A Note on the Barotropic Instability of the Tropical Easterly Current*

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1. Introduction

The stability of barotropic zonal currents in the atmosphere with respect to wave disturbances has been studied by Kuo (1949), Lipps (1962, 1965) and others. But these studies were mostly concerned with the case of westerly zonal currents. The barotropic stability of easterly zonal currents has not been fully examined.

It has been known that there exist synoptic-scale wave disturbances in the trade easterlies in the Pacific and in the Caribbean Sea (Palmer, 1951; Riehl, 1954). These wave disturbances are known to possess the "cold-core" structure. The air to the east of the wave axis, where the most active weather is observed, is colder than the surrounding air. The wave accompanies upward motions to the east of the wave axis and downward motions to the west (Krishnamurti and Baumhefner, 1966; Yanai and Nitta, 1967). The mechanism that maintains this cold-core circulation is still somewhat a puzzle. Palmer (1951) and Yanai (1961) have suggested that kinetic energy of the wave is supplied from the trade easterly current which has a horizontal shear. Charney (1963) argues that in the absence of condensation large-scale motions in the tropics tend to be quasi-horizontal and quasi-non-divergent. If this argument is correct, the barotropic instability may be of some importance in the dynamics of tropical disturbances.

2. Difference between the instability properties of the westerly current and those of the easterly current

For simplicity we discuss the stability of a barotropic zonal current confined between two latitudes based on the linearized vorticity equation

$$\left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x}\right) \nabla^2 \psi + (\beta - U'') \frac{\partial \psi}{\partial x} = 0 \quad (1)$$

where x and y are Cartesian co-ordinates directed toward the east and north respectively, ψ the stream function for the perturbed motion, $\beta = df/dy$, f the Coriolis parameter, $U(y)$ is the basic current and a prime denotes a differentiation with respect to y . We set

$$\psi(x, y, t) = \phi(y) e^{i\alpha(x - ct)},$$

then (1) becomes

$$(U - c)(\phi'' - \alpha^2 \phi) + (\beta - U'')\phi = 0 \quad (2)$$

where α is the wave number and c the phase velocity of the disturbance. c may be complex in general, i.e.,

$$c = c_r + ic_i \quad (3)$$

We impose boundary conditions at two latitudes $y=0$ and $y=2b$ as

$$\phi(0) = \phi(2b) = 0 \quad (4)$$

We divide the interval $2b$ into 20 subdivisions and obtain c and ϕ by the method of finite differences. The validity of the finite difference method and its accuracy were discussed at length in a previous paper (Yanai and Nitta, 1968). In this study we use $\beta = 2.29 \times 10^{-11} \text{m}^{-1} \text{sec}^{-1}$ (on the equator).

To demonstrate the difference between the stability characteristics of the westerly basic current and those of the easterly current, we consider simple velocity profiles given by

$$U_W = U_{max} \sin^2\left(\frac{\pi y}{2b}\right) \quad (5)$$

$$U_E = -U_{max} \sin^2\left(\frac{\pi y}{2b}\right) \quad (6)$$

where U_W and U_E denote westerly and easterly zonal current respectively, and $U_{max} (>0)$ is the maximum zonal wind speed. These profiles have inflection points at $y = y_i = b/2$. The minimum

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critical value of U_{max} denoted by U_c for which the gradient of absolute vorticity $\beta - U''$ vanishes somewhere in the belt is given, in both cases, by $U_c = 2\beta b^2/\pi^2$. When U_{max} is greater than U_c , $\beta - U'' = 0$ is satisfied at $y = y_\kappa$ in the interval $0 < y < b$, and unstable waves may exist.

In the case of the westerly basic current that has a symmetric velocity profile, Kuo (1949) proves that neutral waves whose phase velocity is greater than the maximum zonal current velocity do not exist. On the other hand neutral waves whose phase velocity is less than the minimum zonal current velocity can exist. Kuo further found that disturbances are unstable in the range between two critical wavelengths. The lower critical wavelength denoted by L_κ is the characteristic wavelength of the neutral wave which has a phase velocity $c = U(y_\kappa)$. The upper critical wavelength, L_0 is the minimum wavelength of

the Rossby-Haurwitz wave which has a phase velocity $c = U_{min}$.

