpha^2}{a(1-\sigma^2)} \left\{ 1 - \frac{\ln\sigma}{\mu(1-\mu)} \left(\frac{a}{d} \right)^2 \right\}^{-1}$$

Finally, the rate of vertical heat transport through the unit horizontal cross section per unit time [Eq. (4. 2)] is given by

$$H_\theta = c_p \rho_0 \left(\frac{g}{\theta} \right)^{1/2} S_b^{3/2} \left(\delta - \frac{\sigma^2}{1-\sigma^2} \right)^{3/2} \times \frac{\sigma^2(1-\sigma^2)^{9/2} a^2}{4\alpha^4} \left\{ 1 + \frac{(1-\sigma^2)}{\alpha^2} \frac{a}{d} \right\}^{-3/2} \quad (7)$$

Fig. 1 is the counterpart of Fig. 2a of Asai's paper which represents the vertical heat transport

H_θ as a function of σ^2 , the fraction of the total area covered by the updraft, and a/d , the ratio of the radius of the updraft region to the depth of the cell. Both Eq. (7) and Fig. 1 show that H_θ goes to infinity as (a/d) increases to infinity. In other words, this model, cooperated with the hypothesis of maximum heat transport and under the boundary conditions (3), would not yield a preferred size of steady cumulus convection cell.

This note then concludes that a further study is needed of the proper formulation of the vertical exchange of horizontal momentum, or of some other physical processes through which the development of cumulus clouds with large (a/d) is inhibited.

References

Asai, T. and A. Kasahara, 1967: A theoretical study of the compensating downward motions associated

Fig. 1 Upward heat transport as a function of the ratio of the radius of updraft to its depth (a/d) and of the fraction of the total area covered by the updraft region (σ^2). Solid contour lines show $H_\theta/(c_p \rho_0)$ in units of 10^{-2} deg cm sec $^{-1}$.

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起りやすい定常積雲対流の大きさについて

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