We can show that there is only one mode of unstable wave for both the velocity profiles (5) and (6) and that their lower critical wavelengths are identical, i.e.,

$$L_\kappa = \frac{4b}{\sqrt{3}} = 2.31 b \quad (7)$$

On the other hand, we can show that in the case of easterlies given by (6) the upper critical wavelength does not exist. In other words, there are no long neutral waves of Rossby and Haurwitz and all disturbances are unstable for wavelengths longer than L_κ , providing $U_{max} > U_c$.

The difference between the stability characteristics of the westerly current and those of the easterly current is examined by numerical calculations of the growth rate and the critical curves for instability by the method of finite differences. Figs. 1 and 2 show the curves of the growth rate

Fig. 1. The critical curve and the growth rate of the unstable wave in the westerly basic flow given by (5). The growth rate is given in units of $10^{-11} b \text{ sec}^{-1}$. The numerical value of b (the half width of basic flow) should be given in meters.

Fig. 2. The critical curve and the growth rate of the unstable wave in the easterly basic flow given by (6).

of amplified disturbances for the westerly and the easterly velocity profiles respectively. For the westerly zonal current, unstable waves are confined between two critical wavelengths. On the other hand, all disturbances which have wavelengths longer than $2.31 \bar{b}$ are unstable for the easterly current. The growth rate of amplified disturbances in the easterlies is larger than that in the westerlies. For a moderate horizontal shear of the zonal current, the most preferred scale of unstable waves in the easterlies is larger than that in the westerlies. We remark that the difference of the stability characteristics between the westerly and the easterly currents arises from the β -term in (2). If we choose the x -coordinate directed westward, the form of (2) would be unchanged except for the sign of the β -term. An analogous situation exists in the problem of lee waves produced by obstacle which was discussed by Kasahara (1966).

3. Possibility of instability in the trade easterlies

We next examine the possibility of barotropic instability in the actual zonal currents in the tropics. Fig. 3a shows latitudinal distributions of the average zonal velocity and absolute vorticity at the surface, 1000-, 850-, 700-, and 600-mb levels observed over the Marshall Islands for June 1958. The velocity distributions which resemble half-jets between the equator and 20°N are typical for the western Pacific region. At the surface and at the 1000-mb level, the gradient of absolute vorticity changes its sign at about 8°N and amplified disturbances are expected to occur at these levels. Fig. 3b shows distributions of the average zonal velocity and absolute vorticity observed over the same area for July 1958. The condition $\beta - U'' = 0$ is met at the 700- and 600-mb levels. Fig. 4 shows distributions of the monthly mean zonal velocity and absolute vorticity at the 3-km level over the Line Islands area in the eastern Pacific from April to July 1962. The velocity profiles are nearly symmetric with respect to the jet axis which lies on the equator. It is seen that $\beta - U''$ is positive everywhere except for the velocity profile observed in July.

Using the Marshall Islands data at the surface of June 1958, we made the computation of the growth rate of amplified disturbances by the finite difference method. In this computation, the position of the boundaries is somewhat arbitrary. For an antisymmetric velocity profile similar to

Fig. 3. The mean zonal current and the corresponding absolute vorticity observed over the Marshall Islands at various levels for June 1958 (a) and those for July 1958 (b).

Fig. 4. The mean zonal current and the corresponding absolute vorticity observed at the 3-km level over the Line Islands area from April to July 1962.

the observed profile, the authors examined the dependence of the stability characteristics on the position of boundaries (Yanai and Nitta, 1968). Based on the result of that study and also on an

empirical fact that the amplitude of wave disturbances in the Marshall Islands area attains its maximum at about 10°N and decays off both polewards and equatorwards, we set the boundaries at the equator and at the latitude 20°N . Figs. 5a and 5b show the growth rate and the horizontal structure of the amplified wave respectively. The most preferred scale of the amplified wave is about 2,000 km and the maximum growth rate is $2.1 \times 10^{-6} \text{sec}^{-1}$ which corresponds to an e-folding time 5.2 days. The axis of the amplified wave is directed from southwest to northeast. These characteristics of amplified waves are somewhat similar to those of the easterly wave observed in

the western Pacific. If the daily velocity profiles were used instead of the monthly mean velocity profile, the growth rate could be larger.

Acknowledgments

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Fig. 5. The growth rate of the amplified disturbance for the surface velocity profile of June 1958 (a), and the streamfunction and the $u-v$ correlation of the most preferred wave (b).

熱帯偏東風の順圧不安定について

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CORRIGENDA

“Numerical Simulation of the Blocking Process” By Yukio Kikuchi, in the February 1969 issue of the Journal of the Meteorological Society of Japan (Ser. II, Vol. 47, No. 1) should be corrected as follows.

Page	Column	line	Wrong	Right
30	left	10 from the bottom	$+ \nabla(f \nabla_x = F)$	$+ \nabla(f \nabla_x) = F$
30	left	second from the bottom	$\nabla^2 \phi = \nabla \cdot (f \nabla \phi)$	$\nabla^2 \phi = \nabla \cdot (f \nabla \phi)$
30	right	6 from the top	$i \frac{\partial}{a \cos \phi \partial \lambda}$	$i \frac{\partial}{a \cos \phi \partial \lambda}$
31	left	third from the top	$\times$	X
32	right	6 from the bottom	T'_s	T'_s
32	right	first from the bottom	$a_1 T'_s$	$a_1 T'_s$
33	left	first from the top	$b_1 T'_s$	$b_1 T'_s$
33	right	19 from the bottom	$C = 16.667 \text{ ly day}^{-1} \text{K}^{-1}$	$c = 16.667 \text{ ly day}^{-1} \text{K}^{-1}$
33	right	10 from the bottom	$+(b^2 + c_2 - a_2) T'$	$+(b_2 + c_2 - a_2) T'$
33	right	9 from the bottom	T'_s	T'_s
33	right	7 from the bottom	T'_s	T'_s
33	right	5 from the bottom	T'_s	T'_s
34	left	second from the bottom	Eq. (37) Q	Eq. (37), Q
35	left	fourth from the top	$+(b_0 + c_0 + a_0)$	$+(b_0 + c_0 + a_0)$
35	right	15 from the top	$J(\phi_3 \nabla^2 \phi_3 + f)$	$J(\phi_3, \nabla^2 \phi_3 + f)$
36	left	first from the top	P_n^{-m}	P_n^{-m}
36	left	14 from the bottom	$\frac{d\Psi_\alpha}{dt} -$	$\frac{d\Psi_\alpha}{dt} =$
36	left	7 from the bottom	$-\frac{\rho_2}{R} \sigma D a X a$	$-\frac{\rho_2}{R} \sigma D a X a$
36	right	third from the top	$\int_{-\pi/2}^{\pi/2} p_\alpha \left(m_\beta p_\beta \frac{d\rho_\gamma}{d\phi} \right. \\ \left. - m_\gamma p_\gamma \frac{d\rho_\beta}{d\phi} \right) d\phi$	$\int_{-\pi/2}^{\pi/2} P_\alpha \left(m_\beta P_\beta \frac{dP_\gamma}{d\phi} \right. \\ \left. - m_\gamma P_\gamma \frac{dP_\beta}{d\phi} \right) d\phi$
36	right	fourth from the top	$\int_{-\pi/2}^{\pi/2} p_a p_b p_c \cos \phi d\phi$	$\int_{-\pi/2}^{\pi/2} P_a P_b P_c \cos \phi d\phi$
36	right	9 from the bottom	T_0°	T_0°
42	left	Fig. 17	Burdecki 1955)	Burdecki (1955)
42	right	13 from the bottom	$\frac{\partial \overline{T}_2}{\partial t}$	$\frac{\partial \overline{T}_2}{\partial t}$
42	right	13 from the bottom	$\frac{p_2}{R} \sigma \omega^2$	$\frac{p_2}{R} \sigma \omega_2$
44	left	13 from the bottom	$-\gamma_m \phi_3 \frac{\partial h}{\partial \lambda}$	$-\gamma_m \phi_3 \frac{\partial h}{\partial \lambda}$
45	left	23 from the bottom	Symbols,	Symbols.
46	left	Fig. 23	20 day	30-day
46	right	14 from the bottom	$\nabla^2 \phi_1$	$\nabla^2 \phi_1$
47	left	fourth from the bottom	$1/\Delta p \omega_2 \phi_2$ and $g/\Delta p \phi_2 \tau_2$	$1/\Delta p \omega_2 \phi_2$ and $g/\Delta p \phi_2 \tau_2$
49		Caption of Table 1	component	components
49		„	Oort 1964)	Oort (1964)
49	right	20 from the bottom	at wave number 5).	around this wave number).
49	right	8 from the bottom	figure the	figure, the
50	right	first from the top	From part (1)	From Part (I)
52	left	17 from the bottom	troposphere ocean continent.	troposphere-Ocean-Continent system.
53		和文アブストラクト	菊地	菊池
53		下から5行目	比較した結果	比較した。その結果
53		下から1行目	角運動輸送	角運動量輸送